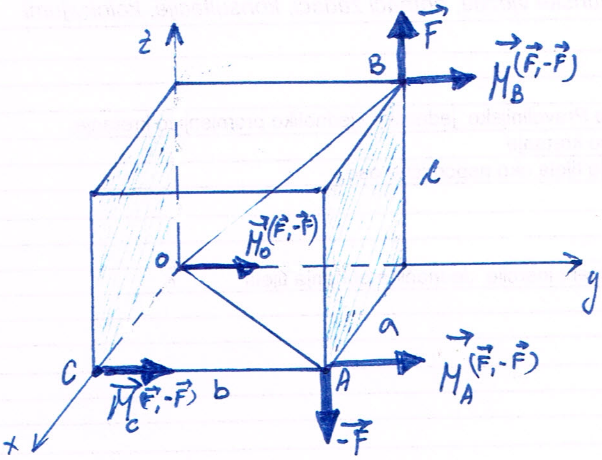
Tehnička mehanika I, Drumski Saobraćaj
IV čas vježbi, 17. 10. 2017.

1. Za kvadar prikazan na slici, opterećen silama $\vec{F}$ i $-\vec{F}$ naći:

a) moment sprega sila $\vec{F}$ i $-\vec{F}$ za tačku O
b) moment sprega sila $\vec{F}$ i $-\vec{F}$ za tačku C
c) moment sprega sila $\vec{F}$ i $-\vec{F}$ za tačku A
d) moment sprega sila $\vec{F}$ i $-\vec{F}$ za tačku B



a) $$\vec{M}_O^{(\vec F,-\vec F)} = \vec{M}_O^{\vec F} + \vec{M}_O^{-\vec F} = \vec{OB}\times\vec{F} + \vec{OA}\times(-\vec{F}) = \begin{vmatrix} \vec i & \vec j & \vec k \\ 0 & b & c \\ 0 & 0 & F \end{vmatrix} + \begin{vmatrix} \vec i & \vec j & \vec k \\ a & b & 0 \\ 0 & 0 & -F \end{vmatrix} =$$

$$= \cancel{(F\cdot b)}\cdot\vec i + 0\cdot\vec j + 0\cdot\vec k + \cancel{(-F\cdot b)}\vec i + (F\cdot a)\cdot\vec j + 0\cdot\vec k = F\cdot a\cdot\vec j$$

b) $$\vec{M}_C^{(\vec F,-\vec F)} = \vec{M}_C^{\vec F} + \vec{M}_C^{(-\vec F)} = \vec{CB}\times\vec{F} + \vec{CA}\times(-\vec{F}) = \begin{vmatrix} \vec i & \vec j & \vec k \\ -a & b & c \\ 0 & 0 & F \end{vmatrix} + \begin{vmatrix} \vec i & \vec j & \vec k \\ 0 & b & 0 \\ 0 & 0 & -F \end{vmatrix} =$$

$$= \cancel{(F\cdot b)}\cdot\vec i + F\cdot a\cdot\vec j + 0\cdot\vec k - \cancel{(F\cdot b)}\vec i + 0\cdot\vec j + 0\cdot\vec k = F\cdot a\cdot\vec j$$

c) $$\vec{M}_A^{(\vec F,-\vec F)} = \vec{AB}\times\vec{F} = \begin{vmatrix} \vec i & \vec j & \vec k \\ a & 0 & c \\ 0 & 0 & F \end{vmatrix} = F\cdot a\cdot\vec j$$

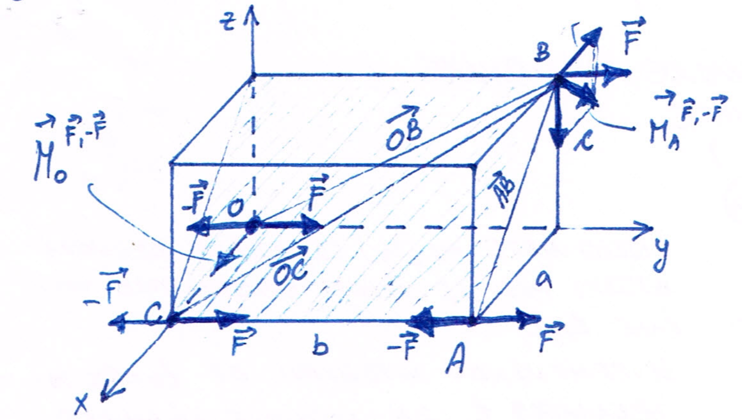
d) $$\vec{M}_B^{(\vec F,-\vec F)} = \vec{BA}\times(-\vec{F}) = \begin{vmatrix} \vec i & \vec j & \vec k \\ -a & 0 & -c \\ 0 & 0 & -F \end{vmatrix} = (-F)\cdot(-a)\cdot\vec j = F\cdot a\cdot\vec j$$

2. Za kvadar prikazan na slici, i opterećen silom $\vec{F}$:

a) Redukovati silu $\vec{F}$ na tačku A

b) Redukovati silu $\vec{F}$ na tačku O

c) Redukovati silu $\vec{F}$ na tačku C



a) Redukcijom sile $\vec{F}$ na tačku A dobijamo silu i moment sile:

- sila u tački A: $\vec{F} = F\vec{j}$

- moment sile u tački A: $\vec{M}_A^{\vec{F},-\vec{F}} = (\vec{AB}\times\vec{F}) = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -a & 0 & c \\ 0 & F & 0 \end{vmatrix} = -F\cdot c\cdot\vec{i} - F\cdot a\cdot$

b) Redukcija sile $\vec{F}$ na tačku O:

$$\vec{F} = F\cdot\vec{j}$$

$$\vec{M}_O^{(\vec{F},-\vec{F})} = (\vec{OB}\times\vec{F}) = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & b & c \\ 0 & F & 0 \end{vmatrix} = -F\cdot c\cdot\vec{i}$$

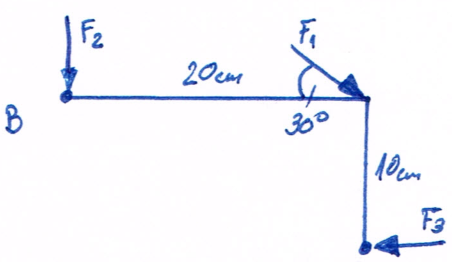
c) Redukcija sile $\vec{F}$ na tačku C:

$$\vec{F} = F\cdot\vec{j}$$

$$\vec{M}_C^{(\vec{F},-\vec{F})} = (\vec{CB}\times\vec{F}) = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -a & b & c \\ 0 & F & 0 \end{vmatrix} = -F\cdot c\cdot\vec{i} - F\cdot a\cdot\vec{k}$$

3. Redukovati dati sistem sila na tačku B. Dato je:

$F_1 = 30N$; $F_2 = 20N$; $F_3 = 10N$



Redukcijom datog sistema sila na tačku B, kao rezultat dobijamo silu F_R i moment M_R.

$$F_R \begin{cases} \sum F_x = -F_3 + F_1\cdot\cos 30^\circ = -10 + 26 = 16\,N \\ \sum F_y = -F_2 - F_1\cdot\sin 30^\circ = -20 - 15 = -35\,N \end{cases}$$

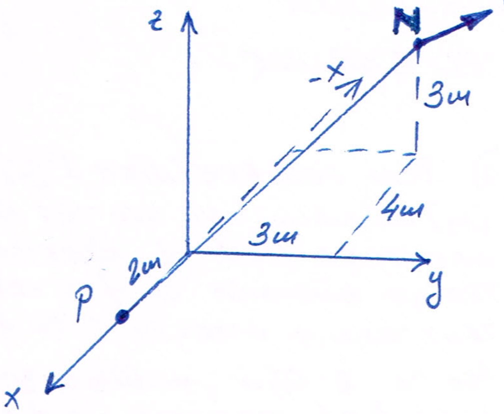
$$\sum M_B = F_3\cdot 10 + F_1\cdot\sin 30^\circ\cdot 20 =$$
$$= 10\cdot 10 + 15\cdot 20 = 400\,Ncm$$

$$F_R = \sqrt{F_x^2 + F_y^2} = \sqrt{16^2 + (-35)^2} = 38{,}48\,N$$

$$M_R = 400\,Ncm$$

4. Odrediti moment date sile za tačku P i za osu x

$\vec F = -5\vec i + 4\vec j + 3\vec k$



$$\vec M_P^{\vec F} = \vec{PN} \times \vec F = \begin{vmatrix} \vec i & \vec j & \vec k \\ -6 & 3 & 3 \\ -5 & 4 & 3 \end{vmatrix} =$$

$$= (9-12)\cdot\vec i - (-18+15)\cdot\vec j + (-24+15)\cdot\vec k$$

$$= -3\vec i + 3\vec j - 9\vec k$$

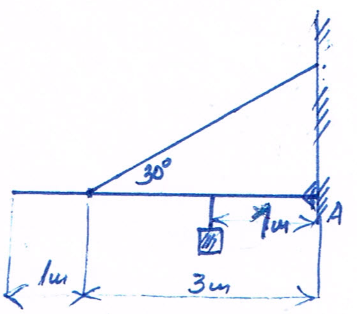
$$\vec{PN} = -2\vec i + 3\vec j - 4\vec i + 3\vec k =$$
$$= -6\vec i + 3\vec j + 3\vec k$$

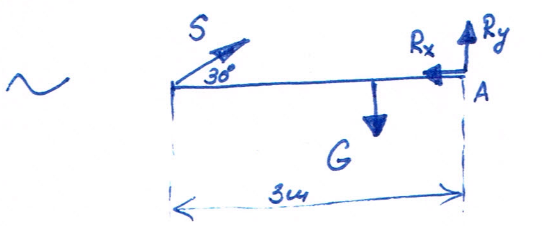
$M_x^F = -3$ – moment sile F za osu x

(*) Projekciju momenta sile F na osu x možemo da izračunamo i tako što ćemo izračunati proizvod projekcije sile F na osu y i najkraćeg rastojanja do ose x (4·3m) i proizvod sile F na osu z i najkraćeg rastojanja do ose x (3·3m). Projekcija sile F na osu y daje negativan moment za osu x.

5. O horizontalni štap zanemarljive težine obješen je teret težine 10 kN. Odrediti silu u užetu i reakciju zgloba A u položaju ravnot

$G = 10kN$





(1) $\Sigma F_x = 0;\quad S\cdot\cos 30° - R_x = 0$

(2) $\Sigma F_y = 0;\quad S\cdot\sin 30° + R_y - G = 0$

(3) $\Sigma M_A = 0;\quad S\cdot\sin 30°\cdot 3 - G\cdot 1 = 0$

(3) $\Rightarrow$ $S\cdot\frac{1}{2}\cdot 3 = G$

$S = \frac{2G}{3} = 6{,}67N$; $\boxed{S = 6{,}67N}$

(2) $\Rightarrow$ $R_y = G - S\cdot\sin 30° = 10 - 6{,}67\cdot\frac{1}{2}$

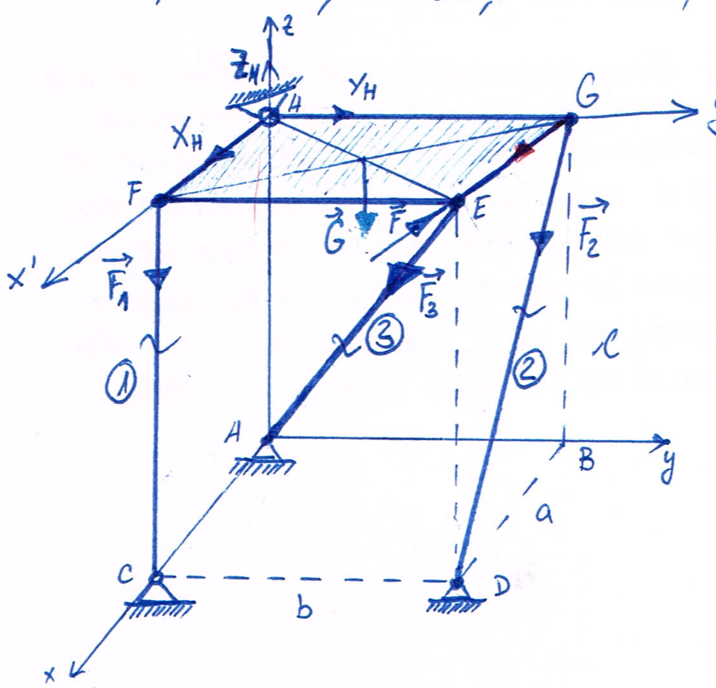
$\boxed{R_y = 6{,}665\,N}$

(1) $\Rightarrow$ $R_x = S\cdot\cos 30° = 6{,}67\cdot\frac{\sqrt3}{2}$

$\boxed{R_x = 5{,}77N}$

6. Homogena pravougaona ploča težine G održava se u ravnotežnom položaju pomoću sfernog ležišta H i tri štapa CF, EA, i GD. Na ploču u tački E djeluje sila F u pravcu EG. Odrediti reakcije, ako je dato

$a=3m$; $b=4m$; $c=12m$; $G=8kN$, $F=10kN$



$$\vec{R}_H = \{X_H, Y_H, Z_H\}$$

$$\vec{F} = \{-F, 0, 0\}$$

$$\vec{G} = \{0, 0, -G\}$$

$$\vec{F_1} = \{0, 0, -F_1\}$$

$$\vec{F_2} = \left\{\frac{F_2 \cdot a}{\sqrt{a^2+c^2}}, 0, \frac{-F_2 \cdot c}{\sqrt{a^2+c^2}}\right\}$$

$$\vec{F_3} = \left\{\frac{-F_3 \cdot a}{\sqrt{a^2+b^2+c^2}}, \frac{-F_3 \cdot b}{\sqrt{a^2+b^2+c^2}}, \frac{-F_3 c}{\sqrt{a^2+b^2+c^2}}\right\}$$

$$\vec{F_2} = F_2 \cdot \vec{e}_{GD} = F_2 \cdot \frac{\vec{GD}}{|\vec{GD}|} = F_2 \cdot \frac{-c \cdot \vec{k} + a \cdot \vec{i}}{\sqrt{(-c)^2 + a^2}} =$$

$$= \frac{-F_2 \cdot c}{\sqrt{a^2+c^2}} \cdot \vec{k} + \frac{a \cdot F_2}{\sqrt{a^2+c^2}} \cdot \vec{i} =$$

$$\vec{F_2} = \frac{F_2 \cdot a}{\sqrt{a^2+c^2}} \cdot \vec{i} - \frac{F_2 \cdot c}{\sqrt{a^2+c^2}} \cdot \vec{k}$$

$$\vec{F_3} = F_3 \cdot \vec{e}_{EA} = F_3 \cdot \frac{\vec{EA}}{|\vec{EA}|} = F_3 \cdot \frac{-a \cdot \vec{i} - b \cdot \vec{j} - c \cdot \vec{k}}{\sqrt{(-a)^2 + (-b)^2 + (-c)^2}} =$$

$$= -\frac{F_3 \cdot a}{\sqrt{a^2+b^2+c^2}} \vec{i} - \frac{F_3 \cdot b}{\sqrt{a^2+b^2+c^2}} \vec{j} - \frac{F_3 \cdot c}{\sqrt{a^2+b^2+c^2}} \vec{k}$$

uslovi ravnoteže:

(1) $\sum F_x = 0$; $X_H - F - \frac{F_3 \cdot a}{\sqrt{a^2+b^2+c^2}} = 0$

(2) $\sum F_y = 0$; $Y_H - \frac{F_3 \cdot b}{\sqrt{a^2+b^2+c^2}} = 0$

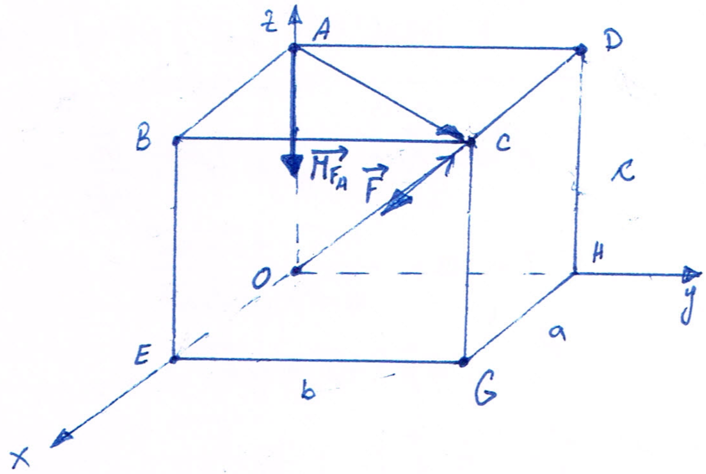
(3) $\sum F_z = 0$; $Z_H - G - F_1 - \frac{F_2 \cdot c}{\sqrt{a^2+c^2}} - \frac{F_3 \cdot c}{\sqrt{a^2+b^2+c^2}} = 0$

(4) $\sum M_{x'} = 0$; $G \cdot \frac{b}{2} + \frac{F_2 \cdot c}{\sqrt{a^2+c^2}} \cdot b + \frac{F_3 \cdot c}{\sqrt{a^2+b^2+c^2}} \cdot b = 0$

(5) $\sum M_{y'} = 0$; $F_1 \cdot a + G \cdot \frac{a}{2} + \frac{F_3 \cdot c}{\sqrt{a^2+b^2+c^2}} \cdot a = 0$

(6) $\sum M_{z'} = 0$; $F \cdot b - F_2 \cdot \frac{a \cdot b}{\sqrt{a^2+c^2}} = 0$

4. Na kvadar prikazan na slici djeluje sila F. Odrediti :
- moment sile F za tačku A
- moment sile F za tačku O
- moment sile F za tačku G
- moment sile F za osu GA . Dato je F, a, b, c



$$\vec{AC} = a\cdot\vec{i} + b\cdot\vec{j}$$

$$\vec{OC} = a\cdot\vec{i} + b\cdot\vec{j} + c\cdot\vec{k}$$

$$\vec{\lambda}_{GA} = \frac{\vec{GA}}{|\vec{GA}|} = \frac{-a\cdot\vec{i} - b\cdot\vec{j} + c\cdot\vec{k}}{\sqrt{a^2+b^2+c^2}} = \frac{-a}{\sqrt{a^2+b^2+c^2}}\cdot\vec{i} + \frac{-b}{\sqrt{a^2+b^2+c^2}}\cdot\vec{j} + \frac{+c}{\sqrt{a^2+b^2+c^2}}$$

a) $$\vec{M}_{F_A} = \vec{AC}\times\vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a & b & 0 \\ F & 0 & 0 \end{vmatrix} = 0\cdot\vec{i} - 0\cdot\vec{j} - F\cdot b\cdot\vec{k}$$

$$M_{F_{Ax}} = 0;\quad M_{F_{Ay}} = 0;\quad M_{F_{Az}} = -F\cdot b$$

b) $$\vec{M}_{F_O} = \vec{OC}\times\vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a & b & c \\ F & 0 & 0 \end{vmatrix} = 0\cdot\vec{i} + F\cdot c\cdot\vec{j} - F\cdot b\cdot\vec{k}$$

$$M_{F_{Ox}} = 0;\quad M_{F_{Oy}} = F\cdot c;\quad M_{F_{Oz}} = -F\cdot b$$

c) $$\vec{M}_{F_G} = \vec{GC}\times\vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 0 & c \\ F & 0 & 0 \end{vmatrix} = 0\cdot\vec{i} + F\cdot c\cdot\vec{j} + 0\cdot\vec{k}$$

$$M_{F_{Gx}} = 0;\quad M_{F_{Gy}} = F\cdot c;\quad M_{F_{Gz}} = 0$$

d) $$M_{GA}^F = \vec{M}_A^F\cdot\vec{\lambda}_{GA} = \vec{M}_G^F\cdot\vec{\lambda}_{GA} =$$

$$= (-F\cdot b\cdot\vec{k})\left(\frac{-a}{\sqrt{a^2+b^2+c^2}}\cdot\vec{i} + \frac{-b}{\sqrt{a^2+b^2+c^2}}\cdot\vec{j} + \frac{c}{\sqrt{a^2+b^2+c^2}}\cdot\vec{k}\right) =$$

$$= \frac{-F\cdot b\cdot c}{\sqrt{a^2+b^2+c^2}}$$

(*) Intenzitet momenta može se izračunati i kao proizvod sile i najkraćeg rastojanja do tačke za koju tražimo moment. Naprimjer : $|\vec{M}_{F_A}| = F\cdot\overline{AD} = F\cdot b$

(**) moment sile F za osu GA bi dobili isti da smo tražili skalarni proizvod $\vec{M}_G^F\cdot\vec{\lambda}_{GA} = (F\cdot c\cdot\vec{j})\left(\frac{-a}{\sqrt{a^2+b^2+c^2}}\cdot\vec{i} + \frac{-b}{\sqrt{a^2+b^2+c^2}}\cdot\vec{j} + \frac{c}{\sqrt{a^2+b^2+c^2}}\cdot\vec{k}\right)$

$$= \frac{-F\cdot c\cdot b}{\sqrt{a^2+b^2+c^2}}$$

5. Na nosač prikazan na slici djeluje sila F. Naći Moment sile F za tačku A.

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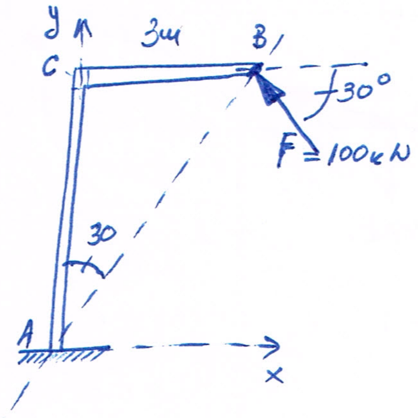


$$\vec{M}_{F_A} = \vec{AB} \times \vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3 & 3\sqrt{3} & 0 \\ -\frac{F\cdot\sqrt{3}}{2} & \frac{F}{2} & 0 \end{vmatrix} =$$

$$= 0\cdot\vec{i} + 0\cdot\vec{j} + \left(50\cdot 3 - (-50\sqrt{3}\cdot 3\sqrt{3})\right)\cdot\vec{k}$$

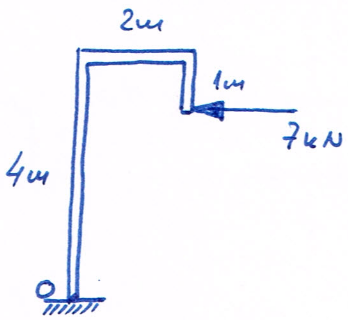
$$= 600\,\vec{k}$$

$$tg\,30^\circ = \frac{CB}{AC} \Rightarrow AC = CB\cdot\frac{1}{tg\,30^\circ} = 3\sqrt{3}$$

$$M_{FA} = 100\cdot 6 \quad (F\cdot\overline{AB})$$

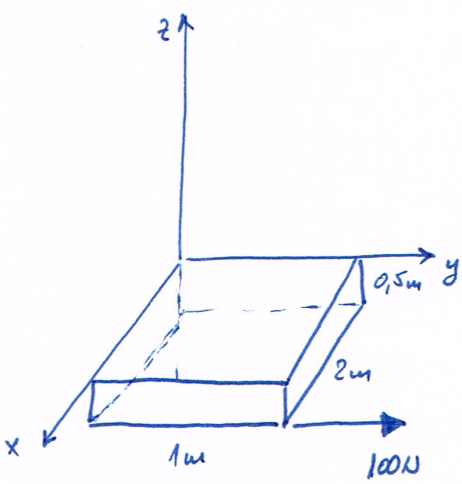
$$\vec{F} = -F\cdot\cos 30^\circ\cdot\vec{i} + F\cdot\sin 30^\circ\cdot\vec{j}$$

6. Odrediti moment date sile za tačku O.



$M_{F_O} = 7\cdot 3 = 21$ kNm

7. Odrediti moment date sile od 100N za sve tri koordinatne ose.



$M_x = 50$ Nm

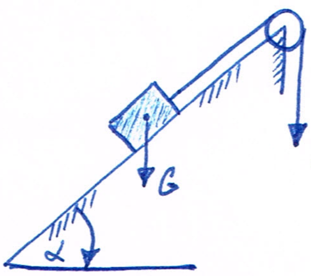
$M_y = 0$ Nm

$M_z = 200$ Nm

Tehnička mehanika I, Drumski Saobraćaj

V čas vježbi, 24. 10. 2017.

1. Odrediti interval vrijednosti intenziteta sile P tako da tijelo težine G koje se nalazi na strmoj ravni nagiba α bude u ravnoteži. Nagib strme ravni je takav da bi se tijelo pod dejstvom sopstvene težine kretalo niz strmu ravan. Koeficijent trenja f je poznat.



Da bi tijelo bilo u ravnoteži treba da je

$$P_{min} \leq P \leq P_{max},$$

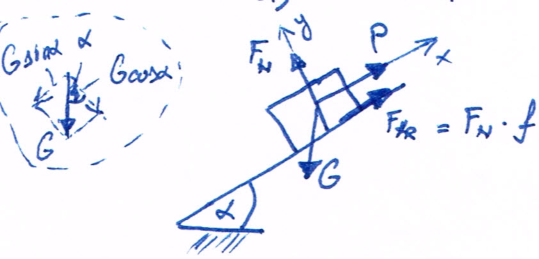
ako je $P < P_{min}$, tijelo se kreće niz strmu ravan

a ako je $P > P_{max}$, tijelo se kreće uz strmu ravan

Razmatraćemo slučajeve kada je $P = P_{min}$ i $P = P_{max}$

1° Tijelo ima tendenciju da se kreće niz strmu ravan

a) $P = P_{min}$



$$\sum F_x = 0; \quad P + F_{tr} - G \cdot \sin\alpha = 0$$

$$\sum F_y = 0; \quad F_N - G \cdot \cos\alpha = 0$$

$$F_N = G \cdot \cos\alpha$$

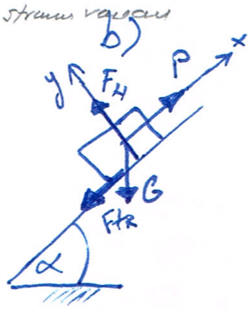
$$P = G \cdot \sin\alpha - F_{tr}$$

$$P = G \cdot \sin\alpha - G \cdot \cos\alpha \cdot f$$

$$P_{min} = P = G(\sin\alpha - \cos\alpha \cdot f)$$

2° Tijelo ima tendenciju da se kreće uz strmu ravan

b) $P = P_{max}$



$$F_{tr} = F_N \cdot f$$

$$\sum F_x = 0; \quad P - F_{tr} - G \cdot \sin\alpha = 0$$

$$\sum F_y = 0; \quad F_N - G \cdot \cos\alpha = 0$$

$$F_N = G \cdot \cos\alpha$$

$$P_{max} = P = F_{tr} + G \cdot \sin\alpha$$

$$P_{max} = G \cdot \cos\alpha \cdot f + G \cdot \sin\alpha$$

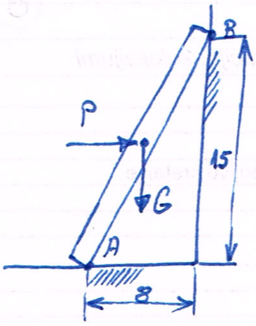
$$P_{max} = G(\sin\alpha + \cos\alpha \cdot f)$$

$$G(\sin\alpha - \cos\alpha \cdot f) \leq P \leq G(\sin\alpha + \cos\alpha \cdot f)$$

2. Greda AB težine G=10kN održava se u položaju prikazanom na slici pod dejstvom sile P. Ako koeficijent trenja f na mjestima kontakta ima vrijednost 0,2 odrediti:

a) minimalnu vrijednost sile P da bi greda AB bila u ravnoteži

b) maksimalnu vrijednost sile P da bi greda AB bila u ravnoteži



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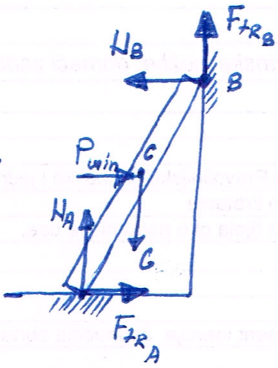

$$\Sigma F_x = 0; \quad P_{min} + F_{tr_A} - N_B = 0 \quad \dots (1)$$

$$\Sigma F_y = 0; \quad N_A + F_{tr_B} - G = 0 \quad \dots (2)$$

$$\Sigma M_C = 0; \quad N_B \cdot 7{,}5 + F_{tr_B} \cdot 4 - N_A \cdot 4 + F_{tr_A} \cdot 7{,}5 = 0 \quad \dots (3)$$

$$F_{trB} = N_B \cdot f$$

$$F_{trA} = N_A \cdot f$$

$$(1) \Rightarrow P_{min} = N_B - N_A \cdot f$$

$$(2) \Rightarrow N_A = G - N_B \cdot f$$

$$(3) \Rightarrow N_B(7{,}5 + f \cdot 4) - N_A(4 - f \cdot 7{,}5) = 0$$

1° Greda ima tendenciju da se spušta pod dejstvom sile G

$$N_B = N_A \cdot \left(\frac{4 - 7{,}5 \cdot f}{7{,}5 + 4f}\right) = 0{,}3\,N_A$$

$$N_A = G - N_A \cdot \left(\frac{4 - 7{,}5 \cdot f}{7{,}5 + 4f}\right) \longrightarrow N_A = \frac{G}{1 + \left(\frac{4 - 7{,}5 \cdot f}{7{,}5 + 4f} \cdot f\right)} = 9{,}43\text{ kN}$$

$$N_B = 2{,}8\text{ kN}; \quad \boxed{P_{min} = 1\text{ kN}}$$

$$P_{min} = N_B - N_A \cdot f = 2{,}8 - 9{,}43 \cdot 0{,}2 = 0{,}91$$

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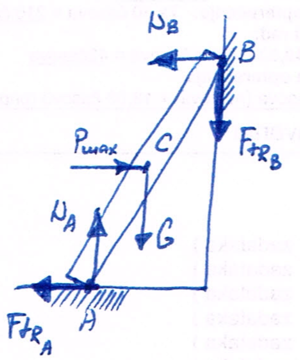


$$\Sigma F_x = 0; \quad P_{max} - F_{tr_A} - N_B = 0 \quad \dots (1)$$

$$\Sigma F_y = 0; \quad N_A - G - F_{tr_B} = 0 \quad \dots (2)$$

$$\Sigma M_C = 0; \quad N_B \cdot 7{,}5 - F_{tr_B} \cdot 4 - N_A \cdot 4 - F_{tr_A} \cdot 7{,}5 = 0 \quad \dots (3)$$

2° Greda ima tendenciju da se podiže pod dejstvom sile P

$$N_B(7{,}5 - 4 \cdot f) - N_A(4 + 7{,}5 \cdot f) = 0$$

$$N_B = N_A \cdot \frac{4 + 7{,}5f}{7{,}5 - 4 \cdot f}$$

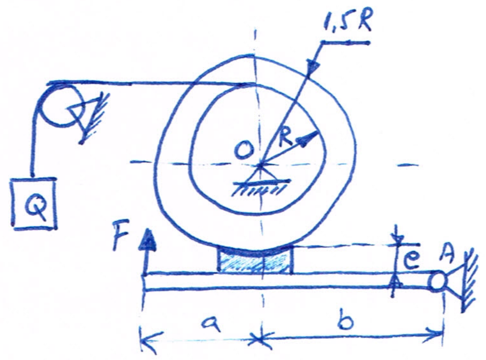
$$N_A = G + N_B \cdot f$$

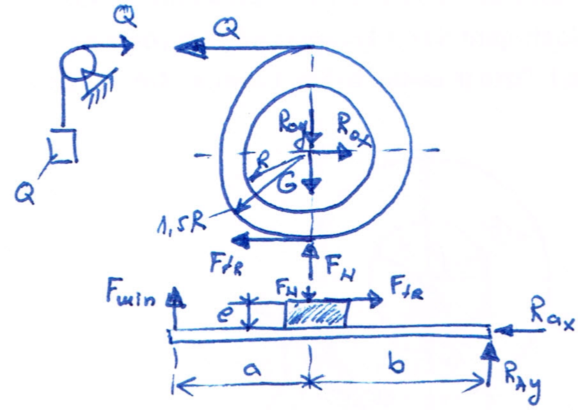
$$N_A = \frac{G}{1 - \frac{4 + 7{,}5f}{7{,}5 - 4 \cdot f} \cdot f} = 12\text{ kN}$$

$$N_B = 10{,}0\text{ kN}; \quad \boxed{P_{max} = 12{,}4\text{ kN}}$$

$$N_B = \frac{N_A - G}{f}; \quad P_{max} = N_B + N_A \cdot f$$

3. Odrediti minimalnu vrijednost sile F, reakcije u osloncima A i O i silu međusobnog pritiska između papuče kočnice i koaksijalnog doboša kočnice prikazane na slici. Težina štapa AB se zanemaruje, dok su težine tereta i doboša Q, odnosno G. Koeficijent trenja između papuče i doboša je f_o. Ostali potrebni podaci dati su na slici.





DOBOŠ:

$$\sum M_O=0;\quad F_{tR}\cdot 1{,}5\cdot R - Q\cdot R=0 \Rightarrow F_{tR}=F_N\cdot f_o;\quad \boxed{F_N=\frac{Q}{1{,}5\cdot f_o}}$$

$$\sum F_x=0;\quad R_{Ox}-Q-F_{tR}=0;\Rightarrow R_{Ox}=Q+F_N\cdot f_o;\quad \boxed{R_{Ox}=\frac{5}{3}Q}$$

$$\sum F_y=0;\quad F_N-R_{Oy}-G=0;\quad R_{Oy}=F_N-G \Rightarrow \boxed{R_{Oy}=\frac{2\cdot Q}{3\cdot f_o}-G}$$

POLUGA:

$$\sum M_A=0;\quad F_{min}(a+b)+F_{tR}\cdot e-F_N\cdot b=0;\Rightarrow F_{min}=(F_N\cdot b-F_{tR}\cdot e)\cdot\frac{1}{a+b}=\frac{2}{3}Q\cdot\frac{b-e\cdot f_o}{(a+b)\cdot f_o}$$

$$\sum F_x=0;\quad F_{tR}-R_{Ax}=0 \Rightarrow \boxed{R_{Ax}=\frac{2}{3}Q}$$

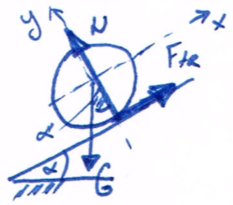
$$\sum F_y=0;\quad F_{min}+R_{Ay}-F_N=0 \Rightarrow \boxed{R_{Ay}=\frac{2}{3}Q\cdot\frac{a+e\cdot f_o}{(a+b)f_o}}$$

4. Točak poluprečnika r, se nalazi na strmoj ravni nagibnog ugla α i koeficijenta trenja μ. Odrediti:

a) ugao α, tako da dođe do klizanja točka

b) krak trenja kotrljanja e, tako da dođe do kotrljanja točka

a)



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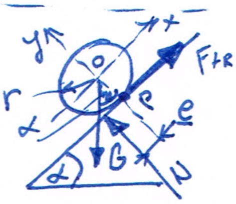
b)



(1) $\sum F_x=0;\ F_{tR}-G\cdot\sin\alpha=0;\ F_{tR}=N\cdot\mu$

(2) $\sum F_y=0;\ N-G\cdot\cos\alpha=0$

(1) $\Rightarrow N=\frac{G\cdot\sin\alpha}{\mu}$

(2) $\Rightarrow N=G\cdot\cos\alpha$

$\cancel{G}\cdot\cos\alpha\cdot\mu=\cancel{G}\cdot\sin\alpha$

$\boxed{\mu=tg\alpha}$

(1)... $\sum F_y=0;\ N-G\cdot\cos\alpha=0;\Rightarrow N=G\cdot\cos\alpha$

(2)... $\sum M_P=0;\ N\cdot e-G\cdot\sin\alpha\cdot r=0$

$\cancel{G}\cdot\cos\alpha\cdot e=\cancel{G}\cdot\sin\alpha\cdot r$

$\boxed{e=tg\alpha\cdot r}$

Tehnička mehanika I, Drumski Saobraćaj
VI čas vježbi, 31. 10. 2017.

1. Za grede prikazane i opterećene kao na slici odrediti reakcije u osloncima, dato je: $F_1 = 20\sqrt{3}$; $F_2 = 40\,kN$; $l = 1\,m$

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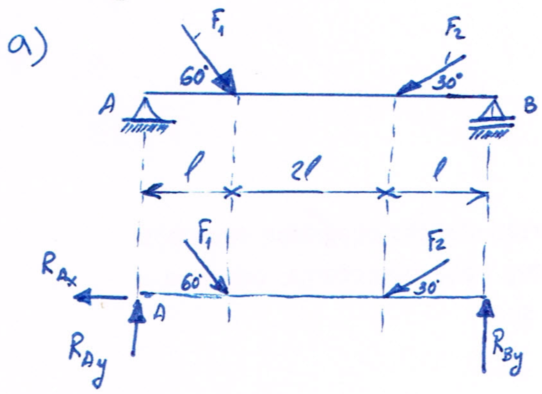
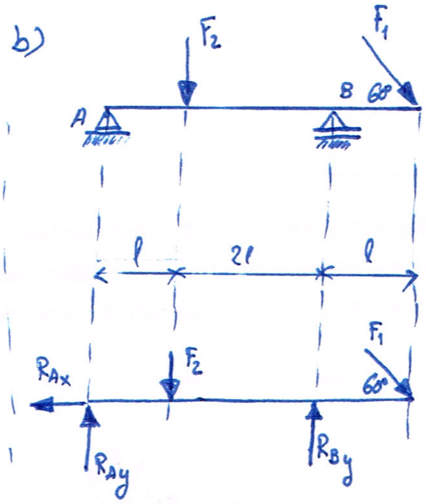
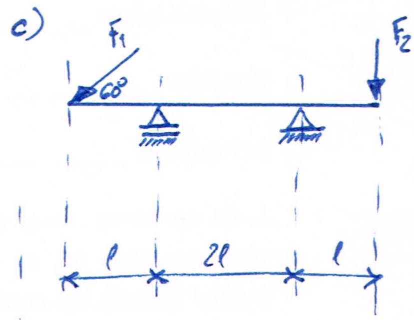
a)



b)



c)



a)

(1) $\sum F_x = 0;\quad F_1\cdot\cos 60^\circ - F_2\cdot\cos 30^\circ - R_{Ax} = 0$

(2) $\sum F_y = 0;\quad R_{Ay} + R_{By} - F_1\cdot\sin 60^\circ - F_2\cdot\sin 30^\circ = 0$

(3) $\sum M_A = 0;\quad R_{By}\cdot 4l - F_2\cdot\sin 30^\circ\cdot 3l - F_1\sin 60^\circ\cdot l = 0$

(3) ⇒ $4\cdot R_{By} - \frac{3\cdot F_2}{2} - F_1\cdot\frac{\sqrt{3}}{2}\cdot 1 = 0$

$R_{By} = \left(\frac{F_1\sqrt{3}}{2} + \frac{F_2\cdot 3}{2}\right)\cdot\frac{1}{4}$

$\boxed{R_{By} = 22{,}5\,kN}$

(2) ⇒ $R_{Ay} = F_2\cdot\frac{1}{2} + F_1\cdot\frac{\sqrt{3}}{2} - R_{By}$

$\boxed{R_{Ay} = 27{,}5\,kN}$

(1) ⇒ $R_{Ax} = \frac{F_1}{2} - F_2\cdot\frac{\sqrt{3}}{2}$

$\boxed{R_{Ax} = 10\sqrt{3}}$

b)

(1) $\sum F_x = 0;\quad F_1\cdot\cos 60^\circ - R_{Ax} = 0$

(2) $\sum F_y = 0;\quad R_{Ay} + R_{By} - F_2 - F_1\cdot\sin 60^\circ = 0$

(3) $\sum M_A = 0;\quad F_1\cdot\sin 60^\circ\cdot 4l - R_{By}\cdot 3l + F_2\cdot l = 0$

(3) $R_{By} = \frac{F_2\cdot l + F_1\cdot\frac{\sqrt{3}}{2}\cdot 4l}{3l}$

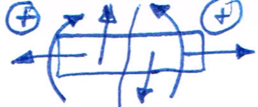
$R_{By} = \frac{F_2 + 2F_1\sqrt{3}}{3}$; $\boxed{R_{By} = 53{,}33\,kN}$

(2) ⇒ $R_{Ay} = F_1\cdot\frac{\sqrt{3}}{2} + F_2 - R_{By}$

$\boxed{R_{Ay} = 16{,}67\,kN}$

(1) ⇒ $R_{Ax} = F_1\cdot\frac{1}{2}$

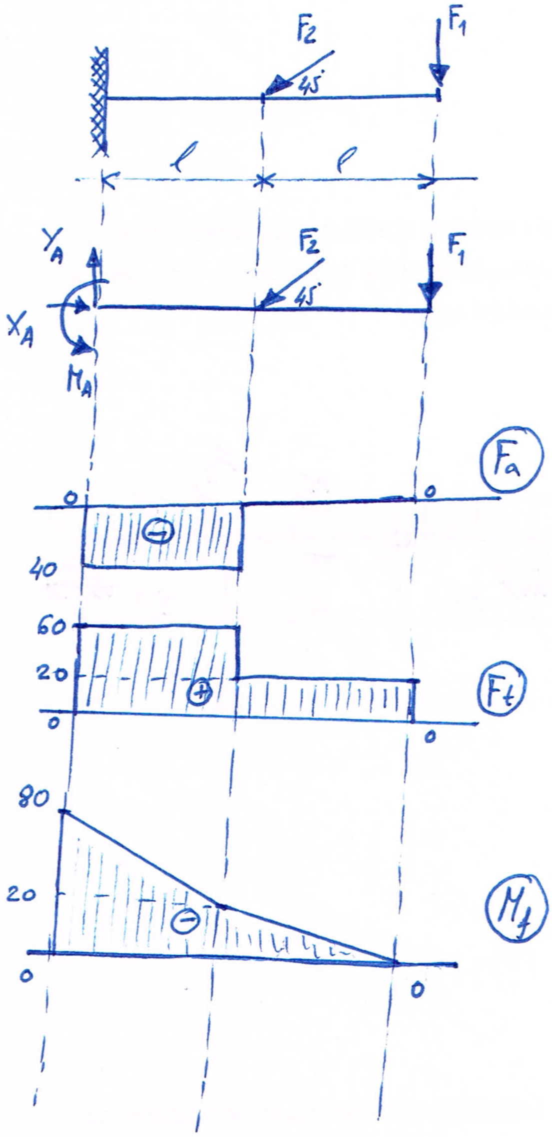
$\boxed{R_{Ax} = 10\sqrt{3}}$



2. Za gredu opterećenu kao na slici: a) odrediti reakcije oslonaca

b) nacrtati statičke dijagrame

Podaci: $F_1 = 20\,kN$; $F_2 = 40\sqrt{2}\,kN$; $l = 1m$



a) (1) $\sum F_x = 0;\quad X_A - F_2 \cdot \cos 45^\circ = 0$

(2) $\sum F_y = 0;\quad Y_A - F_1 - F_2 \cdot \sin 45^\circ = 0$

(3) $\sum M_A = 0;\quad F_1 \cdot 2l + F_2 \cdot \sin 45^\circ \cdot l - M_A = 0$

(1) $\Rightarrow \quad X_A = F_2 \cdot \cos 45^\circ \Rightarrow \boxed{X_A = 40\,kN}$

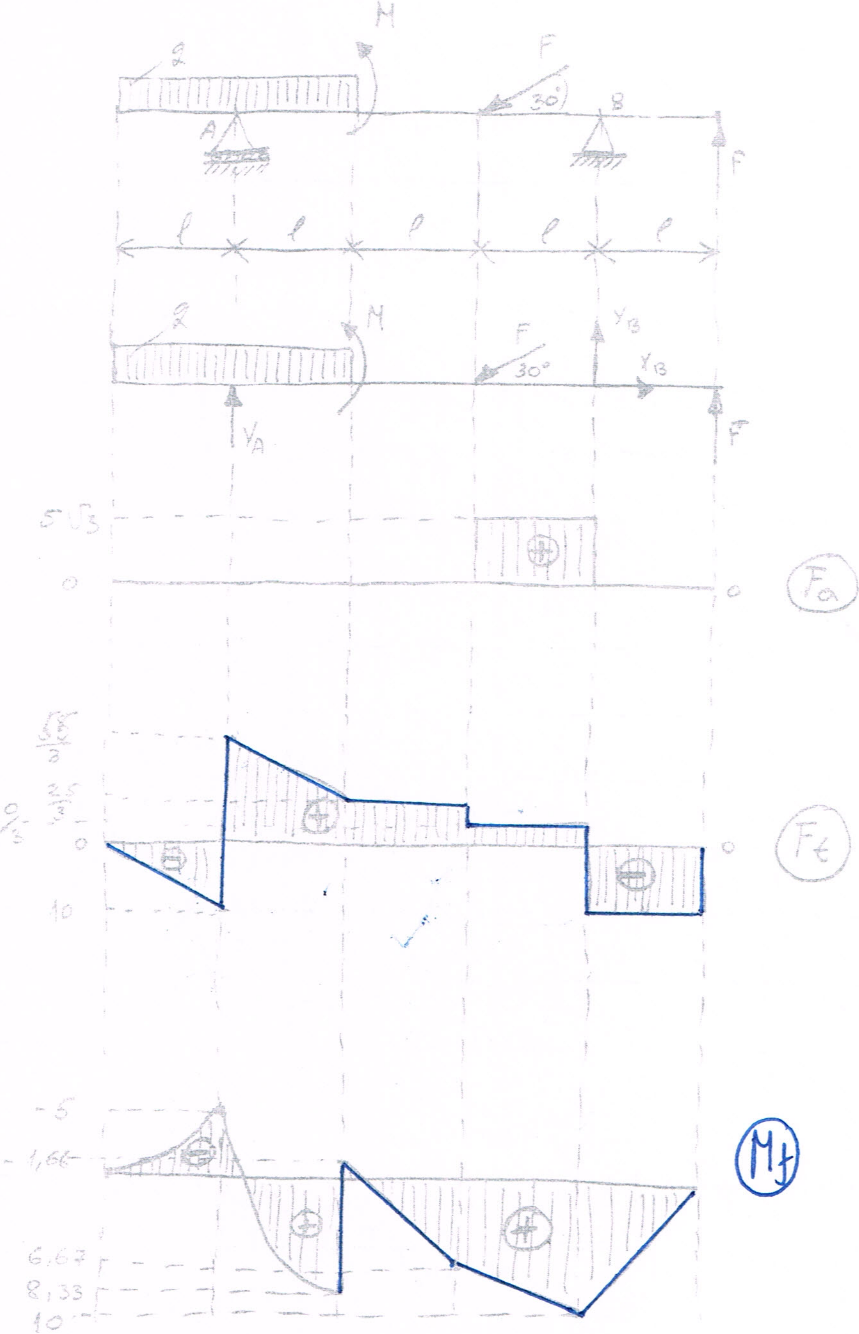
(2) $\Rightarrow \quad Y_A = F_2 \cdot \frac{\sqrt{2}}{2} + F_1$

$\boxed{Y_A = 60\,kN}$

(3) $\Rightarrow \quad M_A = F_1 \cdot 2 + F_2 \cdot \frac{\sqrt{2}}{2}$

$\boxed{M_A = 80\,kN}$

4. Za gredu prikazanu na slici odrediti reakcije veza i nacrtati statičke dijagrame, ako je $l = 1m$; $F = 10kN$; $M = F \cdot l$; $q = F/l$



$\Sigma F_x = 0;$

$-F \cdot \cos 30° + X_B = 0$

$\boxed{X_B = 5\sqrt{3}}$

$\Sigma F_y = 0;$

$-q \cdot 2l + Y_A - F \cdot \sin 30° + Y_B + F = 0$

$Y_A + Y_B = F \cdot \sin 30° - F + 2F$

$Y_A + Y_B = \frac{3F}{2}$

$Y_A + Y_B = \frac{3 \cdot 10}{2}$; $Y_A + Y_B = 15$

$\Sigma M_A = 0;$

$F \cdot 4l + Y_B \cdot 3l - F \cdot \sin 30° \cdot 2l + M = 0;$

$3 \cdot Y_B = F \cdot \sin 30° \cdot 2 - M - F \cdot 4$

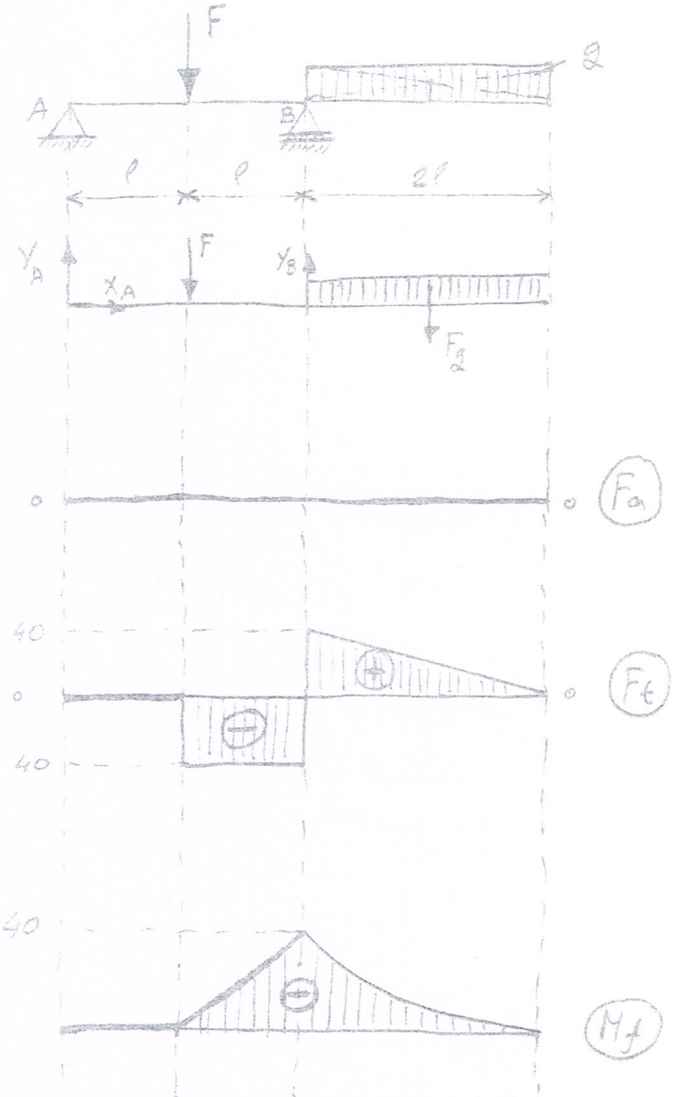
$Y_B = \frac{1}{3}\left(10 \cdot \frac{1}{2} \cdot 2 - 10 - 10 \cdot 4\right)$

$\boxed{Y_B = -\frac{40}{3}}$; $\boxed{Y_A = \frac{85}{3}}$

3. Za gredu opterećenu kao na slici:

- odrediti reakcije oslonaca i nacrtati statičke dijagrame

Podaci: $q = 20\ \frac{kN}{m}$; $F = 40\ kN$



$F_q = q \cdot l$

$F_q = 20\ \frac{kN}{m} \cdot 2m$

$\boxed{F_q = 40\ kN}$

(1) $\sum F_x = 0$; $\boxed{X_A = 0}$

(2) $\sum M_A = 0$;

$F_q \cdot 3l - Y_B \cdot 2l + F \cdot l = 0 \quad /:l$

$3 \cdot F_q - 2 \cdot Y_B + F = 0$

$2 \cdot Y_B = 3 F_q + F$

$Y_B = \frac{1}{2}(3 F_q + F)$

$\boxed{Y_B = 80\ kN}$

(3) $\sum F_y = 0$;

$Y_A + Y_B - F - F_q = 0$

$Y_A + Y_B = F + F_q$

$Y_A + Y_B = 80\ kN$

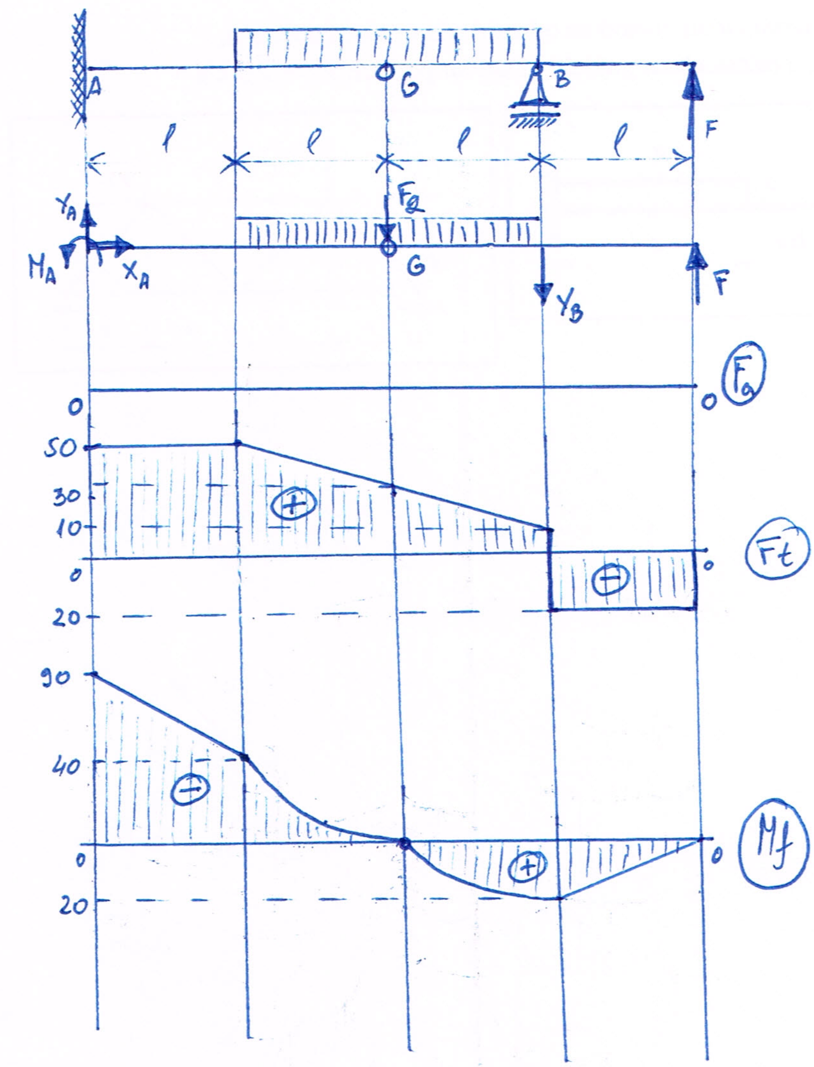
$Y_A = 80 - Y_B$

$\boxed{Y_A = 0}$

Tehnička mehanika I, Drumski Saobraćaj

VII čas vježbi, 07.11.2017

1. Za konzolu prikazanu na slici odrediti reakcije oslonaca i nacrtati statičke dijagrame. Dato je: $F = 20\,kN$; $l = 1\,m$; $q = \frac{F}{l}$



$$F_q = q \cdot 2l = \frac{F}{l} \cdot 2l = 2F$$

$\sum F_x = 0$;

$\sum F_y = 0$; $F - Y_B + Y_A - F_q = 0$

$$Y_A - Y_B = F$$

$\sum M_G^D = 0$;

$$F \cdot 2l - Y_B \cdot l - q \cdot l \cdot \frac{l}{2} = 0$$

$$2F - Y_B - \frac{F}{2} = 0$$

$$Y_B = \frac{3F}{2}; \quad \boxed{Y_B = 30\,kN}$$

$$\boxed{Y_A = 50\,kN}$$

$\sum M_A = 0$;

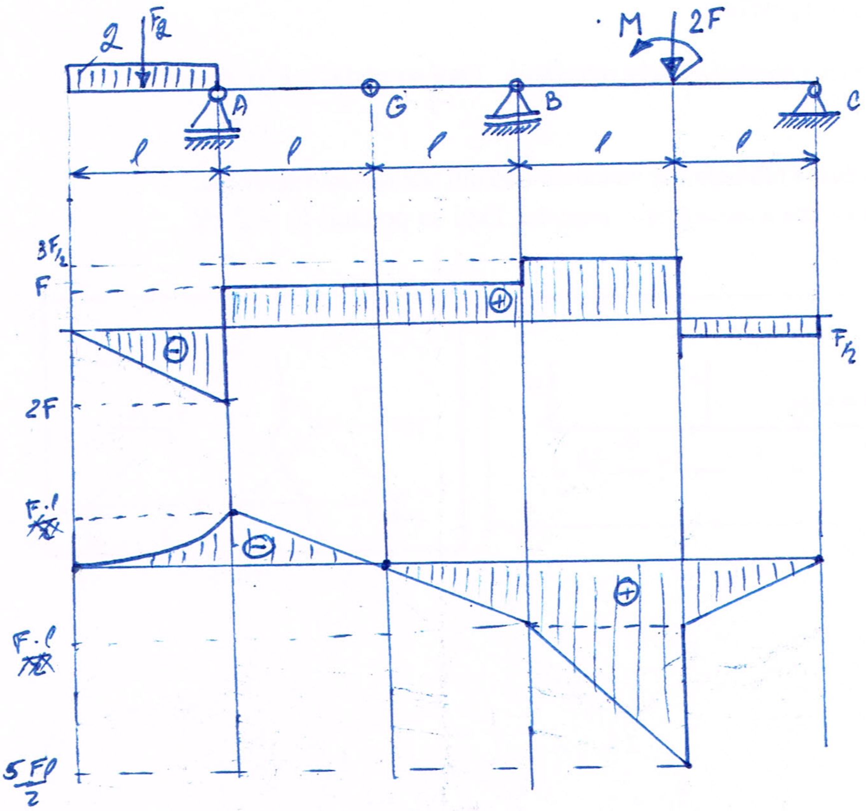
$$F \cdot 4l - Y_B \cdot 3l - F_q \cdot 2l + M_A = 0$$

$$M_A = \cancel{4Fl} + \frac{9Fl}{2} - \cancel{4Fl}$$

$$\boxed{M_A = 90\,kNm}$$

2. Za gredu prikazanu na slici odrediti reakcije oslonaca i nacrtati statičke dijagrame. Dato je: F, l, $M = 2F\cdot l$; $q = \frac{2F}{l}$

$F_q = q\cdot l = 2F$



$\Sigma M_G^l = 0;$

$F_q\cdot\frac{3l}{2} - Y_A\cdot l = 0$

$Y_A = F_q\cdot\frac{3}{2};\quad \boxed{Y_A = 3F}$

$\Sigma M_C = 0;$

$-M + Y_A\cdot 4l + Y_B\cdot 2l - 2F\cdot l - F_q\cdot\frac{9l}{2} = 0$

$2\cdot Y_B = -4\cdot 3F + 2F + 9F + 2F$

$\boxed{Y_B = +\frac{F}{2}}$

$\Sigma F_y = 0;$

$Y_A + Y_B + Y_C - 2F - F_q = 0$

$Y_C = 2F + 2F - 3F - \frac{F}{2}$

$\boxed{Y_C = \frac{F}{2}}$

3. Za okvirni nosač (kam) prikazan i opterećen kao na slici, izračunati reakcije u osloncima i nacrtati statičke dijagrame.

$$\sum M_G^d = 0; \quad R_{By} \cdot 2l - 2Fl = 0$$

$$\boxed{R_{By} = F}$$

$$\sum F_y = 0; \quad R_{Ay} - R_B = 0$$

$$\boxed{R_{Ay} = F}$$

$$\sum F_x = 0; \quad \boxed{R_{Ax} = 0}$$

$$R_{Ax} - F_q = 0$$

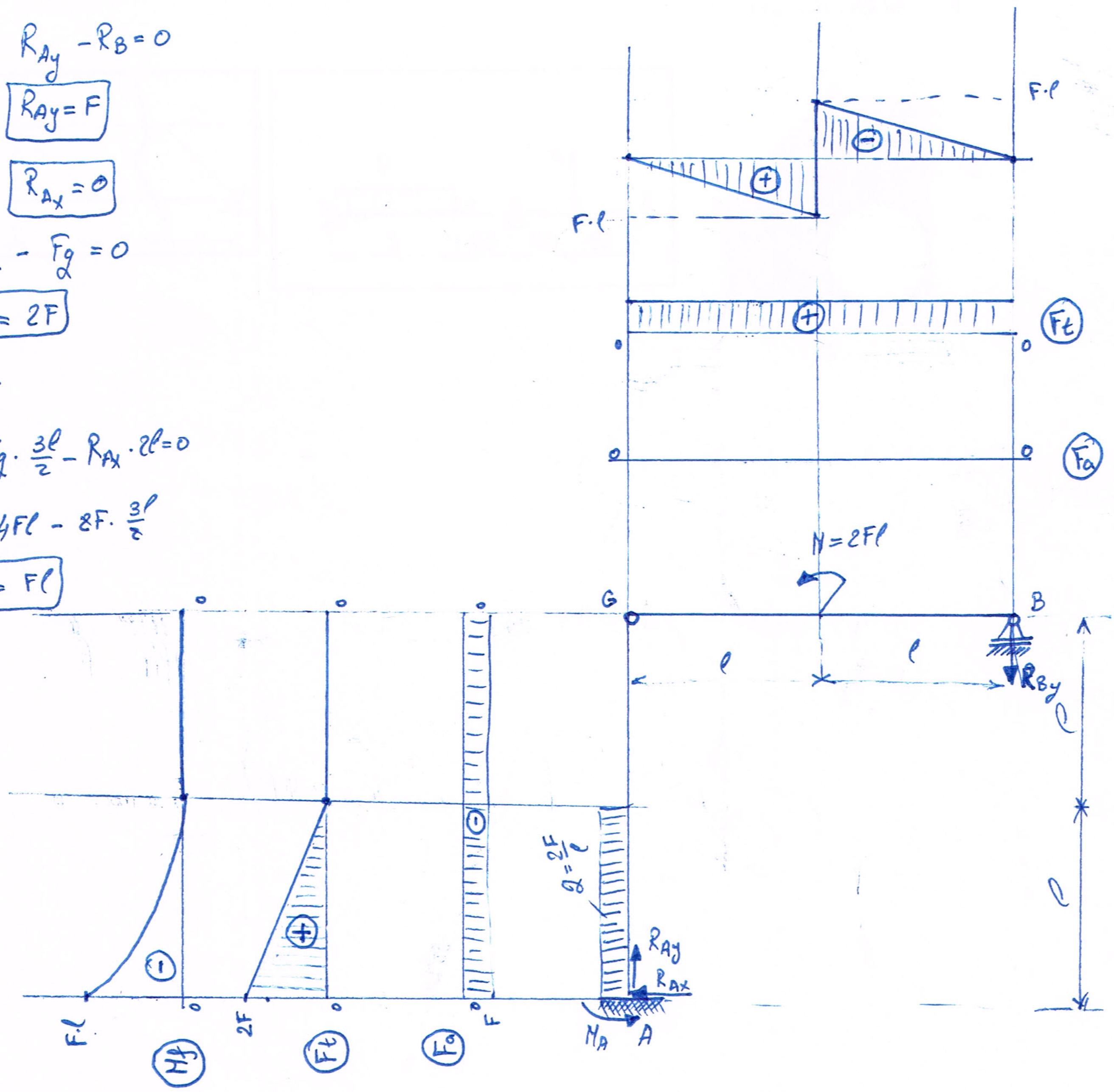
$$\boxed{R_{Ax} = 2F}$$

$$\sum M_G^l = 0;$$

$$M_A + F_q \cdot \frac{3l}{2} - R_{Ax} \cdot 2l = 0$$

$$M_A = 4Fl - 8F \cdot \frac{3l}{2}$$

$$\boxed{M_A = Fl}$$



4. Za ram prikazan na slici, odrediti reakcije oslonaca i nacrtati statičke dijagrame

$\Sigma F_x = 0;$

$R_{Bx} - F = 0$

$\boxed{R_{Bx} = F}$

$\Sigma M_B = 0;$

$R_A \cdot l - F \cdot l - F \cdot l - \frac{2F}{l} \cdot l \cdot \frac{l}{2} = 0$

$\boxed{R_A = 3F}$

$\Sigma F_y = 0;$

$R_A - 2F - R_{By} = 0$

$\boxed{R_{By} = F}$