# Неодређени интеграли вјежбе 13.02.20

$$\int f(x) = F(x) + c$$

F - примитивна функција функције f

$F'(x) = f(x)$

* $\int c \cdot f(x) \cdot dx = c \cdot \int f(x) \cdot dx$
* $\int (f(x) \pm g(x))\, dx = \int f(x) \cdot dx \pm \int g(x) \cdot dx$

## Таблица основних интеграла

- $\int dx = x + c$
- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$
- $\int \frac{1}{x} \cdot dx = \ln |x| + c$
- $\int e^x \cdot dx = e^x + c$
- $\int a^x \cdot dx = \frac{a^x}{\ln a} + c$
- $\int \sin x \cdot dx = -\cos x + c$
- $\int \cos x \cdot dx = \sin x + c$
- $\int \frac{dx}{\cos^2 x} = \operatorname{tg} x + c$
- $\int \frac{dx}{\sin^2 x} = -\operatorname{ctg} x + c$
- $\int \frac{dx}{1+x^2} = \operatorname{arctg} x + c$
- $\int \frac{dx}{1-x^2} = \frac{1}{2} \ln \left| \frac{1-x}{1+x} \right|$

$$\int \frac{dx}{\sqrt{1-x^2}} = \arcsin x + c$$

$$\int \frac{dx}{\sqrt{x^2 \pm 1}} = \ln\left|x + \sqrt{x^2 \pm 1}\right| + c$$

Декомпозиција

1) $$\int (1-x)(1-2x)(1-3x) \cdot dx =$$

$$\int (1-3x+2x^2)(1-3x)\,dx =$$

$$\int (1-3x-3x+9x^2+2x^2-6x^3) \cdot dx$$

$$\int dx - 6\int x \cdot dx + 11\int x^2 \cdot dx - 6\int x^3 \cdot dx$$

$$= x - 6 \cdot \frac{x^2}{2} + 11 \cdot \frac{x^3}{3} - 6 \cdot \frac{x^4}{4} + C$$

2) $$\int \frac{\sqrt{x} - 2\sqrt[3]{x^2} + 1}{\sqrt[4]{x}} \cdot dx = \int \left( \frac{x^{\frac{1}{2}}}{x^{\frac{1}{4}}} - 2 \cdot \frac{x^{\frac{2}{3}}}{x^{\frac{1}{4}}} + \frac{1}{x^{\frac{1}{4}}} \right) \cdot dx$$

$$= \int x^{\frac{1}{4}} dx - 2\int x^{\frac{5}{12}} dx + \int x^{-\frac{1}{4}} \cdot dx =$$

$$= \frac{x^{\frac{5}{4}}}{\frac{5}{4}} - 2 \cdot \frac{x^{\frac{17}{12}}}{\frac{17}{12}} + \frac{x^{\frac{3}{4}}}{\frac{3}{4}} + C$$

$$= \frac{4}{5} \cdot \sqrt[4]{x^5} - \frac{24}{17} \cdot \sqrt[12]{x^{17}} + \frac{4}{3} \cdot \sqrt[4]{x^3} + C$$

3) $\int\left(3\sqrt{x}+\frac{2}{x}\right)^2\cdot dx=\int\left(9x+24\cdot\frac{\sqrt{x}}{x}+\frac{4}{x^2}\right)\cdot dx=$

$=\int 9x\cdot dx+24\int x^{-\frac{1}{2}}dx+4\cdot\int x^{-2}\cdot dx=$

$=9\cdot\frac{x^2}{2}+24\cdot 2\cdot\sqrt{x}+4\cdot\left(-\frac{1}{x}\right)+C$

4) $\int\left(\frac{1}{\sqrt[3]{x^5}}+\sin x\right)dx=\int x^{-\frac{5}{3}}dx+\int\sin x\cdot dx=$

$=-\frac{3}{2}\cdot\frac{1}{\sqrt[3]{x^2}}+(-\cos x)+C$

5) $\int\frac{x^2}{1+x^2}=\int\frac{1+x^2-1}{1+x^2}dx=\int\left(1-\frac{1}{1+x^2}\right)\cdot dx=$

$=\int dx-\int\frac{dx}{1+x^2}=x-\operatorname{arctg}x+C$

6) $\int\frac{x^3+x-2}{x^2+1}\cdot dx=\int\left(x-\frac{2}{x^2+1}\right)\cdot dx=\int x\,dx-2\int\frac{dx}{x^2+1}$

$=\frac{x^2}{2}-2\operatorname{arctg}x+C$

$(x^3+x-2):(x^2+1)=x$

$\underline{x^3\mp x}$

$-2$

$x^3+x-2=x(x^2+1)-2$

$\frac{x^3+x-2}{x^2+1}=x-\frac{2}{x^2+1}$

7) $\int\frac{2^{x+1}-5^{x-1}}{10^x}\cdot dx=\int\frac{2\cdot 2^x}{10^x}\cdot dx-\int\frac{5^x}{5\cdot 10^x}\cdot dx=$

$=2\int\left(\frac{2}{10}\right)^x dx-\frac{1}{5}\int\left(\frac{5}{10}\right)^x\cdot dx=2\int\left(\frac{1}{5}\right)^x\cdot dx-\frac{1}{5}\int\left(\frac{1}{2}\right)^x\cdot dx$

$2\cdot\frac{\left(\frac{1}{5}\right)^x}{\ln\frac{1}{5}}-\frac{1}{5}\cdot\frac{\left(\frac{1}{2}\right)^x}{\ln\frac{1}{2}}+C=$

$=2\cdot\frac{\left(\frac{1}{5}\right)^x}{\ln 5^{-1}}-\frac{1}{5}\cdot\frac{\left(\frac{1}{2}\right)^x}{\ln 2^{-1}}+C=$

$=-2\frac{\left(\frac{1}{5}\right)^x}{\ln 5}+\frac{1}{5}\frac{\left(\frac{1}{2}\right)^x}{\ln 2}$

Метода смјене

8.) $\int tg^2 x\, dx = \int \frac{\sin^2 x}{\cos^2 x} \cdot dx = \int \frac{1-\cos^2 x}{\cos^2 x} \cdot dx =$

$= \int \frac{1}{\cos^2 x} \cdot dx - \int dx = tg x - x + c$

1) $\int \frac{x}{x^2+a}\, dx = \left[ \begin{array}{l} x^2+a=t \\ 2x \cdot dx = dt \\ x\,dx = \frac{1}{2} dt \end{array} \right.$

$= \frac{1}{2} \int \frac{dt}{t} = \frac{1}{2} \ln |t| + c =$

$= \frac{1}{2} \ln |x^2+a| + c$

2.) $\int \sin 2x\, dx = \left[ \begin{array}{l} 2x = t \\ 2dx = dt \\ dx = \frac{1}{2} dt \end{array} \right.$

$\frac{1}{2} \int \sin t\, dt =$

$\frac{1}{2}(-\cos t) + c = -\frac{1}{2} \cos 2x + c$

3.) $\int \frac{dx}{3-x} = \left[ \begin{array}{l} 3-x=t \\ -dx=dt \\ dx=-dt \end{array} \right] = -\int \frac{dt}{t} =$

$= \ln|t| + C = -\ln|3-x| + C$

$1-\frac{1}{5} = \frac{4}{5}$

4) $\int \frac{5}{(x-4)^4}\,dx = \left[\begin{array}{l} x-4=t \\ dx=dt \end{array}\right]$

$= 5\int \frac{dt}{t^4} = 5\cdot\int t^{-4}\,dt = 5\cdot\frac{t^{-3}}{-3} + C =$

$= -\frac{5}{3}\cdot\frac{1}{t^3} + C = -\frac{5}{3}\cdot\frac{1}{(x-4)^3} + C$

5) $\int \frac{x+5}{\sqrt[5]{1-2x}}\,dx = \int \left[\begin{array}{l} 1-2x=t \\ -2dx=dt \\ dx=-\frac{1}{2}dt \end{array}\right]$ $\quad x = \frac{1-t}{2}$

$= \frac{1}{2}\int \frac{\frac{1-t}{2}+5}{\sqrt[5]{t}}\cdot dt = \frac{1}{2}\int \frac{11-t}{2\cdot\sqrt[5]{t}}\cdot dt =$ $\quad -\frac{1}{5}$

$= \frac{11}{4}\int t^{-\frac{1}{5}}\,dt + \frac{1}{4}\int t^{\frac{4}{5}}\,dt =$

$= \frac{11}{4}\cdot\frac{5}{4}\cdot\sqrt[5]{t^4} + \frac{1}{4}\cdot\frac{5}{9}\cdot\sqrt[5]{t^9} + C$ $\quad \frac{3}{2}x^2 = t^2$

$\sqrt{\frac{3}{2}}x = t$

$= \frac{55}{16}\cdot\sqrt[5]{(1-2x)^4} + \frac{5}{36}\cdot\sqrt[5]{(1-2x)^9} + C$

6) $\int \frac{dx}{2+3x^2} = \int \frac{dx}{2\left(1+\frac{3}{2}x^2\right)} = \frac{1}{2}\int \frac{dx}{1+\frac{3}{2}x^2} = \left[\begin{array}{l} \sqrt{\frac{3}{2}}\,x=t \\ \sqrt{\frac{3}{2}}\,dx=dt \\ dx=\sqrt{\frac{2}{3}}\,dt \end{array}\right]$

I

$\frac{1}{2}\cdot\sqrt{\frac{2}{3}}\cdot\int \frac{dt}{1+t^2} = \frac{1}{\sqrt{6}}\cdot \operatorname{arctg} t + C =$

$= \frac{1}{\sqrt{6}}\cdot \operatorname{arctg}\sqrt{\frac{3}{2}}\,x + C$

$$I \quad \int \frac{dx}{2+3x^2} = \left\lceil \begin{array}{l} \sqrt{3}x = \sqrt{2}t \\ dx = \sqrt{\frac{2}{3}}\,dt \end{array} \right\rfloor =$$

$$* \quad 3x^2 = 2t^2 \qquad \sqrt{3x} = \sqrt{2}t \qquad \sqrt{\frac{3}{2}}x = t$$

$$\sqrt{\frac{2}{3}} \int \frac{dt}{2+2t^2} = \frac{1}{\sqrt{6}} \int \frac{dt}{1+t^2} =$$

$$= \frac{1}{\sqrt{6}} \cdot \operatorname{arctg} t + C = \frac{1}{\sqrt{6}} \cdot \operatorname{arctg}\left(\sqrt{\frac{3}{2}}x\right) + C$$

7) $$\int \frac{dx}{\sqrt{3-5x^2}} = \int \frac{dx}{\sqrt{3\left(1-\frac{5}{3}x^2\right)}} =$$

$$\int \frac{dx}{\sqrt{1-x^2}} = \arcsin x \qquad \int \frac{dt}{\sqrt{1-t^2}} = \arcsin t$$

$$= \frac{1}{\sqrt{3}} \int \frac{dx}{\sqrt{1-\frac{5}{3}x^2}} = \left\lceil \begin{array}{l} \sqrt{\frac{5}{3}}x = t \\ dx = \sqrt{\frac{3}{5}}\,dt \end{array} \right\rfloor =$$

$$= \frac{1}{\sqrt{3}} \int \frac{\sqrt{\frac{3}{5}} \cdot dt}{\sqrt{1-t^2}} = \frac{1}{\sqrt{5}} \int \frac{dt}{\sqrt{1-t^2}} =$$

$$= \frac{1}{\sqrt{5}} \cdot \arcsin t + C = \frac{1}{\sqrt{5}} \arcsin\left(\sqrt{\frac{5}{3}}x\right) + C$$

8) $$\int \operatorname{ctg} x\,dx = \int \frac{\cos x}{\sin x} \cdot dx = \left\lceil \begin{array}{l} \sin x = t \\ \cos x\,dx = dt \end{array} \right.$$

$$= \int \frac{dt}{t} = \ln|t| + C = \ln|\sin x| + C$$

**9.**

$$\int \frac{dx}{1+\sin x} = \int \frac{dx}{1+\cos\left(\frac{\pi}{2}-x\right)} = \left[\begin{array}{l} \frac{\pi}{2}-x=t \\ -dx=dt \\ dx=-dt \end{array}\right] =$$

$$= -\int \frac{dt}{1+\cos t} = -\int \frac{dt}{1+\cos^2\frac{t}{2}-\sin^2\frac{t}{2}} =$$

?

$$= -\frac{1}{2}\int \frac{dt}{\cos^2\frac{t}{2}} = \left[\begin{array}{l} \frac{t}{2}=z \\ \frac{1}{2}dt=dz \\ dt=2\cdot dz \end{array}\right]$$

$$= -\frac{1}{2}\cdot 2\int \frac{dz}{\cos^2 z} =$$

$$= -\frac{1}{2}\cdot 2\,tg\,z + C = -\frac{1}{2}\cdot 2\,tg\,\frac{t}{2} + C =$$

$$= -tg\left(\frac{\pi}{4}-\frac{x}{2}\right)+C$$

**10.**

$$\int \frac{dx}{\sin^2x\cos^2x} = \int \frac{dx}{(\sin x\cdot\cos x)^2} = \int \frac{dx}{\left(\underbrace{2\cdot\sin x\cdot\cos x}_{\sin 2x}\cdot\frac{1}{2}\right)^2} =$$

$$4\cdot\int \frac{dx}{(\sin 2x)^2} = 4\cdot\int\frac{dx}{\sin^2 2x} = 4\cdot\frac{1}{2}\cdot(-ctg\,2x)+C =$$

$$= -4\cdot\frac{1}{2}\cdot ctg\,2x + C = -2\,ctg\,2x + C$$

оценка

$$2x=t$$
$$-2dx=dt$$
$$dx=-\frac{dt}{2}$$

**11.**

$$\int \frac{\ln^2 x}{x}\cdot dx = \left[\begin{array}{l} \ln x=t \\ \frac{1}{x}dx=dt \end{array}\right] =$$

$$= \int t^2\cdot dt = \frac{t^3}{3}+C = \frac{\ln^3 x}{3}+C$$

$\frac{1}{5}$

12) $\int \frac{\cos x}{\sqrt[3]{\sin^2 x}} \cdot dx = \left[ \begin{array}{l} \sin x = t \\ \cos x\, dx = dt \end{array} \right] =$

$\int \frac{dt}{\sqrt[3]{t^2}} = \int t^{-\frac{2}{3}} dt = 3 \cdot t^{\frac{1}{3}} + C =$

$= 3 \cdot \sqrt[3]{\sin x} + C$

13) $\int \sqrt{\frac{\ln(x+\sqrt{1+x^2})}{1+x^2}} \cdot dx = \left[ \begin{array}{l} \ln(x+\sqrt{1+x^2}) = t \\ \frac{1}{x+\sqrt{1+x^2}} \left(1 + \frac{1}{2\sqrt{1+x^2}} \cdot 2x\right) dx = \\ \frac{1}{\cancel{x+\sqrt{1+x^2}}} \cdot \frac{\cancel{\sqrt{1+x^2}+x}}{\sqrt{1+x^2}}\, dx = dt \\ \frac{dx}{\sqrt{1+x^2}} = dt \end{array} \right]$

$\int \sqrt{t}\, dt = \int t^{\frac{1}{2}} \cdot dt =$

$= \frac{2}{3} \cdot \sqrt{t^3} + C = \frac{2}{3} \sqrt{\ln^3(x+\sqrt{1+x^2})} + C$

14) $\int \frac{e^{\arcsin x} + x + 1}{\sqrt{1-x^2}}\, dx =$

$= \underbrace{\int \frac{e^{\arcsin x}}{\sqrt{1-x^2}}\, dx}_{I} + \underbrace{\int \frac{x}{\sqrt{1-x^2}}\, dx}_{II} + \int \frac{1}{\sqrt{1-x^2}}\, dx =$

$= \left[ \begin{array}{l} \arcsin x = t \\ \frac{dx}{\sqrt{1-x^2}} = dt \end{array} \right] \quad \left[ \begin{array}{l} 1-x^2 = z \\ -2x\, dx = dz \\ x\, dx = -\frac{1}{2} dz \end{array} \right]$

$$= \int e^{t}\,dt - \frac{1}{2}\int \frac{dz}{\sqrt{z}} + \arcsin x =$$

$$= e^{t} - \frac{1}{2}\cdot 2\cdot \sqrt{z} + \arcsin x + c =$$

$$= e^{\arcsin x} - \frac{1}{2}\cdot 2\cdot \sqrt{1-x^2} + \arcsin x + c$$

$$= e^{\arcsin x} - \sqrt{1-x^2} + \arcsin x + c$$

## Парцијална интеграција

$$\int u\cdot dv = u\cdot v - \int v\cdot du \quad !$$

1) $\int x\cdot e^{x}\cdot dx =$ $\left[\begin{array}{l} u = x \Rightarrow du = dx \\ dv = e^{x}\cdot dx \\ v = \int e^{x}dx = e^{x} \end{array}\right]$

$$x\cdot e^{x} - \int e^{x}\cdot dx = x\cdot e^{x} - e^{x} + c =$$
$$= e^{x}(x-1) + c$$

2) $\int x\cdot \sin x\cdot dx =$ $\left[\begin{array}{l} u = x \Rightarrow du = dx \\ dv = \sin x\cdot dx \Rightarrow v = \int \sin x\,dx = \\ = -\cos x \end{array}\right]$

$$= -x\cdot \cos x - \int(-\cos x)\cdot dx =$$

$-x \cdot \cos x + \int \cos x\, dx =$

$= -x \cdot \cos x + \sin x + C$

3) $\int \ln x\, dx =$ $\left[ \begin{array}{l} u = \ln x \Rightarrow du = \frac{1}{x} dx \\ dV = dx \Rightarrow V = \int dx = x \end{array} \right]$

$= x \cdot \ln x - \int x \cdot \frac{1}{x} dx = x \cdot \ln x - \int dx =$

$= x \cdot \ln x - x + C = x(\ln x - 1) + C$

4) $\int x^2 \cdot \sin 2x\, dx =$ $\left[ \begin{array}{l} u = x^2 \Rightarrow du = 2x\, dx \\ dV = \sin 2x\, dx \Rightarrow V = \int \sin 2x\, dx = \\ = -\frac{1}{2} \cos 2x \end{array} \right]$

$= -\frac{x^2}{2} \cdot \cos 2x + \int x \cdot \cos 2x\, dx =$

$= \left[ \begin{array}{l} u = x \Rightarrow du = dx \\ dV = \cos 2x\, dx \Rightarrow V = \int \cos 2x\, dx = \frac{1}{2} \sin 2x \end{array} \right]$

$-\frac{x^2}{2} \cdot \cos 2x + \frac{x}{2} \sin 2x - \frac{1}{2} \int \sin 2x\, dx =$

$= -\frac{x^2}{2} \cdot \cos 2x + \frac{x}{2} \sin 2x - \frac{1}{2} \cdot \left(-\frac{1}{2} \cos 2x\right) + C =$

$= -\frac{x^2}{2} \cos 2x + \frac{x}{2} \sin 2x + \frac{1}{4} \cos 2x + C$

5) $\int \sqrt{x} \cdot \ln^2 x \cdot dx =$ $\left[ \begin{array}{l} u = \ln^2 x \Rightarrow du = 2 \cdot \ln x \cdot \frac{1}{x} dx \\ dV = \sqrt{x} dx \Rightarrow V = \int \sqrt{x}\, dx = \frac{2}{3} \cdot \sqrt{x^3} \end{array} \right]$

$$= \frac{2}{3} \cdot \sqrt{x^3} \cdot \ln^2 x - \frac{4}{3} \int \sqrt{x} \ln x \, dx =$$

$$= \left[ \begin{array}{l} u = \ln x \Rightarrow du = \frac{1}{x} \cdot dx \\ dV = \sqrt{x} \cdot dx \Rightarrow V = \int \sqrt{x} \cdot dx = \frac{2}{3} \cdot \sqrt{x^3} \end{array} \right]$$

$$= \frac{2}{3} \sqrt{x^3} \cdot \ln^2 x - \frac{4}{3} \left( \frac{2}{3} \sqrt{x^3} \cdot \ln x - \frac{2}{3} \int \sqrt{x} \, dx \right) =$$

$$= \frac{2}{3} \cdot \sqrt{x^3} \cdot \ln^2 x - \frac{8}{9} \sqrt{x^3} \cdot \ln x + \frac{8}{9} \int \sqrt{x} \, dx =$$

$$= \frac{2}{3} \cdot \sqrt{x^3} \cdot \ln^2 x - \frac{8}{9} \sqrt{x^3} \cdot \ln x + \frac{2}{3} \cdot \frac{8}{9} \cdot \sqrt{x^3} + C$$

6) $$\int \frac{x \cdot \cos x}{\sin^3 x} dx = \left[ \begin{array}{l} u = x \Rightarrow du = dx \\ dV = \int \frac{\cos x}{\sin^3 x} dx = \left[ \begin{array}{l} \sin x = t \\ \cos x dx = dt \end{array} \right] \\ = \int \frac{dt}{t^3} = -\frac{1}{2} \cdot \frac{1}{t^2} = -\frac{1}{2} \cdot \frac{1}{\sin^2 x} \end{array} \right]$$

$$= -\frac{x}{2} \cdot \frac{1}{\sin^2 x} + \frac{1}{2} \int \frac{1}{\sin^2 x} dx =$$

$$= -\frac{x}{2} \cdot \frac{1}{\sin^2 x} + \frac{1}{2} \cdot (-\operatorname{ctg} x) + C$$

Знак-!

7) $$\int \operatorname{arctg} x \cdot dx = \left[ \begin{array}{l} u = \operatorname{arctg} x \Rightarrow du = \frac{1}{1+x^2} dx \\ dV = dx \Rightarrow V = \int dx = x \end{array} \right]$$

$$= x \cdot \operatorname{arctg} x - \int \frac{x}{1+x^2} dx = \left[ \begin{array}{l} 1+x^2 = t \\ 2x \, dx = dt \\ x dx = \frac{1}{2} dt \end{array} \right]$$

$$= x \cdot \operatorname{arctg} x - \frac{1}{2} \int \frac{dt}{t} = x \cdot \operatorname{arctg} x - \frac{1}{2} \ln |t| + C$$

$\ln x \cdot \frac{1}{x}$ $\frac{dx}{\sqrt{x^3}}$

$x \cdot \operatorname{arctg} x - \frac{1}{2} \ln|1+x^2| + C$

8. $\int \frac{\arccos x}{\sqrt{(1-x^2)^3}} dx = \begin{bmatrix} \arccos x = t \\ -\frac{1}{\sqrt{1-x^2}} dx = dt \\ x = \cos t \end{bmatrix}$

$= \int \frac{t}{1-\cos^2 t} \cdot (-dt) = -\int \frac{t}{\sin^2 t} dt = \begin{bmatrix} u = t \Rightarrow du = dt \\ v = \int \frac{1}{\sin^2 t} dt = -\operatorname{ctg} t \end{bmatrix}$

$= -\left(-t \cdot \operatorname{ctg} t + \int \operatorname{ctg} t \, dt\right) =$

$= t \cdot \operatorname{ctg} t - \int \frac{\cos t}{\sin t} dt = \begin{bmatrix} \sin t = z \\ \cos t \, dt = dz \end{bmatrix} =$

$= t \cdot \operatorname{ctg} t - \int \frac{dz}{z} =$

$= t \cdot \operatorname{ctg} t - \ln|z| + C =$

$= \arccos x \cdot \operatorname{ctg}(\arccos x) - \ln|\sin(\arccos x)| + C$

9. $I = \int e^{ax} \cdot \sin bx \cdot dx = \begin{bmatrix} u = e^{ax} \Rightarrow du = a \cdot e^{ax} \cdot dx \\ v = \int \sin bx \, dx = -\frac{1}{b} \cdot \cos bx \end{bmatrix}$

$= -\frac{e^{ax}}{b} \cos bx + \frac{a}{b} \int e^{ax} \cdot \cos bx \cdot dx =$

$\begin{bmatrix} u = e^{ax} \Rightarrow du = a \cdot e^{ax} \cdot dx \\ v = \int \cos b \cdot x \, dx = \frac{1}{b} \sin bx \end{bmatrix}$

$$= -\frac{e^{ax}}{b}\cos bx + \frac{a}{b}\left(\frac{1}{b}e^{ax}\cdot \sin bx - \frac{a}{b}\int e^{ax}\underbrace{\cdot \sin bx \cdot dx}_{I}\right)$$

$$I = -\frac{e^{ax}}{b}\cdot \cos bx + \frac{a}{b^2}\cdot e^{ax}\sin bx - \frac{a^2}{b^2}\cdot I$$

$$\left(1+\frac{a^2}{b^2}\right)\cdot I = e^{ax}\cdot\left(\frac{a}{b^2}\cdot \sin bx - \frac{1}{b}\cdot \cos bx\right)$$

$$\left(\frac{a^2+b^2}{b^2}\cdot I\right) = e^{ax}\left(\frac{a\cdot \sin bx - b\cdot \cos bx}{b^2}\right) \Big/ b^2$$

$$I = \frac{a\cdot \sin bx - b\cos bx}{a^2+b^2}\cdot e^{ax}$$

10) $$I = \int \frac{dx}{(x^2+1)^2}$$

$$J = \int \frac{dx}{(x^2+1)} = \left\lceil u = \frac{1}{x^2+1} \Rightarrow du = -\frac{2x}{(x^2+1)^2}\cdot dx \right.$$

$$dV = dx \Rightarrow V = \int dx = x \rfloor$$

$$= \frac{x}{x^2+1} - 2\int \frac{x^2}{(x^2+1)^2}\,dx =$$

$$= \frac{x}{x^2+1} + 2\int \frac{x^2+1-1}{(x^2+1)^2}\,dx =$$

$$= \frac{x}{x^2+1} + 2\underbrace{\int \frac{dx}{x^2+1}}_{J} - 2\cdot \underbrace{\int \frac{dx}{(x^2+1)^2}}_{I}$$

$$J = \frac{x}{x^2+1} + 2J - 2I,$$

$$2I = \frac{x}{x^2+1} + J$$

$$I = \frac{1}{2} \cdot \frac{x}{x^2+1} + \frac{1}{2} \cdot \operatorname{arctg} x + C$$