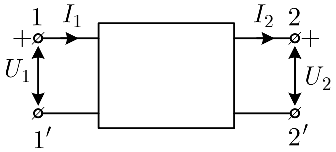
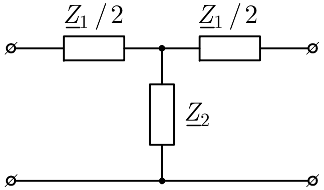
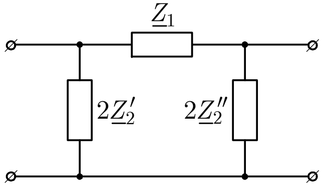
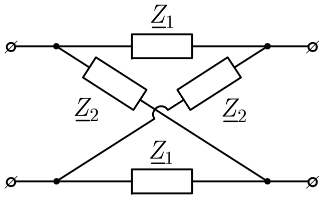
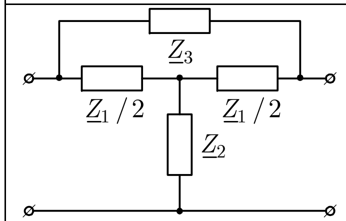
TEORIJA ELEKTRIČNIH KOLA

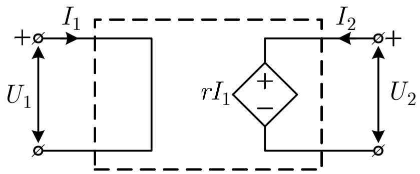
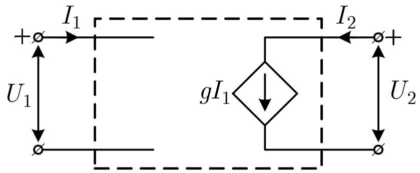
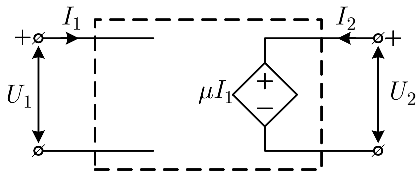
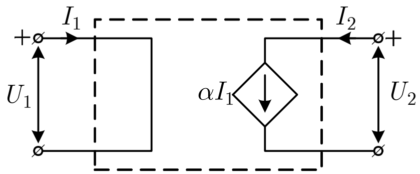
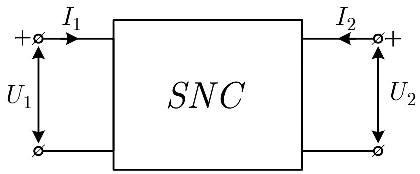
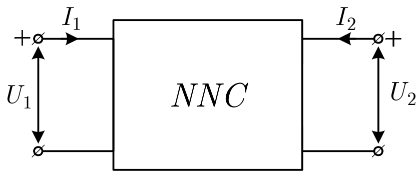
Tablica primarnih parametara mreže sa dva para krajeva

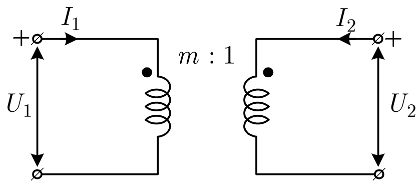
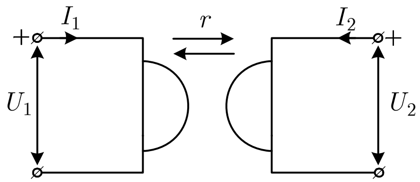
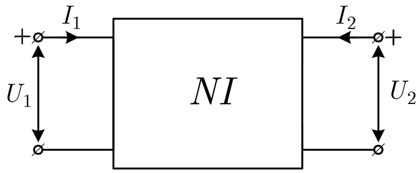
	z - parametri impedanse otvorene mreže	y - parametri admitanse kratko spojene mreže	a - parametri lančani parametri	b - parametri lančani parametri	g - parametri hibridni parametri	h - parametri hibridni parametri
$\begin{bmatrix} U_1 \\ U_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ -I_2 \end{bmatrix}$	$\begin{matrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{matrix}$	$\begin{matrix} \frac{y_{22}}{\lvert y \rvert} & -\frac{y_{12}}{\lvert y \rvert} \\ -\frac{y_{21}}{\lvert y \rvert} & \frac{y_{11}}{\lvert y \rvert} \end{matrix}$	$\begin{matrix} \frac{a_{11}}{a_{21}} & \frac{\lvert a \rvert}{a_{21}} \\ \frac{1}{a_{21}} & \frac{a_{22}}{a_{21}} \end{matrix}$	$\begin{matrix} -\frac{b_{22}}{b_{21}} & -\frac{1}{b_{21}} \\ -\frac{\lvert b \rvert}{b_{21}} & -\frac{b_{11}}{b_{21}} \end{matrix}$	$\begin{matrix} \frac{1}{g_{11}} & -\frac{g_{12}}{g_{11}} \\ \frac{g_{21}}{g_{11}} & \frac{\lvert g \rvert}{g_{11}} \end{matrix}$	$\begin{matrix} \frac{\lvert h \rvert}{h_{22}} & \frac{h_{12}}{h_{22}} \\ -\frac{h_{21}}{h_{22}} & \frac{1}{h_{22}} \end{matrix}$
$\begin{bmatrix} I_1 \\ -I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \end{bmatrix}$	$\begin{matrix} \frac{z_{22}}{\lvert z \rvert} & -\frac{z_{12}}{\lvert z \rvert} \\ -\frac{z_{21}}{\lvert z \rvert} & \frac{z_{11}}{\lvert z \rvert} \end{matrix}$	$\begin{matrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{matrix}$	$\begin{matrix} \frac{a_{22}}{a_{12}} & -\frac{\lvert a \rvert}{a_{12}} \\ -\frac{1}{a_{12}} & \frac{a_{11}}{a_{12}} \end{matrix}$	$\begin{matrix} -\frac{b_{11}}{b_{12}} & \frac{1}{b_{12}} \\ \frac{\lvert b \rvert}{b_{12}} & -\frac{b_{22}}{b_{12}} \end{matrix}$	$\begin{matrix} \frac{\lvert g \rvert}{g_{22}} & \frac{g_{12}}{g_{22}} \\ -\frac{g_{21}}{g_{22}} & \frac{1}{g_{22}} \end{matrix}$	$\begin{matrix} \frac{1}{h_{11}} & -\frac{h_{12}}{h_{11}} \\ \frac{h_{21}}{h_{11}} & \frac{\lvert h \rvert}{h_{11}} \end{matrix}$
$\begin{bmatrix} U_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} U_2 \\ I_2 \end{bmatrix}$	$\begin{matrix} \frac{z_{11}}{z_{21}} & \frac{\lvert z \rvert}{z_{21}} \\ \frac{1}{z_{21}} & \frac{z_{22}}{z_{21}} \end{matrix}$	$\begin{matrix} -\frac{y_{22}}{y_{21}} & -\frac{1}{y_{21}} \\ -\frac{\lvert y \rvert}{y_{21}} & -\frac{y_{11}}{y_{21}} \end{matrix}$	$\begin{matrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{matrix}$	$\begin{matrix} \frac{b_{22}}{\lvert b \rvert} & \frac{-b_{12}}{\lvert b \rvert} \\ \frac{-b_{21}}{\lvert b \rvert} & \frac{b_{11}}{\lvert b \rvert} \end{matrix}$	$\begin{matrix} \frac{1}{g_{21}} & \frac{g_{22}}{g_{21}} \\ \frac{g_{11}}{g_{21}} & \frac{\lvert g \rvert}{g_{21}} \end{matrix}$	$\begin{matrix} -\frac{\lvert h \rvert}{h_{21}} & -\frac{h_{11}}{h_{21}} \\ -\frac{h_{22}}{h_{21}} & -\frac{1}{h_{21}} \end{matrix}$
$\begin{bmatrix} U_2 \\ I_2 \end{bmatrix} = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \begin{bmatrix} U_1 \\ I_1 \end{bmatrix}$	$\begin{matrix} \frac{z_{22}}{z_{12}} & -\frac{\lvert z \rvert}{z_{12}} \\ -\frac{1}{\lvert z \rvert} & \frac{z_{11}}{z_{12}} \end{matrix}$	$\begin{matrix} -\frac{y_{11}}{y_{12}} & \frac{1}{y_{12}} \\ \frac{\lvert y \rvert}{y_{12}} & -\frac{y_{22}}{y_{12}} \end{matrix}$	$\begin{matrix} \frac{a_{22}}{\lvert a \rvert} & \frac{-a_{12}}{\lvert a \rvert} \\ \frac{-a_{21}}{\lvert a \rvert} & \frac{a_{11}}{\lvert a \rvert} \end{matrix}$	$\begin{matrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{matrix}$	$\begin{matrix} -\frac{\lvert g \rvert}{g_{12}} & \frac{g_{22}}{g_{12}} \\ \frac{g_{11}}{g_{12}} & -\frac{1}{g_{12}} \end{matrix}$	$\begin{matrix} \frac{1}{h_{12}} & -\frac{h_{11}}{h_{12}} \\ -\frac{h_{22}}{h_{12}} & \frac{\lvert h \rvert}{h_{12}} \end{matrix}$
$\begin{bmatrix} I_1 \\ U_2 \end{bmatrix} = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} \begin{bmatrix} U_1 \\ -I_2 \end{bmatrix}$	$\begin{matrix} \frac{1}{z_{11}} & -\frac{z_{12}}{z_{11}} \\ \frac{z_{21}}{z_{11}} & \frac{\lvert z \rvert}{z_{11}} \end{matrix}$	$\begin{matrix} \frac{\lvert y \rvert}{y_{22}} & \frac{y_{12}}{y_{22}} \\ -\frac{y_{21}}{y_{22}} & \frac{1}{y_{22}} \end{matrix}$	$\begin{matrix} \frac{a_{21}}{a_{11}} & -\frac{\lvert a \rvert}{a_{11}} \\ \frac{1}{a_{11}} & \frac{a_{12}}{a_{11}} \end{matrix}$	$\begin{matrix} -\frac{b_{21}}{b_{22}} & -\frac{1}{b_{22}} \\ \frac{\lvert b \rvert}{b_{22}} & -\frac{b_{12}}{b_{22}} \end{matrix}$	$\begin{matrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{matrix}$	$\begin{matrix} \frac{h_{22}}{\lvert h \rvert} & -\frac{h_{12}}{\lvert h \rvert} \\ -\frac{h_{21}}{\lvert h \rvert} & \frac{h_{11}}{\lvert h \rvert} \end{matrix}$
$\begin{bmatrix} U_1 \\ -I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ U_2 \end{bmatrix}$	$\begin{matrix} \frac{\lvert z \rvert}{z_{22}} & \frac{z_{12}}{z_{22}} \\ -\frac{z_{21}}{z_{22}} & \frac{1}{z_{22}} \end{matrix}$	$\begin{matrix} \frac{1}{y_{11}} & -\frac{y_{12}}{y_{11}} \\ \frac{y_{21}}{y_{11}} & \frac{\lvert y \rvert}{y_{11}} \end{matrix}$	$\begin{matrix} \frac{a_{12}}{a_{22}} & \frac{\lvert a \rvert}{a_{22}} \\ -\frac{1}{a_{22}} & \frac{a_{21}}{a_{22}} \end{matrix}$	$\begin{matrix} -\frac{b_{12}}{b_{11}} & \frac{1}{b_{11}} \\ -\frac{\lvert b \rvert}{b_{11}} & -\frac{b_{21}}{b_{11}} \end{matrix}$	$\begin{matrix} \frac{g_{22}}{\lvert g \rvert} & -\frac{g_{12}}{\lvert g \rvert} \\ -\frac{g_{21}}{\lvert g \rvert} & \frac{g_{11}}{\lvert g \rvert} \end{matrix}$	$\begin{matrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{matrix}$

Tablica primarnih i sekundarnih parametara simetričnih mreža sa dva para krajeva

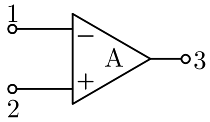
	PRIMARNI PARAMETRI			SEKUNDARNI PARAMETRI	IMPEDANSE
	$\underset{\sim}{a}$	$\underset{\sim}{y}$	$\underset{\sim}{z}$		
	$\underline{a}_{11} = 1 + \frac{\underline{Z}_1}{2\underline{Z}_2}$ $\underline{a}_{12} = \underline{Z}_1\left(1 + \frac{\underline{Z}_1}{4\underline{Z}_2}\right)$ $\underline{a}_{21} = \frac{1}{\underline{Z}_2}$ $\underline{a}_{22} = 1 + \frac{\underline{Z}_1}{2\underline{Z}_2}$	$\underline{y}_{11} = \frac{2\underline{Z}_1 + 4\underline{Z}_2}{\underline{Z}_1(\underline{Z}_1 + 4\underline{Z}_2)}$ $\underline{y}_{12} = -\frac{4\underline{Z}_2}{\underline{Z}_1(\underline{Z}_1 + 4\underline{Z}_2)}$ $\underline{y}_{21} = \frac{4\underline{Z}_2}{\underline{Z}_1(\underline{Z}_1 + 4\underline{Z}_2)}$ $\underline{y}_{22} = -\frac{2\underline{Z}_1 + 4\underline{Z}_2}{\underline{Z}_1(\underline{Z}_1 + 4\underline{Z}_2)}$	$\underline{z}_{11} = \frac{\underline{Z}_1}{2} + \underline{Z}_2$ $\underline{z}_{12} = -\underline{Z}_2$ $\underline{z}_{21} = \underline{Z}_2$ $\underline{z}_{22} = -\left(\frac{\underline{Z}_1}{2} + \underline{Z}_2\right)$	$\underline{Z}_C = \underline{Z}_C^T = \sqrt{\underline{Z}_1\underline{Z}_2\left(1 + \frac{\underline{Z}_1}{4\underline{Z}_2}\right)}$ $\underline{\Gamma}_C = 2\ln\left(\sqrt{1 + \frac{\underline{Z}_1}{4\underline{Z}_2}} + \sqrt{\frac{\underline{Z}_1}{4\underline{Z}_2}}\right)$ $\mathrm{ch}\underline{\Gamma} = 1 + \frac{\underline{Z}_1}{2\underline{Z}_2}$ $\mathrm{sh}\frac{\underline{\Gamma}}{2} = \sqrt{\frac{\underline{Z}_1}{4\underline{Z}_2}}$	$\frac{\underline{Z}_1}{2} = \frac{\underline{a}_{11} - 1}{\underline{a}_{21}} =$ $= \underline{Z}_T\frac{\mathrm{ch}\underline{\Gamma} - 1}{\mathrm{sh}\underline{\Gamma}} = \underline{Z}_T\mathrm{th}\frac{\underline{\Gamma}}{2}$ $\underline{Z}_2 = \frac{1}{\underline{a}_{21}} = \frac{\underline{Z}_T}{\mathrm{sh}\underline{\Gamma}}$
	$\underline{a}_{11} = 1 + \frac{\underline{Z}_1}{2\underline{Z}_2}$ $\underline{a}_{12} = \underline{Z}_1$ $\underline{a}_{21} = \frac{1}{\underline{Z}_2}\left(1 + \frac{\underline{Z}_1}{4\underline{Z}_2}\right)$ $\underline{a}_{22} = 1 + \frac{\underline{Z}_1}{2\underline{Z}_2}$	$\underline{y}_{11} = \frac{1}{\underline{Z}_1} + \frac{1}{2\underline{Z}_2}$ $\underline{y}_{12} = -\frac{1}{\underline{Z}_1}$ $\underline{y}_{21} = \frac{1}{\underline{Z}_1}$ $\underline{y}_{22} = -\left(\frac{1}{\underline{Z}_1} + \frac{1}{2\underline{Z}_2}\right)$	$\underline{z}_{11} = \frac{(2\underline{Z}_1 + 4\underline{Z}_2)\underline{Z}_2}{\underline{Z}_1 + 4\underline{Z}_2}$ $\underline{z}_{12} = -\frac{4\underline{Z}_2^2}{\underline{Z}_1 + 4\underline{Z}_2}$ $\underline{z}_{21} = \frac{4\underline{Z}_2^2}{\underline{Z}_1 + 4\underline{Z}_2}$ $\underline{z}_{22} = -\frac{(2\underline{Z}_1 + 4\underline{Z}_2)\underline{Z}_2}{\underline{Z}_1 + 4\underline{Z}_2}$	$\underline{Z}_C = \underline{Z}_C^\Pi = \sqrt{\underline{Z}_1\underline{Z}_2\left(\frac{1}{1 + \frac{\underline{Z}_1}{4\underline{Z}_2}}\right)}$ $\underline{\Gamma}_C = 2\ln\left(\sqrt{1 + \frac{\underline{Z}_1}{4\underline{Z}_2}} + \sqrt{\frac{\underline{Z}_1}{4\underline{Z}_2}}\right)$ $\mathrm{ch}\underline{\Gamma} = 1 + \frac{\underline{Z}_1}{2\underline{Z}_2}$ $\mathrm{sh}\frac{\underline{\Gamma}}{2} = \sqrt{\frac{\underline{Z}_1}{4\underline{Z}_2}}$	$\underline{Z}_1 = \underline{a}_{12} = \underline{Z}_\Pi\mathrm{sh}\underline{\Gamma}$ $2\underline{Z}_2 = \frac{\underline{a}_{12}}{\underline{a}_{11} - 1} =$ $= \underline{Z}_\Pi\frac{\mathrm{sh}\underline{\Gamma}}{\mathrm{ch}\underline{\Gamma} - 1} =$ $= \underline{Z}_\Pi\mathrm{cth}\frac{\underline{\Gamma}}{2}$
	$\underline{a}_{11} = \frac{\underline{Z}_2 + \underline{Z}_1}{\underline{Z}_2 - \underline{Z}_1}$ $\underline{a}_{12} = \frac{2\underline{Z}_2\underline{Z}_1}{\underline{Z}_2 - \underline{Z}_1}$ $\underline{a}_{21} = \frac{2}{\underline{Z}_2 - \underline{Z}_1}$ $\underline{a}_{22} = \frac{\underline{Z}_2 + \underline{Z}_1}{\underline{Z}_2 - \underline{Z}_1}$	$\underline{y}_{11} = \frac{\underline{Z}_2 + \underline{Z}_1}{2\underline{Z}_2\underline{Z}_1}$ $\underline{y}_{12} = -\left(\frac{\underline{Z}_2 - \underline{Z}_1}{2\underline{Z}_2\underline{Z}_1}\right)$ $\underline{y}_{21} = \frac{\underline{Z}_2 - \underline{Z}_1}{2\underline{Z}_2\underline{Z}_1}$ $\underline{y}_{22} = -\left(\frac{\underline{Z}_2 + \underline{Z}_1}{2\underline{Z}_2\underline{Z}_1}\right)$	$\underline{z}_{11} = \frac{1}{2}(\underline{Z}_1 + 4\underline{Z}_2)$ $\underline{z}_{12} = -\frac{1}{2}(\underline{Z}_1 - 4\underline{Z}_2)$ $\underline{z}_{21} = \frac{1}{2}(\underline{Z}_1 - 4\underline{Z}_2)$ $\underline{z}_{22} = -\frac{1}{2}(\underline{Z}_1 + 4\underline{Z}_2)$	$\underline{Z}_C = \underline{Z}_T = \sqrt{\underline{Z}_1\underline{Z}_2}$ $\underline{\Gamma}_C = 2\ln\left(\sqrt{1 + \frac{\underline{Z}_1}{4\underline{Z}_2}} + \sqrt{\frac{\underline{Z}_1}{4\underline{Z}_2}}\right)$ $\mathrm{ch}\underline{\Gamma} = \frac{\underline{Z}_2 + \underline{Z}_1}{\underline{Z}_2 - \underline{Z}_1}$ $\mathrm{th}\frac{\underline{\Gamma}}{2} = \sqrt{\frac{\underline{Z}_1}{\underline{Z}_2}}$	$\underline{Z}_1 = \frac{\underline{a}_{11} - 1}{\underline{a}_{21}} = \underline{Z}_C\mathrm{th}\frac{\underline{\Gamma}}{2}$ $\underline{Z}_2 = \frac{\underline{a}_{11} + 1}{\underline{a}_{21}} = \underline{Z}_C\mathrm{cth}\frac{\underline{\Gamma}}{2}$ ili $\underline{Z}_1 = \frac{\underline{a}_{11} + 1}{\underline{a}_{21}} = \underline{Z}_C\mathrm{cth}\frac{\underline{\Gamma}}{2}$ $\underline{Z}_2 = \frac{\underline{a}_{11} - 1}{\underline{a}_{21}} = \underline{Z}_C\mathrm{th}\frac{\underline{\Gamma}}{2}$
				$\underline{Z}_C = \sqrt{\frac{\underline{Z}_1\underline{Z}_3(\underline{Z}_1 + 4\underline{Z}_2)}{4(\underline{Z}_1 + \underline{Z}_3)}}$ $\mathrm{ch}\underline{\Gamma} = 1 + \frac{\underline{Z}_1}{2\underline{Z}_2}\left(\frac{\underline{Z}_3}{\underline{Z}_2 + \underline{Z}_3 + \frac{\underline{Z}_1^2}{4\underline{Z}_2}}\right)$	

Parametri nekih idealnih aktivnih mreža sa dva para krajeva

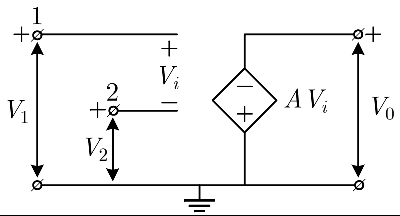
	Naziv	Simbol	z	y	g	h	a
KONTROLISANI IZVORI	strujom kontrolisani naponski izvor **SKNI**		$\begin{matrix} 0 & 0 \\ r & 0 \end{matrix}$	-------	-------	-------	$\begin{matrix} 0 & 0 \\ \frac{1}{r} & 0 \end{matrix}$
	naponom kontrolisani strujni izvor **NKSI**		-------	$\begin{matrix} 0 & 0 \\ g & 0 \end{matrix}$	-------	-------	$\begin{matrix} 0 & -\frac{1}{g} \\ 0 & 0 \end{matrix}$
	naponom kontrolisani naponski izvor **NKNI**		-------	-------	$\begin{matrix} 0 & 0 \\ \mu & 0 \end{matrix}$	-------	$\begin{matrix} \frac{1}{\mu} & 0 \\ 0 & 0 \end{matrix}$
	strujom kontrolisani strujni izvor **SKSI**		-------	-------	-------	$\begin{matrix} 0 & 0 \\ \alpha & 0 \end{matrix}$	$\begin{matrix} 0 & 0 \\ 0 & -\frac{1}{\alpha} \end{matrix}$
KONVERTORI	strujni negativni konvertor **SNC**		-------	-------	$\begin{matrix} 0 & \frac{1}{k} \\ \frac{1}{k} & 0 \end{matrix}$	$\begin{matrix} 0 & k \\ k & 0 \end{matrix}$	$\begin{matrix} k & 0 \\ 0 & -\frac{1}{k} \end{matrix}$
	Naponski negativni konvertor **NNC**		-------	-------	$\begin{matrix} 0 & -\frac{1}{k} \\ -\frac{1}{k} & 0 \end{matrix}$	$\begin{matrix} 0 & -k \\ -k & 0 \end{matrix}$	$\begin{matrix} -k & 0 \\ 0 & \frac{1}{k} \end{matrix}$

	pozitivni konvertor **IDEALNI TRANSFORMATOR**		-------	-------	$\begin{matrix} 0 & -\frac{1}{m} \\ \frac{1}{m} & 0 \end{matrix}$	$\begin{matrix} 0 & m \\ -m & 0 \end{matrix}$	$\begin{matrix} m & 0 \\ 0 & -\frac{1}{m} \end{matrix}$
INVERTORI	pozitivni invertor **ŽIRATOR**		$\begin{matrix} 0 & \mp r \\ \pm r & 0 \end{matrix}$	$\begin{matrix} 0 & \frac{1}{\pm r} \\ \frac{1}{\mp r} & 0 \end{matrix}$	-------	-------	$\begin{matrix} 0 & \pm r \\ \frac{1}{\pm r} & 0 \end{matrix}$
	negativni invertor		$\begin{matrix} 0 & \mp r \\ \mp r & 0 \end{matrix}$	$\begin{matrix} 0 & \frac{1}{\mp r} \\ \frac{1}{\mp r} & 0 \end{matrix}$	-------	-------	$\begin{matrix} 0 & \pm r \\ \frac{1}{\mp r} & 0 \end{matrix}$

SIMBOL



EKVIVALENTNA ŠEMA



$$V_o = A(V_2 - V_1) = -A\,V_i$$

TABLICA ZAVISNOSTI IZMEĐU PRIMARNIH PARAMETARA:

$\underline{z}_{12} = \underline{z}_{21}$; $\underline{y}_{12} = \underline{y}_{21}$; $|\underline{a}| = \underline{a}_{11}\underline{a}_{22} - \underline{a}_{12}\underline{a}_{21} = 1$; $|\underline{b}| = \underline{b}_{11}\underline{b}_{22} - \underline{b}_{12}\underline{b}_{21} = 1$; $\underline{g}_{12} = -\underline{g}_{21}$; $\underline{h}_{12} = -\underline{h}_{21}$

TABLICA ZAVISNOSTI IZMEĐU PRIMARNIH PARAMETARA SIMETRIČNIH MREŽ:

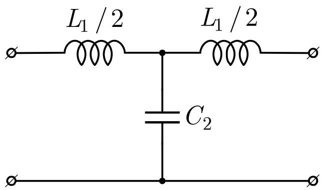
$\underline{z}_{11} = \underline{z}_{22}$; $\underline{y}_{11} = \underline{y}_{22}$; $\underline{a}_{11} = \underline{a}_{22}$; $\underline{b}_{11} = \underline{b}_{22}$; $|\underline{g}| = \underline{g}_{11}\underline{g}_{22} - \underline{g}_{12}\underline{g}_{21} = 1$; $|\underline{h}| = \underline{h}_{11}\underline{h}_{22} - \underline{h}_{12}\underline{h}_{21} = 1$;

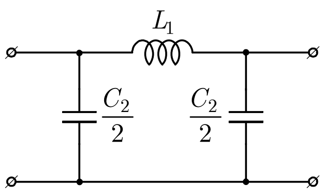
$|\underline{y}| = \underline{y}_{11}\underline{y}_{22} - \underline{y}_{12}\underline{y}_{21}$; $|\underline{z}| = \underline{z}_{11}\underline{z}_{22} - \underline{z}_{12}\underline{z}_{21}$; $|\underline{g}| = \underline{g}_{11}\underline{g}_{22} - \underline{g}_{12}\underline{g}_{21}$; $|\underline{h}| = \underline{h}_{11}\underline{h}_{22} - \underline{h}_{12}\underline{h}_{21}$.

Filtri

Filtri sa konstantnim proizvodom redne i otočne impedanse $\underline{Z}_1\underline{Z}_2 = \text{const}$ nazivaju se **K**-filtri.

K-filtri niskih učestanosti:





$$\underline{Z}_1 = j\omega L_1 = \omega L_1 e^{j\frac{\pi}{2}}$$

$$\underline{Z}_2 = \frac{1}{j\omega C_2} = \frac{1}{\omega C_2} e^{-j\frac{\pi}{2}}$$

$$\underline{Z}_1\underline{Z}_2 = \frac{L_1}{C_2} = R^2 = \text{const}$$

$$\underline{Z}_C^{\mathrm{T}}\underline{Z}_C^{\Pi} = \underline{Z}_1\underline{Z}_2 = R^2$$

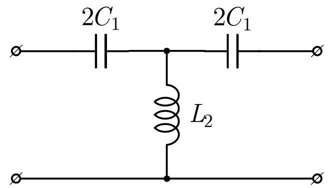
$$\frac{\underline{Z}_1}{4\underline{Z}_2} = \frac{1}{4}L_1C_2\omega^2 e^{j\pi} = Ne^{j\pi}$$

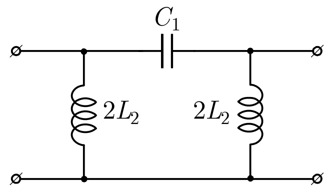
$$\underline{\Gamma}_C = 2\ln\left(\sqrt{1+Ne^{j\pi}} + \sqrt{Ne^{j\pi}}\right)$$

granica propusnog i nepropusnog opsega

$$\mathrm{N{=}1} \Rightarrow \omega{=}\omega_c = \frac{2}{\sqrt{L_1C_2}}$$

K-filtri visokih učestanosti





$$\underline{Z}_1 = \frac{1}{j\omega C_1} = \frac{1}{\omega C_1} e^{-j\frac{\pi}{2}}$$

$$\underline{Z}_2 = j\omega L_2 = \omega L_2 e^{j\frac{\pi}{2}}$$

$$\underline{Z}_1\underline{Z}_2 = \frac{L_2}{C_1} = R^2 = \text{const}$$

$$\underline{Z}_C^{\mathrm{T}}\underline{Z}_C^{\Pi} = \underline{Z}_1\underline{Z}_2 = R^2$$

$$\frac{\underline{Z}_1}{4\underline{Z}_2} = \frac{1}{4}\frac{1}{L_2C_1\omega^2} e^{-j\pi} = Ne^{-j\pi}$$

$$\underline{\Gamma}_C = 2\ln\left(\sqrt{1+Ne^{-j\pi}} + \sqrt{Ne^{-j\pi}}\right)$$

granica propusnog i nepropusnog opsega

$$\mathrm{N{=}1} \Rightarrow \omega{=}\omega_c = \frac{1}{2\sqrt{L_2C_1}}$$

Campbell-ova jednačina: $\mathrm{ch}\underline{\Gamma}_C = 1 + \frac{\underline{Z}_1}{4\underline{Z}_2}$; $\mathrm{ch}\underline{\Gamma}_C = \mathrm{ch}(A_C + jB_C) = \mathrm{ch}A_C\cos B_C + j\mathrm{sh}A_C\sin B_C$

Trigonometrijske i hiperbolne funkcije

$e^{jx} = \cos x + j\sin x \quad e^{-jx} = \cos x - j\sin x$

$$\cos x = \frac{e^{jx}+e^{-jx}}{2} \qquad \sin x = \frac{e^{jx}-e^{-jx}}{2j}$$

$$\cos \underline{z} = \frac{e^{j\underline{z}}+e^{-j\underline{z}}}{2} \qquad \sin \underline{z} = \frac{e^{j\underline{z}}-e^{-j\underline{z}}}{2j}$$

Veza između trigonometrijskih i hiperbolnih funkcija

$\mathrm{sh}\,\underline{z} = -j\sin \underline{z} \qquad \mathrm{sh}\,j\underline{z} = j\sin\underline{z}$

$\mathrm{ch}\,\underline{z} = \cos\underline{z} \qquad \mathrm{ch}\,j\underline{z} = \cos\underline{z}$

$\mathrm{th}\,j\underline{z} = -j\tan\underline{z} \qquad \mathrm{th}\,j\underline{z} = j\tan\underline{z}$

$\mathrm{cth}\,\underline{z} = j\,\mathrm{cth}\,j\underline{z} \qquad \mathrm{cth}\,j\underline{z} = -j\,\mathrm{cth}\,\underline{z}$

Hiperbolične funkcije

$$\mathrm{ch}\underline{z} = \frac{e^{\underline{z}}+e^{-\underline{z}}}{2} \qquad \mathrm{sh}\underline{z} = \frac{e^{\underline{z}}-e^{-\underline{z}}}{2}$$

$$\mathrm{th}\underline{z} = \frac{\mathrm{sh}\underline{z}}{\mathrm{ch}\underline{z}} \quad \text{za } \underline{z} \neq (2k+1)\frac{j\pi}{2} \quad (k = 0, \pm1, \pm2, \ldots)$$

$$\mathrm{cth}\underline{z} = \frac{\mathrm{ch}\underline{z}}{\mathrm{sh}\underline{z}} = \frac{1}{\mathrm{th}\underline{z}} \quad \text{za } \underline{z} \neq jk\pi \quad (k = 0, \pm1, \pm2, \ldots)$$

Izvodi trigonometrijskih i hiperbolnih funkcija

$(\sin\underline{z})' = \cos\underline{z} \qquad (\cos\underline{z})' = -\sin\underline{z}$

$$(\tan\underline{z})' = \frac{1}{\cos^2\underline{z}} \qquad (\cot\underline{z})' = -\frac{1}{\sin^2\underline{z}}$$

$(\mathrm{sh}\,\underline{z})' = \mathrm{ch}\,\underline{z} \qquad (\mathrm{ch}\,\underline{z})' = \mathrm{sh}\,\underline{z}$

$$(\mathrm{th}\,\underline{z})' = \frac{1}{\mathrm{ch}^2\underline{z}} \qquad (\mathrm{cth}\,\underline{z})' = -\frac{1}{\mathrm{sh}^2\underline{z}}$$

$$\arcsin\underline{z} = \frac{1}{j}\log\left(j\underline{z} \pm \sqrt{1-\underline{z}^2}\right)$$

$$\arccos\underline{z} = \frac{1}{j}\log\left(\underline{z} \pm \sqrt{1-\underline{z}^2}\right)$$

$$\arctan\underline{z} = \frac{1}{j2}\log\frac{j-\underline{z}}{j+\underline{z}}$$

$$\mathrm{arccot}\,\underline{z} = \frac{1}{j2}\log\frac{\underline{z}+j}{\underline{z}-j}$$

$$\mathrm{Arsh}\,\underline{z} = \log\left(\underline{z} \pm \sqrt{\underline{z}^2+1}\right)$$

$$\mathrm{Arch}\,\underline{z} = \log\left(\underline{z} \pm \sqrt{\underline{z}^2-1}\right)$$

$$\mathrm{Arth}\,\underline{z} = \frac{1}{2}\log\frac{1+\underline{z}}{1-\underline{z}}$$

$$\mathrm{Arcth}\,\underline{z} = \frac{1}{2}\log\frac{\underline{z}+1}{\underline{z}-1}$$

Fourier-ov red za periodične funkcije

Složenoperiodična funkcija $f(t)=f(t+T)$ može se predstaviti Fourier-ovim redom koji predstavlja sumu prosotperiodičnih funkcija u obliku: $f(t)=\frac{a_0}{2}+\sum_{n=1}^{\infty}(a_n\cos n\omega_0 t+b_n\sin n\omega_0 t)$:

gdje je:

$$a_0=\frac{2}{T}\int_{t_0}^{t_0+T}f(t)\,dt;\quad a_n=\frac{2}{T}\int_{t_0}^{t_0+T}f(t)\cos n\omega_0 t\,dt;\quad n=0,1,2,...$$

$$b_n=\frac{2}{T}\int_{t_0}^{t_0+T}f(t)\sin n\omega_0 t\,dt;\quad n=1,2,3,...$$

1. Ako je funkcija $f(t)$ parna funkcija tj. $f(-t)=f(t)$ Fourier-ov red će sadržati konstantni član i kosinusne članove:

$$a_n=\frac{4}{T}\int_0^{T/2}f(t)\cos n\omega_0 t,\quad n=0,1,2,...$$

$$b_n=0,\quad n=1,2,3,...$$

2. Ako je funkcija $f(t)$ neparna funkcija tj. $f(-t)=-f(t)$ Fourier-ov red će sadržati samo sinusne članove:

$$b_n=\frac{4}{T}\int_0^{T/2}f(t)\sin n\omega_0 t,\quad n=1,2,3,...$$

$$a_n=0,\quad n=0,1,2,...$$

3. Ako je negativni talas vremenske funkcije ogledalska slika pozitivnog talasa tj. ako je $f(t+\frac{T}{2})=-f(t)$ onda:

$$a_n=0,\quad \text{za } n\text{-parno}$$

$$a_n=\frac{4}{T}\int_0^{T/2}f(t)\cos n\omega_0 t,\quad n\text{-neparno}$$

$$b_n=0,\quad n\text{-parno}$$

$$b_n=\frac{4}{T}\int_0^{T/2}f(t)\sin n\omega_0 t,\quad n\text{-neparno}$$

4. Ako funkcija $f(t)$ ispunjava uslov iz trećeg slučaja i uz to je simetrična u odnosu na koordinatni početak, tj. neparna, njen Fourier-ov red će imati samo neparne sinusne članove: $b_{2n+1}=\frac{8}{T}\int_0^{T/4}f(t)\sin(2n+1)\omega_0 t,\quad n=1,2,3,...$

Drugi oblici trigonometrijskog Fourier-ovog reda su: $f(t)=\frac{a_0}{2}+\sum_{n=1}^{\infty}A_n\cos(n\omega_0 t+\theta_n)$; $a_n\cos n\omega_0 t+b_n\sin n\omega_0 t=A_n\cos(n\omega_0 t+\theta_n)$; $A_n=\sqrt{a_n^2+b_n^2}$; $\theta_n=-\arctan\frac{b_n}{a_n}$;

ili $f(t)=\frac{a_0}{2}+\sum_{n=1}^{\infty}A_n\sin(n\omega_0 t+\theta_n')$; $\theta_n'=\theta_n+\frac{\pi}{2}$

Kompleksni oblik Fourier-ovog reda.

Polazeći od relacija: $\cos n\omega_0 t=\frac{1}{2}(e^{jn\omega_0 t}+e^{-jn\omega_0 t})$ i $\sin n\omega_0 t=\frac{1}{2j}(e^{jn\omega_0 t}-e^{-jn\omega_0 t})$ dobijamo:

$f(t)=\frac{a_0}{2}+\sum_{n=1}^{\infty}\left[\left(\frac{a_n-jb_n}{2}\right)e^{jn\omega_0 t}+\left(\frac{a_n+jb_n}{2}\right)e^{-jn\omega_0 t}\right]$. Uvodeći novi koeficijent $\underline{c}_n=\frac{a_n-jb_n}{2}$ dobijamo

$$\underline{c}_n=\frac{1}{T}\int_{-T/2}^{T/2}f(t)[\cos n\omega_0 t-j\sin n\omega_0 t]dt=\frac{1}{T}\int_{-T/2}^{T/2}f(t)e^{-jn\omega_0 t}dt;\quad \underline{c}_n^*=\frac{1}{T}\int_{-T/2}^{T/2}f(t)[\cos n\omega_0 t+j\sin n\omega_0 t]dt$$

Ako n zamijenimo sa $-n$ dobijamo $\underline{c}_{-n}=\frac{a_n+jb_n}{2}=\underline{c}_n^*$; $\frac{a_0}{2}=\frac{1}{T}\int_{-T/2}^{T/2}f(t)dt=c_0$.

Sada možemo pisati: $f(t) = c_0 + \sum_{n=1}^{\infty} \underline{c}_n e^{jn\omega_0 t} + \sum_{n=1}^{\infty} \underline{c}_{-n} e^{-jn\omega_0 t}$. Pošto je $c_0 = \underline{c}_n \Big|_{n=0}$ dobijamo:

$f(t) = \sum_{n=0}^{\infty} \underline{c}_n e^{jn\omega_0 t} + \sum_{n=-1}^{-\infty} \underline{c}_n e^{jn\omega_0 t}$ pa konačno dobijamo kompleksni oblik `Fourier`-ovog reda:

$$\boxed{f(t) = \sum_{n=-\infty}^{\infty} \underline{c}_n e^{jn\omega_0 t}; \quad \underline{c}_n = \frac{1}{T} \int_{-T/2}^{T/2} f(t) e^{-jn\omega_0 t} dt}$$

$\frac{1}{T} \int_{-T/2}^{T/2} f^2(t) dt = \left(\frac{a_0}{2}\right)^2 + \sum_{n=1}^{\infty} \frac{A_n^2}{2} = \left(\frac{a_0}{2}\right)^2 + \sum_{n=1}^{\infty} \frac{a_n^2 + b_n^2}{2}$ **-PARSERVALOVA JEDNAČINA**

Snage u kolima sa složenoperiodičnim eksitacijama

Aktivna snaga: $P = \sum_k P_k = \sum_k U^{(k)} I^{(k)} \cos \varphi^{(k)}$

Reaktivna snaga: $Q = \sum_k Q_k = \sum_k U^{(k)} I^{(k)} \sin \varphi^{(k)}$

Prividna snaga: $S = UI; \quad S^2 = P^2 + Q^2 + D^2$

Snaga izobličenja: $D = \sqrt{\sum_{\substack{k=0;l=0 \\ k \neq l}}^{\infty} \left[\left(U^{(k)}\right)^2 \left(I^{(l)}\right)^2 - 2U^{(k)}U^{(l)}I^{(k)}I^{(l)} \cos\left(\varphi^{(k)} - \varphi^{(l)}\right) + \left(U^{(l)}\right)^2 \left(I^{(k)}\right)^2 \right]}$

Fourier-ova transformacija

Aperiodične funkcije koje zadovoljavaju uslov: $\int_{-\infty}^{+\infty} |f(t)| dt < \infty$ imaju Fourier-ovu transformaciju:

$F(j\omega) = \int_{-\infty}^{+\infty} f(t) e^{-j\omega t} dt$.

Spektralna funkcija $F(j\omega)$ ima: moduo - $|F(j\omega)| = F(\omega); \quad F(\omega) = F(-\omega)$ (amplitudski spektar)

argument - $\arg F(j\omega) = \Psi(\omega); \; \Psi(\omega) = -\Psi(\omega)$ (fazni spektar)

Inverzna Fourier-ova transformacija:

$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} F(j\omega) e^{j\omega t} d\omega$$

Osnovne osobine Fourier-ove transformacije

Osobina linearnosti	$F\{c_1 f_1(t) + c_2 f_2(t)\} = c_1 F_1(j\omega) + c_2 F_2(j\omega)$
Teorema sklairanja	$F\left\{f\left(\frac{t}{a}\right)\right\} = \lvert a \rvert F(j\omega a) \quad a \equiv \text{const}$
Teorema kašnjenja	$F\{f(t-\tau)\} = e^{-j\omega\tau} F(j\omega)$ $F\left\{e^{j\omega_0 t} f(t)\right\} = F(j\omega - j\omega_0)$
Teorema simetričnosti	Ako imamo $F(jt)$ tada je Fourier-ova transformacija $2\pi f(-\omega)$. ako zamijenimo mjesta promjenljivim ω i t tada dobijamo teoremu simetričnosti.
Promjena znaka argumenta	$F\{f(-t)\} = F(-j\omega)$
Teorema o diferenciranju u vremenskom domenu	$F\left\{\frac{d^n}{dt_n}[f(t)]\right\} = (j\omega)^n F(j\omega); \quad n = 0,1,2,...$

Teorema o diferenciranju u kompleksnom domenu	$F\{t^n f(t)\} = (-1)^n \dfrac{d^n}{d(j\omega)^n} F(j\omega)$
Konvolucija dvije funkcije u vremenskom domenu	$F\{f_1(t) * f_2(t)\} = F_1(j\omega)F_2(j\omega)$
Konvolucija dvije funkcije u frekvencijskom domenu	$F\{f_1(t)f_2(f)\} = \dfrac{1}{2\pi} F_1(j\omega) * F_2(j\omega)$
Osobina mudulacije	$F\{f(t)\cos\omega_0 t\} = \dfrac{1}{2}[F(j\omega + j\omega_0) + F(j\omega - j\omega_0)]$ $F\{f(t)\sin\omega_0 t\} = \dfrac{1}{j2}[F(j\omega - j\omega_0) - F(j\omega + j\omega_0)]$
Paservalova teorema u Fourier-ovoj transformaciji	$\int_{-\infty}^{+\infty} f_1(t)f_2^*(t)dt = \dfrac{1}{2\pi}\int_{-\infty}^{+\infty} F_1(j\omega)F_2^*(j\omega)d\omega$ Ako je $f_1(t) = f_2(t) = f(t)$ dobijamo teoremu Releja: $\int_{-\infty}^{+\infty} f^2(t)dt = \dfrac{1}{2\pi}\int_{-\infty}^{+\infty} \lvert F_1(j\omega)\rvert^2 d\omega$

Parovi Fourier-ove transformacije

	$F(j\omega)$	$f(t)$
1.	1	$\delta(t)$ - impulsna funkcija
2.	$2\pi\delta(\omega)$	1
3.	$\pi\delta(\omega) + \dfrac{1}{j\omega}$	$h(t)$ - jedinična funkcija
4.	$\dfrac{2}{j\omega}$	$\operatorname{sgn} t$
5.	$-j\pi[\delta(\omega - \omega_0) - \delta(\omega + \omega_0)]$	$\sin\omega_0 t$
6.	$\pi[\delta(\omega - \omega_0) + \delta(\omega + \omega_0)]$	$\cos\omega_0 t$
7.	$\dfrac{\pi}{2}[\delta(\omega + \omega_0) + \delta(\omega - \omega_0)] + \dfrac{j\omega}{\omega_0^2 - \omega^2}$	$[\cos\omega_0 t]h(t)$
8.	$j\dfrac{\pi}{2}[\delta(\omega + \omega_0) - \delta(\omega - \omega_0)] + \dfrac{\omega_0}{\omega_0^2 - \omega^2}$	$[\sin\omega_0 t]h(t)$
9.	$2\pi e^{a\omega}h(-\omega);\quad a > 0$	$\dfrac{1}{a + jt}$
10.	$2\pi A \quad \lvert\omega\rvert < \dfrac{a}{2}$ $0 \quad \lvert\omega\rvert > \dfrac{a}{2}$	$\dfrac{2A}{t}\sin\dfrac{at}{2}$
11.	$\dfrac{2a}{\omega^2 + a^2}$	$e^{-a\lvert t\rvert};\quad a > 0$
12.	$\pi e^{-\lvert\omega\rvert}$	$\dfrac{1}{t^2 + 1}$
13.	$2\pi e^{-a\lvert\omega\rvert}$	$\dfrac{1}{a^2 + t^2}$
14.	$\dfrac{1}{a + j\omega}$	$e^{-at}h(t);\quad a > 0$
15.	$e^{-ja\omega}$	$\delta(t - a)$
16.	$\dfrac{n!}{(a + j\omega)^{n+1}}$	$t^n e^{-at}h(t)$
17.	$2\pi\delta(\omega - a)$	e^{-jat}
18.	$\dfrac{2\sin a\omega}{\omega}$	$p_a(t)$

19.	$p_a(\omega)$	$\frac{\sin at}{\pi t}$
20.	$2\pi e^{a\omega}h(-\omega); \quad a > 0$	$\frac{1}{a + jt}$
21.	$\frac{1}{\pi}\frac{\sin(\omega - 2)}{\omega - 2}$	$\frac{e^{j2t}}{2\pi}, \quad -1 < t < 1; \;\; 0 - \text{van tog intervala}$
22.	$\frac{a + j\omega}{(a + j\omega)^2 + b^2}$	$e^{-at}\cos(bt)h(t)$
23.	$\frac{b}{(a + j\omega)^2 + b^2}$	$e^{-at}\sin(bt)h(t)$
24.	$\frac{1 - e^{-2(a+j\omega)}}{a + j\omega}$	$e^{-at}[h(t) - h(t-2)]$
25.	$\frac{F(j\omega)}{j\omega} + \pi F(0)\delta(\omega)$	$\int\limits_{-\infty}^{t} f(t)dt$
26.	$\frac{2(a^2 - \omega^2)}{(a^2 + \omega^2)^2}$	$\lvert t\rvert e^{-\lvert a\rvert t}$

Osnovne osobine Laplace-ove transformacije	
Laplace-ova transformacija	$L\{f(t)\} = F(s) = \int\limits_{0}^{\infty} f(t)e^{-st}dt$
Inverzna Laplace-ova transformacija	$f(t) = L^{-1}\{F(s)\} = \frac{1}{2\pi j}\int\limits_{C_0-j\infty}^{C_0+j\infty} F(s)e^{st}ds = \sum \text{Res}F(s)e^{st}$
Diferenciranje originala	$L\left\{\frac{df}{dt}\right\} = sF(s) - f(0)$ $L\left\{\frac{d^n f}{dt^n}\right\} = s^n F(s) - \sum\limits_{k=1}^{n} f^{(k-1)}(0)s^{(k-1)}$
Integracija originala	$L\left\{\int\limits_{0}^{t} f(t)dt\right\} = \frac{F(s)}{s}$
Diferenciranje transformacije (diferenciranje u kompleksnom domenu)	$\frac{dF(s)}{ds} = L\{-tf(t)\}$ $\frac{d^n F(s)}{ds^n} = L\{(-t)^n f(t)\}$
Integracija transformacije (integracija u kompleksnom domenu)	$\int\limits_{s}^{\infty} F(s)ds = L\left\{\frac{1}{t}f(t)\right\}$
Teorema sličnosti	$L\{f(at)\} = \frac{1}{a}F\left(\frac{s}{a}\right)$
Teorema kašnjenja	$L\{f(t-\tau\} = e^{-s\tau}F(s)$
Teorema pomjeraja	$F(s-a) = L\{e^{at}f(t)\}$
Granične vrijednosti	$f(0) = \lim\limits_{s\to\infty} sF(s)$ $f(\infty) = \lim\limits_{s\to 0} sF(s)$
Teorema konvolucije u vremenskom domenu	$F_1(s)F_2(s) = L\left\{\int\limits_{0}^{t} f_1(\tau)f_2(t-\tau)d\tau\right\} = L\left\{\int\limits_{0}^{t} f_2(\tau)f_1(t-\tau)d\tau\right\}$
DuHamel-ov integral	$sF_1(s)F_2(s) = L\left\{\frac{d}{dt}\int\limits_{0}^{t} f_1(\tau)f_2(t-\tau)d\tau\right\} = L\left\{\frac{d}{dt}\int\limits_{0}^{t} f_2(\tau)f_1(t-\tau)d\tau\right\}$

Teorema konvolucije u kompleksnom domenu	$L\{f(t)g(t)\} = \frac{1}{2\pi j} F(s) * G(s)$ $F(s) * G(s) = \int_{\sigma - j\omega}^{\sigma + j\omega} F(z)G(s-z)dz = \int_{\sigma - j\omega}^{\sigma + j\omega} F(s-z)G(z)dz$
Teorema razlaganja (prosti korijeni $F_2(s) = 0$)	$L^{-1}\left\{\frac{F_1(s)}{F_2(s)}\right\} = \sum_{k=1}^{n} \frac{F_1(s_k)}{F'_2(s_k)} e^{s_k t}$

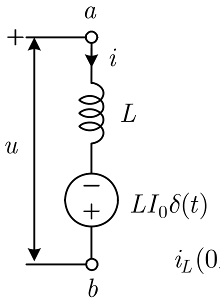
Traženje inverzne Laplace-ove transformacije	
Inverzna Laplace-ova transformacija	$f(t) = \frac{1}{2\pi j} \oint e^{st} F(s) ds = \sum \mathrm{Res}\left[F(s)e^{st}\right]$
Teorema razlaganja	$\frac{F_1(s)}{F_2(s)} = \frac{a_0 s^m + a_1 s^{m-1} + \ldots + a_m}{b_0 s^n + b_1 s^{n-1} + \ldots + b_n} \quad m < n$
Za slučaj kada $F_2(s) = 0$ ima proste korjene $F_2'(s_k) = \left.\frac{dF_2(s)}{ds}\right\|_{s=s_k}$	$L^{-1}\left\{\frac{F_1(s)}{F_2(s)}\right\} = \sum_{k=1}^{n} \frac{F_1(s_k)}{F_2'(s_k)} e^{s_k t}$
Za slučaj kada $F_3(s) = 0$ ima proste korjene i nema nula korjena	$L^{-1}\left\{\frac{F_1(s)}{sF_3(s)}\right\} = \frac{F_1(0)}{F_3(0)} + \sum_{k}^{n} \frac{F_1(s_k)e^{s_k t}}{s_k F_3'(s_k)}$
Za slučaj višestrukih korjena $F_2(s) = 0$	$L^{-1}\left\{\frac{F_1(s)}{F_2(s)}\right\} = \sum_{k=1}^{n} \frac{1}{(m_k - 1)!} \left[\frac{d^{m_k - 1}}{dt^{m_k - 1}} \frac{(s - s_k)^{m_k} F_1(s)}{F_2(s)} e^{s_k t}\right]_{s=s_k}$
Za slučaj kada $F_2(s) = 0$ ima dva konjugovano kompleksna korjena s i s^*	$L^{-1}\left\{\frac{F_1(s)}{F_2(s)}\right\} = 2\,\mathrm{Re} \frac{F_1(s)}{F_2'(s)} e^{st} = 2\,\mathrm{Re} \frac{F_1(s^*)}{F_2'(s^*)} e^{s^* t}$

Parovi Laplace-ove transformacije

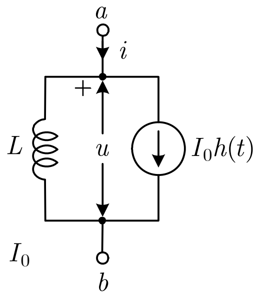
	$F(s)$	$f(t); \quad f(t) = 0$ za $t < 0$
1.	1	$\delta(t)$ - impulsna funkcija
2.	$\frac{1}{s}$	$h(t)$ - jedinična funkcija
3.	$\frac{1}{s^2}$	t - usponska funkcija $r(t)$
4.	$\frac{1}{s^n}$	$\frac{t^{n-1}}{(n-1)!}$; n je cio broj
5.	$\frac{1}{s+a}$	e^{-at}
6.	$\frac{1}{(s+a)^2}$	te^{-at}
7.	$\frac{s}{(s+a)^2}$	$e^{-at}(1 - at)$
8.	$\frac{1}{s(s+a)}$	$\frac{1}{a}(1 - e^{-at})$
9.	$\frac{1}{s(s+a)^2}$	$\frac{1}{a^2}\left[1 - e^{-at}(1 + at)\right]$
10.	$\frac{1}{s^2(s+a)}$	$\frac{1}{a^2}\left[at - e^{-at}(1 - at)\right]$
11.	$\frac{1}{(s+a)(s+b)} \quad a \neq b$	$\frac{1}{(a-b)}\left[e^{-at} - e^{-bt}\right]$

12.	$\dfrac{s}{(s+a)(s+b)} \quad a \neq b$	$\dfrac{1}{(a-b)}\left[ae^{-at} - be^{-bt}\right]$
13.	$\dfrac{1}{s(s+a)(s+b)} \quad a \neq b$	$\dfrac{1}{ab(a-b)}\left[be^{-at} - ae^{-bt}\right] + \dfrac{1}{ab}$
14.	$\dfrac{\omega}{s^2+\omega^2}$	$\sin\omega t$
15.	$\dfrac{s}{s^2+\omega^2}$	$\cos\omega t$
16.	$\dfrac{1}{s\left(s^2+\omega^2\right)}$	$\dfrac{1}{\omega^2}[1-\cos\omega t]$
17.	$\dfrac{1}{s^2-\beta^2}$	$\dfrac{1}{\beta}\sinh\beta t$
18.	$\dfrac{s}{s^2-\beta^2}$	$\cosh\beta t$
19.	$\dfrac{s+a}{(s+a)^2+\omega^2}$	$e^{-at}\cos\omega t$
20.	$\dfrac{\omega}{(s+a)^2+\omega^2}$	$e^{-at}\sin\omega t$
21.	$\dfrac{1}{s\left(s^2-\beta^2\right)}$	$\dfrac{1}{\beta^2}[\cosh\beta t-1]$
22.	$\dfrac{1}{(s+a)\left(s^2+\omega^2\right)}$	$\dfrac{1}{\omega\left(a^2+\omega^2\right)}\left[a\sin\omega t - \omega\cos\omega t + \omega e^{at}\right]$ ili $\dfrac{1}{\omega\sqrt{\left(a^2+\omega^2\right)}}\left\{\sin(\omega t-\theta) + \sin\theta\, e^{at}\right\}, \quad \theta = \arctan\left(\dfrac{\omega}{a}\right)$
23.	$\dfrac{s}{(s+a)\left(s^2+\omega^2\right)}$	$\dfrac{1}{\left(a^2+\omega^2\right)}\left[a\cos\omega t - \omega\sin\omega t - ae^{-at}\right]$ ili $\dfrac{1}{\sqrt{\left(a^2+\omega^2\right)}}\left\{\cos(\omega t-\theta) - \dfrac{a}{\omega}\sin\theta\, e^{at}\right\}, \quad \theta = \arctan\left(\dfrac{\omega}{a}\right)$
24.	$\dfrac{1}{s(s+a)\left(s^2+\omega^2\right)}$	$\left(\dfrac{1}{a\omega^2}\right)\left\{1 - \dfrac{1}{\left(a^2+\omega^2\right)}\left[a^2\cos\omega t + a\omega\sin\omega t + \omega^2 e^{-at}\right]\right\}$ ili $\left(\dfrac{1}{a\omega^2}\right)\left\{1 - \cos\theta\cos(\omega t-\theta) - \sin^2\theta\, e^{at}\right\}, \quad \theta = \arctan\left(\dfrac{\omega}{a}\right)$
25.	$\dfrac{1}{(s+a)\left(s^2-\beta^2\right)}$	$\dfrac{1}{\beta\left(a^2-\beta^2\right)}\left[a\sinh\beta t - \beta\cosh\beta t + \beta e^{-at}\right]$
26.	$\dfrac{s}{(s+a)\left(s^2-\beta^2\right)}$	$\dfrac{1}{\left(a^2-\beta^2\right)}\left[a\cosh\beta t - \beta\sinh\beta t - ae^{-at}\right]$
27.	$\dfrac{1}{s(s+a)\left(s^2-\beta^2\right)}$	$\dfrac{1}{\beta^2}\left\{\dfrac{1}{\left(a^2-\beta^2\right)}\left[a^2\cosh\beta t - a\beta\sinh\beta t - \beta^2 e^{-at}\right] - 1\right\}$
28.	$\dfrac{1}{s^2+2\alpha s+\omega_0^2}$	$\omega_0 > \alpha; \quad \dfrac{1}{\omega_1}e^{-\alpha t}\sin\omega_1 t, \quad \omega_1 = \sqrt{\left(\omega_0^2-\alpha^2\right)}$ $\omega_0 < \alpha; \quad \dfrac{1}{\beta}e^{-\alpha t}\sinh\beta t, \quad \beta = \sqrt{\left(\alpha^2-\omega_0^2\right)}$
29.	$\dfrac{s}{s^2+2\alpha s+\omega_0^2}$	$\omega_0 > \alpha; \quad e^{-\alpha t}\left[\cos\omega_1 t - \left(\dfrac{\alpha}{\omega_1}\right)\sin\omega_1 t\right], \quad \omega_1 = \sqrt{\left(\omega_0^2-\alpha^2\right)}$ $\omega_0 < \alpha; \quad e^{-\alpha t}\left[\cosh\beta t - \left(\dfrac{\alpha}{\beta}\right)\sinh\beta t\right], \quad \beta = \sqrt{\left(\alpha^2-\omega_0^2\right)}$
30.	$\dfrac{1}{s\left(s^2+2\alpha s+\omega_0^2\right)}$	$\omega_0 > \alpha; \quad \left(\dfrac{1}{\omega_0^2}\right)\left\{1 - e^{-\alpha t}\left[\cos\omega_1 t - \left(\dfrac{\alpha}{\omega_1}\right)\sin\omega_1 t\right]\right\}, \quad \omega_1 = \sqrt{\left(\omega_0^2-\alpha^2\right)}$ $\omega_0 < \alpha; \quad \left(\dfrac{1}{\omega_0^2}\right)\left\{1 - e^{-\alpha t}\left[\cosh\beta t - \left(\dfrac{\alpha}{\beta}\right)\sinh\beta t\right]\right\}, \quad \beta = \sqrt{\left(\alpha^2-\omega_0^2\right)}$

Uzimanje u obzir početnih uslova u kalemu i kondenzatoru preko ekvivalentnih šema sa nezavisnim strujnim i naponskim generatorima.

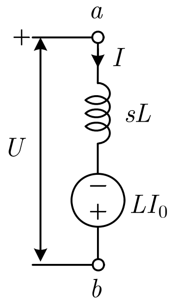


$i_L(0_-) = I_0$

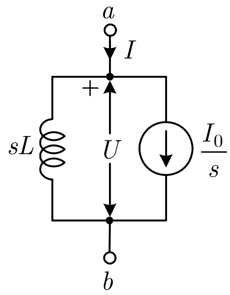


$$u(t) = -LI_0\frac{dh(t)}{dt} + L\frac{di}{dt} = -LI_0\delta(t) + L\frac{di}{dt}$$

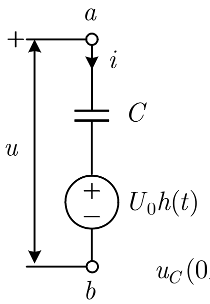
$$i(t) = \frac{1}{L}\int_{-\infty}^{+\infty} u(\tau)d\tau = I_0h(t) + \frac{1}{L}\int_0^t u(\tau)d\tau$$



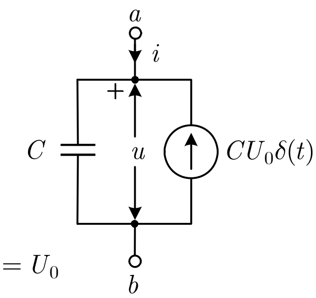
$$U = sLI - LI_0$$



$$I = \frac{U}{sL} + \frac{I_0}{s}$$

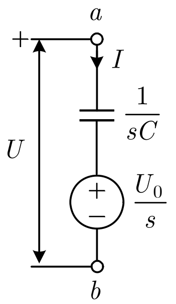


$u_C(0_-) = U_0$

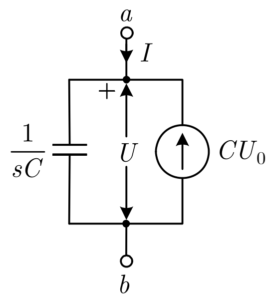


$$u(t) = \frac{1}{C}\int_{-\infty}^{+\infty} i(\tau)d\tau = U_0h(t) + \frac{1}{C}\int_0^t i(\tau)d\tau$$

$$i(t) = -CU_0\delta(t) + C\frac{du(t)}{dt}$$



$$U = \frac{I}{sC} + \frac{U_0}{s}$$



$$I = sCU - CU_0$$

Različiti oblici zapisivanja DuHamel-ovog integrala

prvi:

$$i(t) = u(0)g(t) + \int_0^t u'(\tau)g(t-\tau)d\tau$$

drugi:

$$i(t) = u(0)g(t) + \int_0^t u'(t-\tau)g(\tau)d\tau$$

treći:

$$i(t) = g(0)u(t) + \int_0^t g'(t-\tau)u(\tau)d\tau$$

četvrti:

$$i(t) = g(0)u(t) + \int_0^t g'(\tau)u(t-\tau)d\tau$$

peti:

$$i(t) = \frac{d}{dt}\int_0^t u(t-\tau)g(\tau)d\tau$$

šesti:

$$i(t) = \frac{d}{dt}\int_0^t u(\tau)g(t-\tau)d\tau$$

Neki korisni integrali

$$\int \sin^2 t\,dt = \frac{1}{2}t - \frac{1}{4}\sin 2t\,dt + C$$

$$\int \cos^2 t\,dt = \frac{1}{2}t + \frac{1}{4}\sin 2t\,dt + C$$

$$\int \frac{dt}{a^2 + t^2} = \frac{1}{2}\tan^{-1}\frac{t}{a} + C$$

$$\int te^{at}dt = \frac{1}{a^2}(at-1)e^{at} + C$$

$$\int e^{at}\sin bt\,dt = \frac{e^{at}}{a^2+b^2}(a\sin bt - b\cos bt) + C$$

$$\int e^{at}\cos bt\,dt = \frac{e^{at}}{a^2+b^2}(a\cos bt + b\sin bt) + C$$

$$\int t\sin bt\,dt = \frac{1}{b^2}\sin bt - \frac{t}{b}\cos bt + C$$

$$\int t\cos bt\,dt = \frac{1}{b^2}\cos bt + \frac{t}{b}\sin bt + C$$

$$\int_0^{2\pi/\omega} \sin(\omega t + \alpha)dt = 0$$

$$\int_0^{2\pi/\omega} \cos(\omega t + \alpha)dt = 0$$

$$\int_0^{2\pi/\omega} \sin(n\omega t + \alpha)dt = 0; \quad n - \text{cio broj}$$

$$\int_0^{2\pi/\omega} \cos(n\omega t + \alpha)dt = 0; \quad n - \text{cio broj}$$

$$\int_0^{2\pi/\omega} \sin(m\omega t + \alpha)\cos(n\omega t + \alpha)dt = 0; \quad m, n - \text{cijeli brojevi}$$

$$\int_0^{2\pi/\omega} \sin^2(\omega t + \alpha)dt = \frac{\pi}{\omega}$$

$$\int_0^{2\pi/\omega} \cos^2(\omega t + \alpha)dt = \frac{\pi}{\omega}$$

$$\int_0^{2\pi/\omega} \cos(m\omega t + \alpha)\cos(n\omega t + \beta)dt = 0; \quad m \neq n, m, n - \text{cijeli brojevi}$$

$$\int_0^{2\pi/\omega} \cos(m\omega t + \alpha)\cos(n\omega t + \beta)dt = \frac{\pi\cos(\alpha - \beta)}{\omega}; \quad m = n, m, n - \text{cijeli brojevi}$$