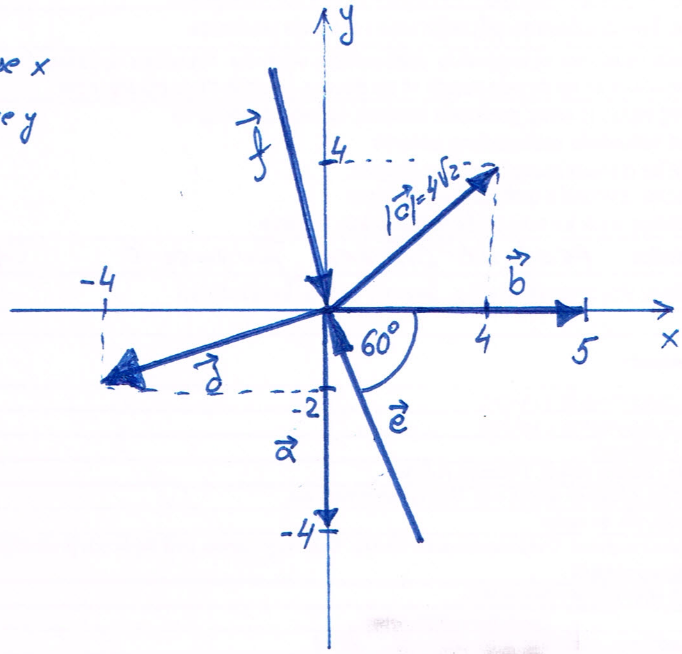
Tehnička mehanika I, Drumski Saobraćaj

I čas vježbi, 26. 09. 2017

1. Vektore sa slike izraziti u Dekartovim koordinatama:

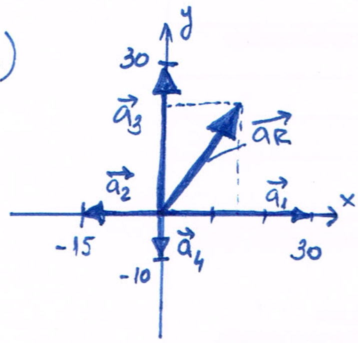
$\vec{i}$ - jedin. vektor ose x
$\vec{j}$ - jedin. vektor ose y



$$\vec{a} = 0\cdot\vec{i} - 4\cdot\vec{j}$$
$$\vec{b} = 5\cdot\vec{i} + 0\cdot\vec{j}$$
$$\vec{c} = (4\sqrt{2}\cdot\cos 45^\circ)\,\vec{i} + (4\sqrt{2}\cdot\sin 45^\circ)\,\vec{j} = 4\vec{i} + 4\vec{j}$$
$$\vec{d} = -4\vec{i} + (-2)\cdot\vec{j}$$
$$\vec{e} = -e\cdot\cos 60^\circ\,\vec{i} + e\sin 60^\circ\,\vec{j} = -\frac{1}{2}e\cdot\vec{i} + \frac{e\sqrt{3}}{2}\cdot\vec{j}$$
$$\vec{f} = f\cos\theta\cdot\vec{i} - f\sin\theta\cdot\vec{j}$$

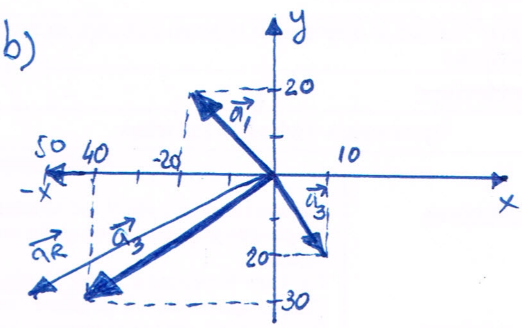
2. Za date vektore naći:
- rezultujući vektor
- intenzitet rezultujućeg vektora

a)



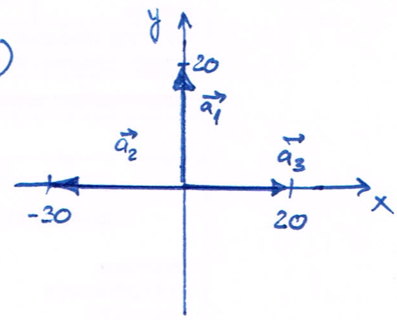
$$\vec{a_1} = 30\cdot\vec{i} + 0\cdot\vec{j}$$
$$\vec{a_2} = -15\cdot\vec{i} + 0\cdot\vec{j}$$
$$\vec{a_3} = 0\cdot\vec{i} + 30\cdot\vec{j}$$
$$\vec{a_4} = 0\cdot\vec{i} + (-10\cdot\vec{j})$$
$$\vec{a_R} = \vec{a_1} + \vec{a_2} + \vec{a_3} + \vec{a_4} = (30\vec{i} + 0\cdot\vec{j}) + (-15\vec{i} + 0\cdot\vec{j}) + (0\cdot\vec{i} + 30\cdot\vec{j}) + (0\cdot\vec{i} + (-10\cdot\vec{j})) = 15\vec{i} + 20\cdot\vec{j}$$
$$|\vec{a_R}| = \sqrt{a_{R_x}^2 + a_{R_y}^2} = \sqrt{15^2 + 20^2} = 25$$

b)



$$\vec{a_1} = -20\vec{i} + 20\vec{j}$$
$$\vec{a_2} = 10\vec{i} - 20\vec{j}$$
$$\vec{a_3} = -40\vec{i} - 30\vec{j}$$
$$\vec{a_R} = \vec{a_1} + \vec{a_2} + \vec{a_3} = (-20\vec{i} + 20\vec{j}) + (10\vec{i} - 20\vec{j}) + (-40\vec{i} - 30\vec{j}) = -50\vec{i} - 30\vec{j}$$
$$a_{R_x} = -50;\quad a_{R_y} = -30$$
$$a_R = \sqrt{a_{R_x}^2 + a_{R_y}^2} = \sqrt{(-50)^2 + (-30)^2} = 10\sqrt{34}$$

c)



$$\vec{a_1} = 0\cdot\vec{i} + 20\cdot\vec{j}$$
$$\vec{a_2} = -30\cdot\vec{i} + 0\cdot\vec{j}$$
$$\vec{a_3} = 20\cdot\vec{i} + 0\cdot\vec{j}$$
$$\vec{a_R} = \vec{a_1} + \vec{a_2} + \vec{a_3} = (0\cdot\vec{i} + 20\cdot\vec{j}) + (-30\vec{i} + 0\cdot\vec{j}) + (20\vec{i} + 0\cdot\vec{j}) = -10\vec{i} + 20\vec{j}$$
$$|\vec{a}| = \sqrt{(-10)^2 + 20^2} = \sqrt{500} = 10\sqrt{5}$$

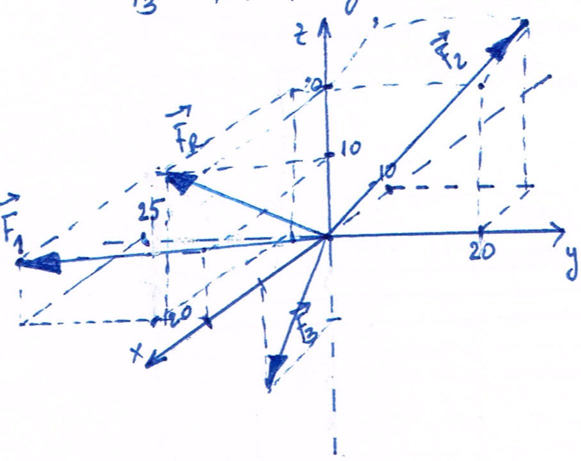
3. Zadate vektore nacrtati u Dekartovom koordinatnom sistemu, a zatim odrediti:

- intenzitet rezultante
- kosinuse uglova koje rezultanta zaklapa sa koord. osama

a) $\vec{F_1} = 20\vec{i} - 25\vec{j} + 10\vec{k}$

$\vec{F_2} = -10\vec{i} + 20\vec{j} + 20\vec{k}$

$\vec{F_3} = 10\vec{i} + 0\cdot\vec{j} + (-10\cdot\vec{k})$



$\vec{F_R} = \vec{F_1} + \vec{F_2} + \vec{F_3} =$

$= (20\vec{i} - 25\vec{j} + 10\vec{k}) +$

$+ (-10\vec{i} + 20\vec{j} + 20\vec{k}) +$

$+ (10\vec{i} + 0\cdot\vec{j} + (-10\vec{k})) =$

$= 20\vec{i} - 5\vec{j} + 20\vec{k}$

$F_R = \sqrt{F_{Rx}^2 + F_{Ry}^2 + F_{Rz}^2}$

$F_{Rx} = 20; \quad F_{Ry} = -5; \quad F_{Rz} = 20$

$F_R = \sqrt{20^2 + (-5)^2 + 20^2} =$

$= 28{,}72$

$\cos\alpha = \frac{F_{Rx}}{F_R} = \frac{20}{28{,}72} = 0{,}69$

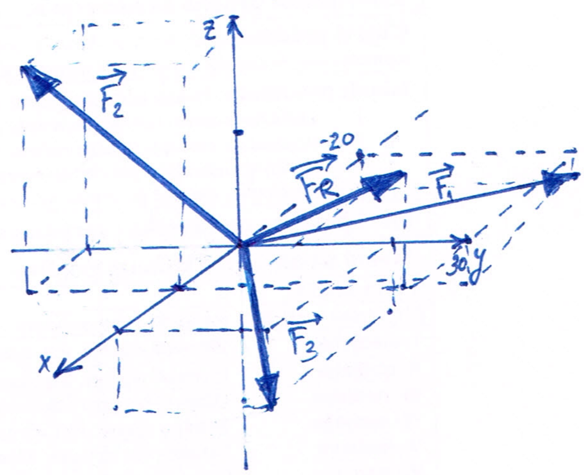
$\cos\beta = \frac{F_{Ry}}{F_R} = \frac{-5}{28{,}72} = 0{,}52$

$\cos\gamma = \frac{F_{Rz}}{F_R} = \frac{20}{28{,}72} = 0{,}69$

b) $\vec{F_1} = -20\vec{i} + 30\vec{j} - 5\vec{k}$

$\vec{F_2} = 10\vec{i} - 20\vec{j} + 30\vec{k}$

$\vec{F_3} = 20\vec{i} + 20\vec{j} - 10\vec{k}$



$\vec{F_R} = \vec{F_1} + \vec{F_2} + \vec{F_3} =$

$= (-20\vec{i} + 30\vec{j} - 5\vec{k}) +$

$+ (10\vec{i} - 20\vec{j} + 30\vec{k}) +$

$+ (20\vec{i} + 20\vec{j} - 10\vec{k}) =$

$= 10\vec{i} + 30\vec{j} + 15\vec{k}$

$F_{Rx} = 10; \quad F_{Ry} = 30; \quad F_{Rz} = 15$

$F_R = \sqrt{F_{Rx}^2 + F_{Ry}^2 + F_{Rz}^2} =$

$= \sqrt{10^2 + 30^2 + 15^2} =$

$= 35$

$\cos\alpha = \frac{F_{Rx}}{F_R} = \frac{10}{35} = 0{,}28$

$\cos\beta = \frac{F_{Ry}}{F_R} = \frac{30}{35} = 0{,}86$

$\cos\gamma = \frac{F_{Rz}}{F_R} = \frac{15}{35} = 0{,}43$

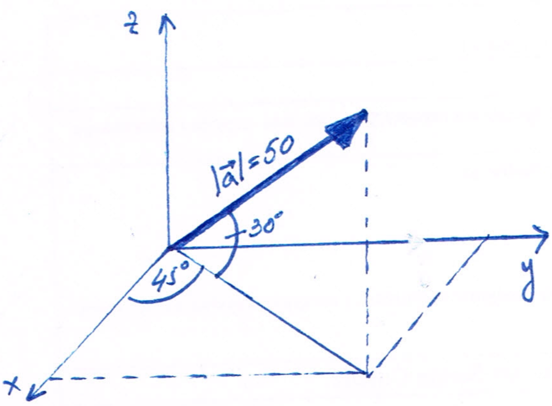
4. Odrediti projekciju datog vektora :

a)- na osu x

b)- na osu y

a) dato je: $\theta = 30^\circ$; $\alpha = 45^\circ$

$|\vec{a}| = 50$



$a_{xy} = a \cdot \cos 30^\circ$

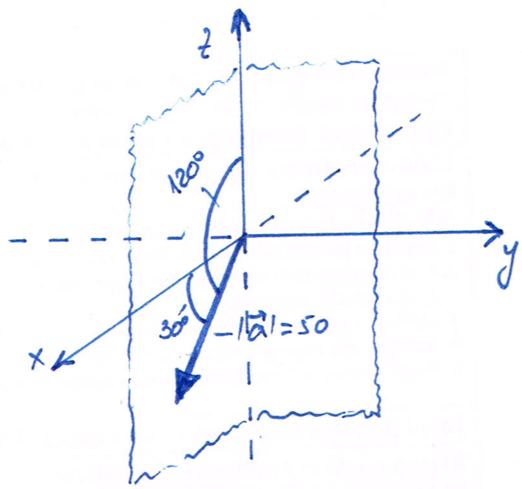
$a_x = a_{xy} \cdot \cos 45^\circ$

$a_x = a \cdot \cos 30^\circ \cdot \cos 45^\circ$

$= 50 \cdot \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} =$

$= 28{,}93$

b) dato je : $\alpha = 30^\circ$; $\gamma = 120^\circ$; $|\vec{a}| = 50$



$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$\cos^2\beta = 1 - \cos^2\alpha - \cos^2\gamma$

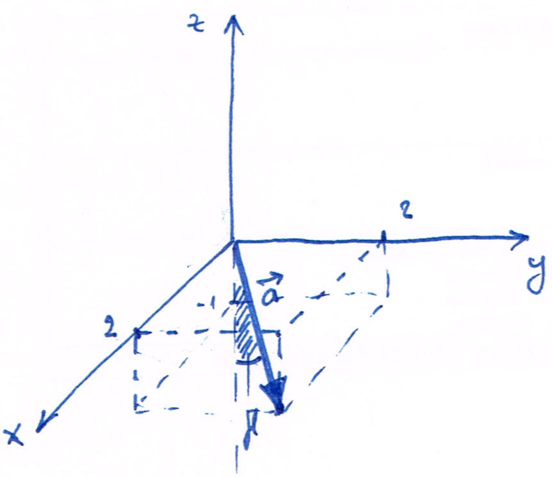
$\cos^2\beta = 1 - \cos^2 30^\circ - \cos^2 120^\circ$

$\cos\beta = \sqrt{1 - 0{,}75 - 0{,}25}$

$\cos\beta = 0$; $\cos\beta = \frac{a_y}{a}$

$a_y = a \cdot \cos\beta = 0$

5. Odrediti $\cos\gamma$. (Ugao između vektora $\vec{a}$ i ose z)

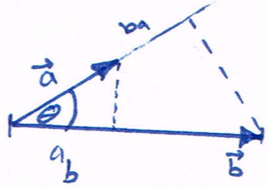


$a_x = 2$; $a_y = 2$; $a_z = -1$

$|\vec{a}| = \sqrt{a_x^2 + a_y^2 + a_z^2} = \sqrt{2^2 + 2^2 + (-1)^2} = 3$

$\cos\gamma = \frac{a_z}{|\vec{a}|} = \frac{-1}{3} = -0{,}33$

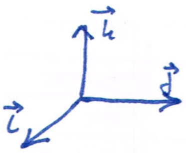
⊛ skalarni proizvod dva vektora



$$\vec{a}\cdot\vec{b} = |\vec{a}|\cdot|\vec{b}|\cdot\cos\theta$$

$$a_b = |\vec{a}|\cdot\cos\theta\ ;\ b_a = |\vec{b}|\cdot\cos\theta$$

$$\vec{a}\cdot\vec{b} = |\vec{b}|\cdot a_b = \vec{a}\cdot b_a$$



$$\cos 90^\circ = 0$$
$$\cos 0^\circ = 1$$

$$\vec{i}\cdot\vec{j} = \vec{j}\cdot\vec{k} = \vec{k}\cdot\vec{i} = 0$$
$$\vec{i}\cdot\vec{i} = \vec{j}\cdot\vec{j} = \vec{k}\cdot\vec{k} = 1$$

6. Izračunati skalarni proizvod vektora

a) $(5\vec{i})\cdot(3\vec{j}) = 5\cdot 3\cdot\vec{i}\cdot\vec{j} = 15\cdot 0 = 0$

b) $(-3\vec{j})\cdot(-2\vec{k}) = (-3)\cdot(-2)\cdot\vec{j}\cdot\vec{k} = 6\cdot 0 = 0$

c) $7\vec{i}\cdot 3\vec{i} = 7\cdot 3\cdot\vec{i}\cdot\vec{i} = 21\cdot 1 = 21$

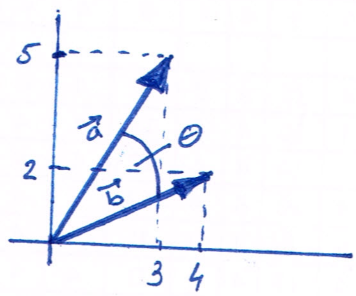
d) $(3\vec{i}-2\vec{j}+4\vec{k})(2\vec{i}-4\vec{j}+3\vec{k}) = 6\cdot\vec{i}\cdot\vec{i} - \cancelto{0}{4\vec{i}\cdot\vec{j}} + \cancelto{0}{8\vec{k}\cdot\vec{i}} - \cancelto{0}{12\vec{i}\cdot\vec{j}} + 8\vec{j}\cdot\vec{j} - \cancelto{0}{16\vec{j}\cdot\vec{k}} + \cancelto{0}{9\vec{i}\cdot\vec{k}} - \cancelto{0}{6\vec{j}\cdot\vec{k}} + 12\cdot\vec{k}\cdot\vec{k} =$

$= 6+8+12 = 26$

7. Izračunati ugao između vektora $\vec{a}$ i $\vec{b}$

a)
$$\vec{a} = 3\vec{i}+5\vec{j}$$
$$\vec{b} = 4\vec{i}+2\vec{j}$$

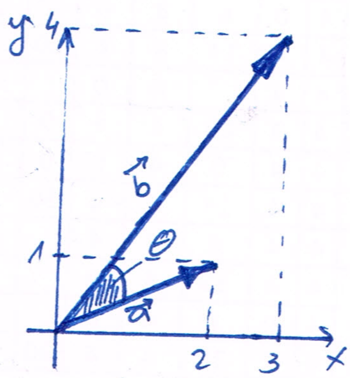
$$|\vec{a}| = \sqrt{3^2+5^2} = \sqrt{34}$$
$$|\vec{b}| = \sqrt{4^2+2^2} = \sqrt{20}$$



$$\vec{a}\cdot\vec{b} = |\vec{a}|\cdot|\vec{b}|\cdot\cos\theta$$
$$\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}|\cdot|\vec{b}|}$$
$$\cos\theta = \frac{(3\vec{i}+5\vec{j})\cdot(4\vec{i}+2\vec{j})}{\sqrt{34}\cdot\sqrt{20}}$$
$$\cos\theta = \frac{12+10}{\sqrt{640}}\ ;\ \theta = \arccos\frac{22}{\sqrt{640}} = 32^\circ$$

b)
$$\vec{a} = 2\vec{i}+\vec{j}$$
$$\vec{b} = 3\vec{i}+4\vec{j}$$

$$|\vec{a}| = \sqrt{2^2+1^2} = \sqrt{5}$$
$$|\vec{b}| = \sqrt{3^2+4^2} = 5$$



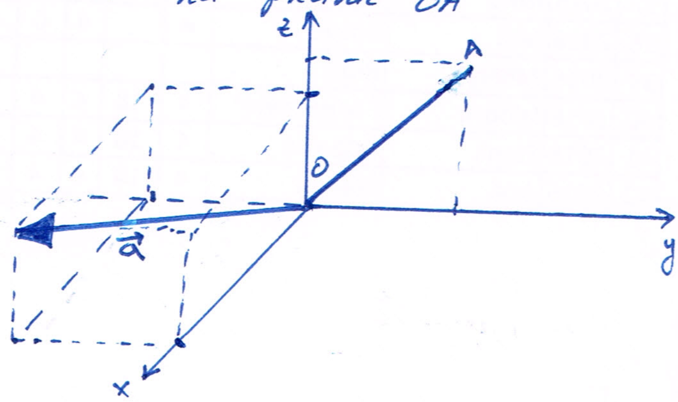
$$\vec{a}\cdot\vec{b} = |\vec{a}|\cdot|\vec{b}|\cdot\cos\theta$$
$$\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}|\cdot|\vec{b}|} = \frac{(2\vec{i}+\vec{j})(3\vec{i}+4\vec{j})}{\sqrt{5}\cdot 5} =$$
$$= \frac{10}{5\sqrt{5}} = \frac{2\sqrt{5}}{5}\ ;\ \theta = \arccos\frac{2\sqrt{5}}{5} = \ldots$$

8. Izračunati projekciju vektora $\vec{a}$ na pravac $\overrightarrow{OA}$

$$\vec{a} = 5\vec{i}-4\vec{j}+3\vec{k}$$

$$a_{OA} = \vec{a}\cdot\vec{e}_{OA}\ ;\quad |\overrightarrow{OA}| = \sqrt{4^2+4^2}$$

$$\vec{e}_{OA} = \frac{\overrightarrow{OA}}{|\overrightarrow{OA}|} = \frac{4\vec{j}+4\vec{k}}{\sqrt{4^2+4^2}} = \frac{\sqrt{2}}{2}\vec{j}+\frac{\sqrt{2}}{2}\vec{k}$$

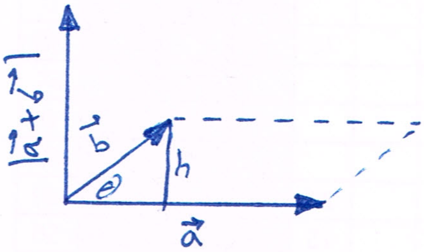


$$a_{OA} = (5\vec{i}-4\vec{j}+3\vec{k})\cdot\left(\frac{\sqrt{2}}{2}\vec{j}+\frac{\sqrt{2}}{2}\vec{k}\right)$$
$$= -2\sqrt{2}+\frac{3\sqrt{2}}{2} = -\frac{\sqrt{2}}{2}$$

Tehnička mehanika I, Drumski saobraćaj

II čas vježbi, 03.10.2017

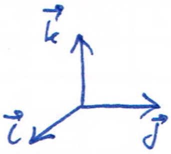
⊛ vektorski proizvod



$|\vec a \times \vec b| = |\vec a| \cdot |\vec b| \cdot \sin\theta$; $h = |\vec b| \cdot \sin\theta$

$|\vec a \times \vec b| = |\vec a| \cdot h$

$|\vec a \times \vec b| = P_{\square}$; $|\vec a \times \vec b| \perp \vec a$; $\vec a \times \vec b \perp \vec b$



$\sin 0° = 0$

$\sin 90° = 1$

$\vec i \times \vec j = \vec k$; $\vec j \times \vec i = -\vec k$; $\vec i \times \vec i = 0$

$\vec j \times \vec k = \vec i$; $\vec k \times \vec j = -\vec i$; $\vec j \times \vec j = 0$

$\vec k \times \vec i = \vec j$; $\vec i \times \vec k = -\vec j$; $\vec k \times \vec k = 0$

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1. Izračunati vektorski proizvod i skalarni proizvod vektora.

a) $\vec a_1 = \{10, 20, 30\}$

$\vec a_2 = \{-15, 5, -10\}$

$\vec a_1 = 10\vec i + 20\vec j + 30\vec k$

$\vec a_2 = -15\vec i + 5\vec j - 10\vec k$

$\vec a_1 \cdot \vec a_2 = (10\vec i + 20\vec j + 30\vec k) \cdot (-15\vec i + 5\vec j - 10\vec k) =$

$= -150 + 100 - 300 = -350$

$$|\vec a_1 \times \vec a_2| = \begin{vmatrix} \vec i & \vec j & \vec k \\ 10 & 20 & 30 \\ -15 & 5 & -10 \end{vmatrix} =$$

$= (20\cdot(-10)) - (30\cdot 5)\cdot\vec i$

$- (10\cdot(-10)) - (30\cdot(-15))\cdot\vec j$

$+ (5\cdot 10) - (20\cdot(-15))\cdot\vec k =$

$= -350\vec i - (-100 + 450)\cdot\vec j + 350\vec k$

$= -350\vec i - 350\vec j + 350\vec k$

$$|\vec a_2 \times \vec a_1| = \begin{vmatrix} \vec i & \vec j & \vec k \\ -15 & 5 & -10 \\ 10 & 20 & 30 \end{vmatrix} =$$

$= 350\vec i + 350\vec j - 350\vec k$

b) $\vec a_1 = \vec i + \vec j + \vec k$

$\vec a_2 = -\vec i - \vec j - \vec k$

$\vec a_1 \cdot \vec a_2 = (\vec i + \vec j + \vec k)(-\vec i - \vec j - \vec k) = -3$

$$\vec a_1 \times \vec a_2 = \begin{vmatrix} \vec i & \vec j & \vec k \\ 1 & 1 & 1 \\ -1 & -1 & -1 \end{vmatrix} = 0\cdot\vec i - 0\cdot\vec j + 0\cdot\vec k$$

$\vec a_2 \times \vec a_1 = 0\cdot\vec i - 0\cdot\vec j + 0\cdot\vec k$

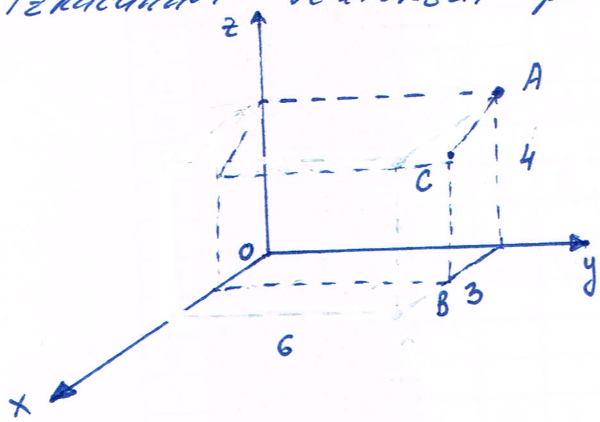
c) $\vec a_1 = 4\vec i - 2\vec j$

$\vec a_2 = 2\vec i + 3\vec j - 2\vec k$

$\vec a_1 \cdot \vec a_2 = (4\vec i - 2\vec j)(2\vec i + 3\vec j - 2\vec k) = 8 - 6 = 2$

$$|\vec a_1 \times \vec a_2| = \begin{vmatrix} \vec i & \vec j & \vec k \\ 4 & -2 & 0 \\ 2 & 3 & -2 \end{vmatrix} = 4\vec i + 8\vec j + 16\vec k$$

2. Vektor $\vec{a}$ ima intenzitet 5 i leži duž pravca OA. Naći jedinični vektor pravca OA. Napisati vektor $\vec{a}$ kao proizvod intenziteta i jediničnog vektora. Naći komponente vektora $\vec{a}$ u pravcu koordinatnih osa. Odrediti ugao koji vektor $\vec{a}$ zaklapa sa osom y. Izračunati vektorski proizvod $\vec{r}_{CO}$ i $\vec{a}$.



a) $\vec{e}_{OA} = \frac{\vec{OA}}{|\vec{OA}|} = \frac{6\vec{j}+4\vec{k}}{\sqrt{6^2+4^2}} = \frac{6\vec{j}+4\vec{k}}{\sqrt{52}}$

$\vec{e}_{OA} = \frac{6}{\sqrt{52}}\vec{j} + \frac{4}{\sqrt{52}}\cdot\vec{k}$

b) $\vec{a} = |\vec{a}|\cdot\vec{e}_{OA} = 5\left(\frac{6}{\sqrt{52}}\vec{j} + \frac{4}{\sqrt{52}}\vec{k}\right)$

$= \frac{30}{\sqrt{52}}\cdot\vec{j} + \frac{20}{\sqrt{52}}\cdot\vec{k}$

c) $\vec{a}_x = 0\cdot\vec{i}\,;\ \vec{a}_y = \frac{30}{\sqrt{52}}\cdot\vec{j}\,;\ \vec{a}_z = \frac{20}{\sqrt{52}}\cdot\vec{k}$

$a_x = 0\,;\ a_y = \frac{30}{\sqrt{52}}\,;\ a_z = \frac{20}{\sqrt{52}}$

d) $\cos\beta = \frac{a_y}{|\vec{a}|} = \frac{\frac{30}{\sqrt{52}}}{5} = \frac{6}{\sqrt{52}}$

$\beta = \arccos\left(\frac{6}{\sqrt{52}}\right) = \dots$

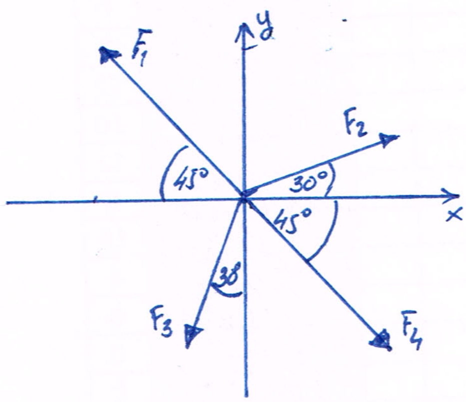
e) $\vec{r}_{CO} = -3\vec{i} - 6\vec{j} - 4\vec{k}$

$\vec{a} = \frac{30}{\sqrt{52}}\vec{j} + \frac{20}{\sqrt{52}}\vec{k}$

$|\vec{r}_{CO}\times\vec{a}| = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -3 & -6 & -4 \\ 0 & \frac{30}{\sqrt{52}} & \frac{20}{\sqrt{52}} \end{vmatrix} = \left(\frac{-120}{\sqrt{52}} + \frac{120}{\sqrt{52}}\right)\cdot\vec{i} - -\left(\frac{-60}{\sqrt{52}} + 0\right)\cdot\vec{j} + +\left(\frac{-90}{\sqrt{52}}\right)\vec{k} =$

$= \frac{60}{\sqrt{52}}\cdot\vec{j} - \frac{90}{\sqrt{52}}\cdot\vec{k}$

3. Odrediti rezultantu datih sila



$F_1 = 14,2 = 10\sqrt{2}$ kN

$F_2 = 15$ kN

$F_3 = 8$ kN

$F_4 = 18$ kN

$\vec{F}_1 = -14,2\cdot\cos 45^\circ\cdot\vec{i} + 14,2\cdot\sin 45^\circ\cdot\vec{j} =$

$= -10\vec{i} + 10\vec{j}$

$\vec{F}_2 = 15\cdot\cos 30^\circ\cdot\vec{i} + 15\cdot\sin 30^\circ\cdot\vec{j} =$

$= 13\vec{i} + 7,5\vec{j}$

$\vec{F}_3 = -8\cdot\sin 30^\circ\cdot\vec{i} - 8\cdot\cos 30^\circ\cdot\vec{j} =$

$= -4\vec{i} - 7\vec{j}$

$\vec{F}_4 = 18\cos 45^\circ\cdot\vec{i} - 18\cdot\sin 45^\circ\vec{j} = 12,72\vec{i} - 12,72\vec{j}$

$\vec{F}_R = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_4 =$

$= (-10\vec{i} + 10\vec{j}) +$

$+ (13\vec{i} + 7,5\vec{j}) +$

$+ (-4\vec{i} - 7\vec{j}) +$

$+ (12,72\vec{i} - 12,72\vec{j}) =$

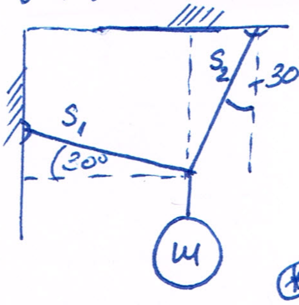
$= 11,72\vec{i} - 2,22\vec{j}$

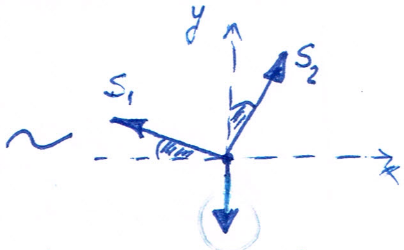
$F_{Rx} = 11,72\,;\ F_{Ry} = -2,22$

$F_R = \sqrt{F_{Rx}^2 + F_{Ry}^2} = 12$ kN$;\ \tan\theta = \frac{F_{Ry}}{F_{Rx}} = \frac{11,}{-2,}$

4. Kugla mase 10 kg se održava u ravnoteži pomoću dva užeta S_1 i S_2 koja se nalaze pod uglom od 30° redom u odnosu na vertikalu tj. horizontalu (slika). Naći sile u užadima S_1 i S_2

$m = 10kg$; $g = 9{,}81\ m/s^2$





$S_{1x} = -S_1 \cdot \cos 30°$

$S_{1y} = S_1 \cdot \sin 30°$

$S_{2x} = S_2 \cdot \sin 30°$

$S_{2y} = S_2 \cdot \cos 30°$

(*) da bi tijelo bilo u ravnoteži, rezultanta datih sila mora biti jednaka nuli, tj: $\vec{S_1}+\vec{S_2}+m\vec{g}=\vec{0}$. To znači da projekcije rezultante na ose x i y moraju biti jednake nuli, pa je:

$-S_{1x} + S_{2x} = 0$; $S_{1y} + S_{2y} - m \cdot g = 0$

$S_{1x} = S_{2x}$

$S_1 \cdot \cos 30° = S_2 \cdot \sin 30°$

$S_1 = S_2 \cdot tg 30°$

$S_1 \cdot \sin 30° + S_2 \cos 30° - mg = 0$

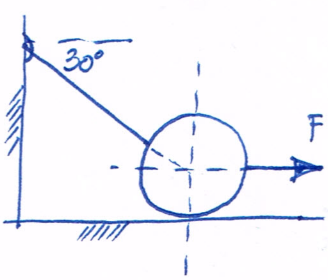
$S_2 \cdot tg 30° \cdot \sin 30° + S_2 \cdot \cos 30° = m \cdot g$

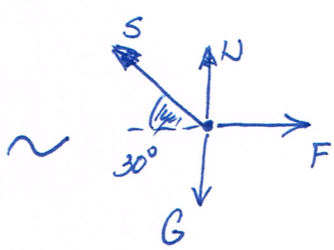
$S_2 (\sin 30° \cdot tg 30° + \cos 30°) = mg$

$$S_2 = \frac{m \cdot g}{\sin 30° \cdot tg 30° + \cos 30°} = \frac{98{,}1}{0{,}5 \cdot 0{,}57 + 0{,}86}$$

$S_2 = 86{,}05 N$; $S_1 = 49{,}05 N$

5. Kugla mase 50 kg miruje na glatkoj vodoravnoj podlozi. Ako na kuglu djeluje sila 500 N kao na slici, koliko iznosi normalna reakcija podloge N.





$F - S \cdot \cos 30° = 0$

$N + S \cdot \sin 30° - G = 0$

$$S = \frac{F}{\cos 30°} = 581{,}4 N$$

$N = G - S \cdot \sin 30° =$

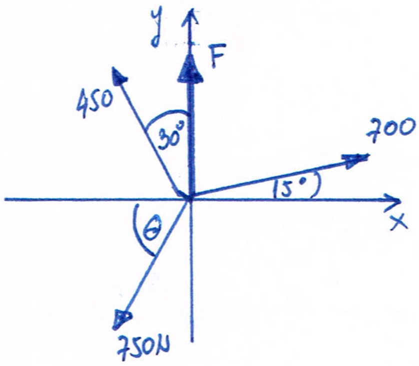
$= 500 - 290{,}69 =$

$= 209{,}30 N$

Tehnička mehanika I, Drumski saobraćaj

III čas vježbi, 10.10.2017

1. Odrediti intenzitet sile F i ugao θ prikazan na slici, ako je sistem u ravnoteži



$\sum F_x = 0;$

$$700 \cdot \cos 15 - 450 \cdot \sin 30^\circ - 750 \cdot \cos\theta = 0$$

$$\cos\theta = \frac{450 \cdot \sin 30^\circ - 700 \cdot \cos 15^\circ}{-750} = 0{,}601$$

$$\boxed{\theta = 53{,}03^\circ}$$

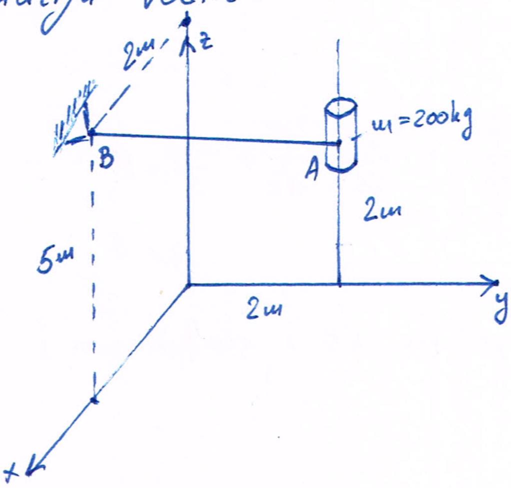
$\sum F_y = 0;$

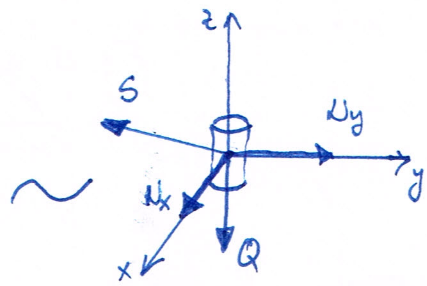
$$F + 450 \cdot \cos 30^\circ + 700 \cdot \sin 15^\circ - 750 \cdot \sin\theta = 0$$

$$F = 750 \cdot \sin 53{,}03^\circ - 700 \cdot \sin 15^\circ - 450 \cdot \cos 30^\circ \Rightarrow$$

$$\boxed{F = 28{,}28\,N}$$

2. Klizač mase 200kg pridržava se na vertikalnoj vodici pomoću užeta kao što je prikazano na slici. Odrediti silu u užetu i normalnu reakciju vodice.





$m = 200\,kg$

$g = 9{,}81\ m/s^2$

$N_x = ?;\ N_y = ?;\ S = ?$

3 nepoznate, 3 uslova ravnot

$Q = m \cdot g = 1962\,N$

$$\vec{N_x} = N_x \cdot \vec{i}$$

$$\vec{N_y} = N_y \cdot \vec{j}$$

$$\vec{Q} = -Q \cdot \vec{k}$$

$$\vec{S} = S \cdot \vec{e}_{AB} = S \cdot \frac{\vec{AB}}{|\vec{AB}|} =$$

$$= S \cdot \frac{2\vec{i} - 2\vec{j} + 3\vec{k}}{\sqrt{2^2 + (-2)^2 + 3^2}} =$$

$$= \frac{2S}{\sqrt{17}} \cdot \vec{i} - \frac{2S}{\sqrt{17}} \cdot \vec{j} + \frac{3S}{\sqrt{17}} \cdot \vec{k}$$

* postavljamo uslove ravnoteže za ose x, y, z

(1) $\sum F_x = 0;\quad N_x + \frac{2S}{\sqrt{17}} = 0$

(2) $\sum F_y = 0;\quad N_y - \frac{2S}{\sqrt{17}} = 0$

(3) $\sum F_z = 0;\quad -Q + \frac{3S}{\sqrt{17}} = 0$

$$(3) \Rightarrow \quad Q = \frac{3S}{\sqrt{17}} \Rightarrow S = \frac{Q \cdot \sqrt{17}}{3}$$

$$\boxed{S = 2695{,}5\,N}$$

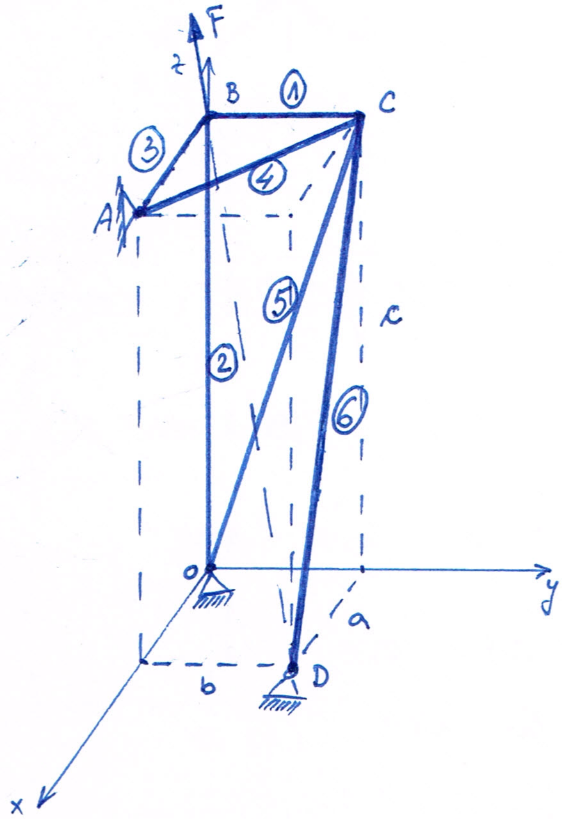
zamjenom S u (2) i u (1) dobijamo:

$$N_y = \frac{2S}{\sqrt{17}};\quad \boxed{N_y = 1308\,N}$$

$$N_x = -\frac{2S}{\sqrt{17}};\quad \boxed{N_x = -1308\,N}$$

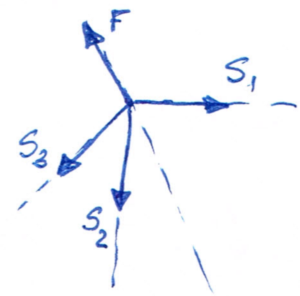
– znak "–" znači d je pogrešno pret stavljen smjer sile N_x

3. Odrediti sile u štapovima date konstrukcije sastavljene od 6 zglobno vezanih, lakih štapova. Na konstrukciju djeluje sila $\vec{F}$, u tački B. Dato je: $a = 3m$; $b = 4m$; $c = 12m$; $F = 13kN$

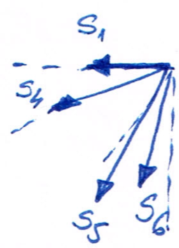


Sistem je u ravnoteži ako su sile koje djeluju na čvorove B i C u ravnoteži

Čvor B:



Čvor C:



Čvor B:

$$\vec{F} = F \cdot \vec{e}_{DB} = F \cdot \frac{\vec{DB}}{|\vec{DB}|} = F \cdot \frac{-3\vec{i} - 4\vec{j} + 12\vec{k}}{\sqrt{(-3)^2 + (-4)^2 + 12^2}} = -3\vec{i} - 4\vec{j} + 12\vec{k}$$

$\vec{S}_1 = S_1 \cdot \vec{j}$

$\vec{S}_2 = -S_2 \cdot \vec{k}$

$\vec{S}_3 = S_3 \cdot \vec{i}$

uslovi ravnoteže:

(1) $\Sigma F_x = 0$; $-3 + S_3 = 0$

(2) $\Sigma F_y = 0$; $S_1 - 4 = 0$

(3) $\Sigma F_z = 0$; $-S_2 + 12 = 0$

$S_1 = 4kN$; $S_2 = 12kN$

$S_3 = 3kN$

Čvor C:

$\vec{S}_1 = -S_1 \cdot \vec{j} = -4 \cdot \vec{j}$

$\vec{S}_4 = S_4 \cdot \vec{e}_{CA} = S_4 \cdot \frac{\vec{CA}}{|\vec{CA}|} = S_4 \cdot \frac{3\vec{i} - 4\vec{j}}{\sqrt{3^2 + 4^2}} = \frac{3S_4}{5} \cdot \vec{i} - \frac{4 \cdot S_4}{5} \cdot \vec{j}$

$\vec{S}_5 = S_5 \cdot \vec{e}_{CO} = S_5 \cdot \frac{\vec{CO}}{|\vec{CO}|} = S_5 \cdot \frac{-4\vec{j} - 12\vec{k}}{\sqrt{4^2 + 12^2}} = -\frac{\sqrt{10}}{10} \cdot S_5 \cdot \vec{j} - \frac{3\sqrt{10}}{10} \cdot S_5 \cdot$

$\vec{S}_6 = S_6 \cdot \vec{e}_{CD} = S_6 \cdot \frac{\vec{CD}}{|\vec{CD}|} = S_6 \cdot \frac{3\vec{i} - 12\vec{k}}{\sqrt{3^2 + (-12)^2}} = \frac{\sqrt{17}}{17} \cdot S_6 \cdot \vec{i} - \frac{4\sqrt{17}}{17} S_6$

uslovi ravnoteže:

(4) $\Sigma F_x = 0$; $\frac{3}{5} S_4 + \frac{\sqrt{17}}{17} S_6 = 0$

(5) $\Sigma F_y = 0$; $-4 - \frac{4S_4}{5} - \frac{\sqrt{10}}{10} \cdot S_5 = 0$

(6) $\Sigma F_z = 0$; $-\frac{3\sqrt{10}}{10} \cdot S_5 - \frac{4\sqrt{17}}{17} \cdot S_6 = 0$

Rješavanjem sistema jednačina dobijamo:

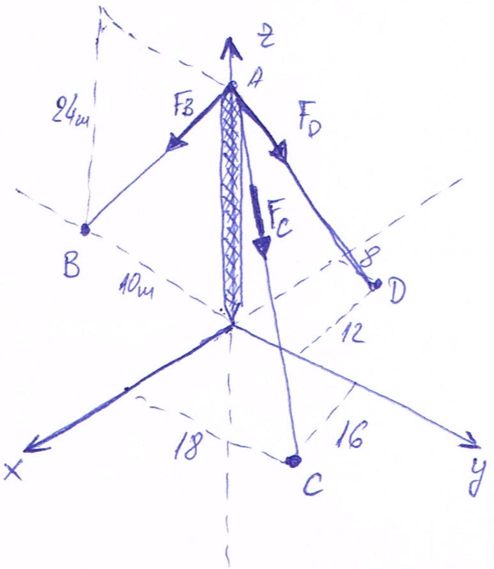
$S_6 = \frac{3\sqrt{17}}{2} kN$

$S_5 = -2\sqrt{10} kN$

$S_4 = -2,5 kN$

* Znak minus u proračunu znači da su smjerovi sila u štapovima 4 i 5 pogrešno pretpostavljeni, što znači da ~~se~~ treba promijeniti smjer. To dalje znači da su ~~su~~ štapovi 4 i 5 opterećeni na ~~istezanje~~ pritisak, a štapovi 1, 2, 3, 6 su opterećeni na istezanje ~~pritisak~~

4. Antenski stub je učvršćen za podlogu pomoću tri užeta kako to pokazuje slika. Ako sile u užadima iznose: $F_B = 520N$, $F_C = 680N$, $F_D = 56N$ odrediti rezultantu sila po intenzitetu, kao i uglove koje ona zaklapa sa koordinatnim osama.



$$\vec{F_B} = F_B \cdot \vec{e}_{AB} = F_B \cdot \frac{\vec{AB}}{|\vec{AB}|} = F_B \cdot \frac{-10\vec{j} - 24\vec{k}}{\sqrt{10^2+24^2}} = -\frac{5}{13} F_B \vec{j} - \frac{12}{13}$$

$$\vec{F_B} = -200\vec{j} - 480 \cdot \vec{k}$$

$$\vec{F_C} = F_C \cdot \vec{e}_{AC} = F_C \cdot \frac{\vec{AC}}{|\vec{AC}|} = F_C \cdot \frac{16\vec{i} + 18\vec{j} - 24\vec{k}}{\sqrt{16^2+18^2+24^2}} =$$

$$= 170\vec{i} + 360\vec{j} - 480 \cdot \vec{k}$$

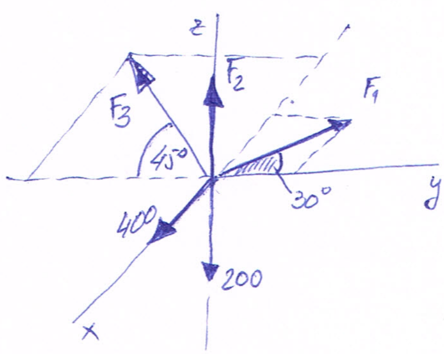
$$\vec{F_D} = F_D \cdot \vec{e}_{AD} = F_D \cdot \frac{\vec{AD}}{|\vec{AD}|} = F_D \cdot \frac{-12\vec{i} + 8\vec{j} - 24\vec{k}}{\sqrt{12^2+8^2+24^2}} =$$

$$= -24\vec{i} + 16 \cdot \vec{j} - 48\vec{k}$$

$$\vec{F_R} = \vec{F_1} + \vec{F_2} + \vec{F_3} = 146\vec{i} - 54 \cdot \vec{j} - 1008\vec{k}$$

$$|\vec{F_R}| = 1020\,N$$

5. J Odrediti intenzitet sila F_1, F_2 i F_3 iz uslova ravnoteže tačke A.



$$(1)\quad \sum F_x = 0; \quad 400 - F_3 \cdot \underbrace{\sin 45^\circ}_{\frac{\sqrt{2}}{2}} - F_1 \cdot \underbrace{\sin 30^\circ}_{1/2} = 0$$

$$(2)\quad \sum F_y = 0; \quad -F_3 \cdot \underbrace{\cos 45^\circ}_{\frac{\sqrt{2}}{2}} + F_1 \cdot \underbrace{\cos 30^\circ}_{\frac{\sqrt{3}}{2}} = 0$$

$$(3)\quad \sum F_z = 0; \quad F_2 - 200 = 0$$

$F_2 = 200\,N$; $F_1 = 292{,}8\,N$; $F_3 = 358{,}6\,N$