An inverse result for Wang's theorem on extremal trees

Ivan Damnjanović
Faculty of Electronic Engineering, University of Niš (Niš, Serbia)
Diffine LLC (San Diego, California, USA)

Žarko Ranđelović
Centre for Mathematical Sciences, University of Cambridge (Cambridge, UK)

ivan.damnjanovic@elfak.ni.ac.rs

Abstract

It was recently noted by Damnjanović et al. [MATCH Commun. Math. Comput. Chem. **90** (2023), 197–202] that the problem of finding a tree which minimizes or maximizes the Sombor index among all the trees with a given degree sequence fits within the framework of results by Hua Wang from [Cent. Eur. J. Math. **12** (2014), 1656–1663]. Here, we extend these results by providing an inverse for the aforementioned theorem by Wang. In other words, for any fixed symmetric function f satisfying a monotonicity condition that

$$f(x,a)+f(y,b)>f(y,a)+f(x,b) \quad \text{for any } x>y \text{ and } a>b,$$

we characterize precisely the set of all the trees minimizing or maximizing the sum $f(\deg x, \deg y)$ over all the adjacent pairs of vertices x and y, among the trees with a given degree sequence.

Keywords

tree, degree sequence, adjacent vertices, graph invariant, algorithm, extremal problem