

Vježbe 2

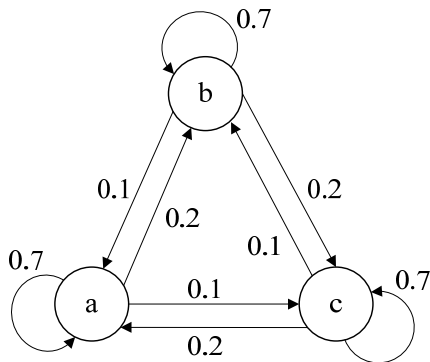
1. Dat je Markovljev izvor I reda sa alfabetom $X=\{a,b,c\}$ i uslovnim vjerovatnoćama datim tabelom:

$$\begin{bmatrix} 0.7 & 0.1 & 0.2 \\ 0.2 & 0.7 & 0.1 \\ 0.1 & 0.2 & 0.7 \end{bmatrix}$$

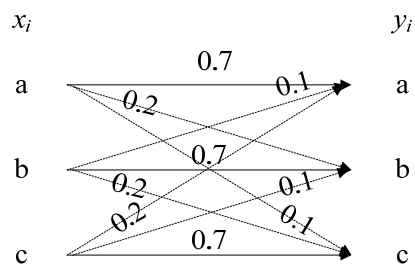
- Prikazati graf ovog sistema;
- Prikazati trellis ovog sistema;
- Prikazati vjerovatnoće simbola u stacionarnom stanju.

Rješenje:

a)



b)



c)

$$\begin{bmatrix} p'(a) \\ p'(b) \\ p'(c) \end{bmatrix} = \begin{bmatrix} 0.7 & 0.1 & 0.2 \\ 0.2 & 0.7 & 0.1 \\ 0.1 & 0.2 & 0.7 \end{bmatrix} \begin{bmatrix} p'(a) \\ p'(b) \\ p'(c) \end{bmatrix}$$

$$0.7p'(a) + 0.1p'(b) + 0.2p'(c) = p'(a)$$

$$0.2p'(a) + 0.7p'(b) + 0.1p'(c) = p'(b)$$

$$0.1p'(a) + 0.2p'(b) + 0.7p'(c) = p'(c)$$

$$-0.3p'(a) + 0.1p'(b) + 0.2p'(c) = 0$$

$$0.2p'(a) - 0.3p'(b) + 0.1p'(c) = 0 \quad / \times (-2)$$

$$0.1p'(a) + 0.2p'(b) - 0.3p'(c) = 0$$

Sabiranjem prve i druge jednačine dobijamo:

$$-0.7p'(a) + 0.7p'(b) = 0 \Rightarrow p'(a) = p'(b)$$

Iz treće jednačine dobijamo:

$$0.3p'(c) = 0.1p'(a) + 0.2p'(b) \Rightarrow p'(c) = p'(a) \Rightarrow p'(a) = p'(b) = p'(c)$$

Imajući u vidu da je:

$$p'(a) + p'(b) + p'(c) = 1$$

lako zaključujemo da je:

$$p'(a) = p'(b) = p'(c) = \frac{1}{3}$$

2. Pretpostaviti da su vjerovatnoće pojavljivanja simbola u prethodnom zadatku $p(a)=p(b)=2p(c)$.

- Odrediti vjerovatnoće pojave pojedinih simbola na izlazu iz kanala,
- Odrediti entropije ulaznog i izlaznog skupa simbola, $H(X)$ i $H(Y)$.

Rješenje:

a) Vjerovatnoće na ulazu su:

$$p(a) = p(b) = 2p(c)$$

$$p(a) + p(b) + p(c) = 1$$

$$5p(c) = 1 \Rightarrow p(c) = 0.2$$

$$\Rightarrow p(a) = p(b) = 0.4$$

Imajući u vidu da je:

$$\begin{bmatrix} 0.7 & 0.1 & 0.2 \\ 0.2 & 0.7 & 0.1 \\ 0.1 & 0.2 & 0.7 \end{bmatrix} = \begin{bmatrix} p(y_1|x_1) & p(y_1|x_2) & p(y_1|x_3) \\ p(y_2|x_1) & p(y_2|x_2) & p(y_2|x_3) \\ p(y_3|x_1) & p(y_3|x_2) & p(y_3|x_3) \end{bmatrix}$$

možemo računati:

$$\begin{aligned} [p(x_i, y_j)] &= [p(x_i)p(y_j|x_i)] = \begin{bmatrix} p(x_1)p(y_1|x_1) & p(x_1)p(y_2|x_1) & p(x_1)p(y_3|x_1) \\ p(x_2)p(y_1|x_2) & p(x_2)p(y_2|x_2) & p(x_2)p(y_3|x_2) \\ p(x_3)p(y_1|x_3) & p(x_3)p(y_2|x_3) & p(x_3)p(y_3|x_3) \end{bmatrix} = \\ &= \begin{bmatrix} 0.4 \times 0.7 & 0.4 \times 0.2 & 0.4 \times 0.1 \\ 0.4 \times 0.1 & 0.4 \times 0.7 & 0.4 \times 0.2 \\ 0.2 \times 0.2 & 0.2 \times 0.1 & 0.2 \times 0.7 \end{bmatrix} = \begin{bmatrix} 0.28 & 0.08 & 0.04 \\ 0.04 & 0.28 & 0.08 \\ 0.04 & 0.02 & 0.14 \end{bmatrix} \end{aligned}$$

Pošto je:

$$p(y_j) = \sum_{i=1}^N p(x_i)p(y_j|x_i),$$

to su vjerovatnoće na izlazu: $p(y_1)=0.36$; $p(y_2)=0.38$ i $p(y_3)=0.26$;

$$b) H(X) = -\sum_{i=1}^N p(x_i) \log p(x_i) = -[0.4 \times \log 0.4 + 0.4 \times \log 0.4 + 0.2 \times \log 0.2] = 1.52 \text{ bit / simbol}$$

$$H(Y) = -\sum_{i=1}^N p(y_i) \log p(y_i) = -[0.36 \times \log 0.36 + 0.38 \times \log 0.38 + 0.26 \times \log 0.26] = 1.56 \text{ bit / simbol}$$