Redovi sa pozitivnim članovima

8. Ispitati konvergenciju reda

a) $\sum_{n=1}^{\infty} \frac{1}{2n-1}$ $a_n = \frac{1}{2n-1}$, Neka je $b_n = \frac{1}{n}$

$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{n}{2n-1} = \frac{1}{2} > 0 \Rightarrow$ Redovi suma $\sum_{n=1}^{\infty} b_n$ i $\sum_{n=1}^{\infty} a_n$ su ekvikonvergentni

Red $\sum_{n=1}^{\infty} \frac{1}{n}$ divergira kao hiperharmonijski red za koji je $p=1$, pa divergira i red $\sum_{n=1}^{\infty} \frac{1}{2n-1}$

b) $\sum_{n=1}^{\infty} \frac{1}{n\sqrt{n+1}}$ $a_n = \frac{1}{n\sqrt{n+1}}$, Neka je $b_n = \frac{1}{n\sqrt{n}}$

$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{n\sqrt{n}}{n\sqrt{n+1}} = 1 > 0 \Rightarrow$ redovi $\sum_{n=1}^{\infty} a_n$ i b_n su ekvikonvergentni

Red $\sum_{n=1}^{\infty} b_n$ je konvergentan kao hiperharmonijski za koji je $p = \frac{3}{2} > 1$, pa konvergira i red $\sum_{n=1}^{\infty} a_n$

Ispitati konvergenciju reda $\sum_{n=2}^{\infty} \frac{\sqrt{n+2}-\sqrt{n}}{n^{\alpha}}$

$a_n = \frac{\sqrt{n+2}-\sqrt{n}}{n^{\alpha}}$ → racionališemo

$a_n = \frac{n+2-n}{n^{\alpha}(\sqrt{n+2}+\sqrt{n})} = \frac{2}{n^{\alpha}(\sqrt{n+2}+\sqrt{n})}$ Neka je $b_n = \frac{2}{n^{\alpha}\sqrt{n}}$

$\lim_{n\to\infty} \frac{a_n}{b_n} = \frac{\frac{\cancel{2}}{\cancel{n^{\alpha}}(\sqrt{n+2}+\sqrt{n})}}{\frac{\cancel{2}}{\cancel{n^{\alpha}}\sqrt{n}}} = \lim_{n\to\infty} \frac{\sqrt{n}}{\sqrt{n}\left(\sqrt{1+\frac{2}{n}}+1\right)} = \frac{1}{2} > 0$ ovi redovi su ekvikonvergentni

Red $\sum_{n=2} \frac{1}{n^{\alpha+\frac{1}{2}}}$ konvergira za $\alpha + \frac{1}{2} > 1$, $\alpha > \frac{1}{2}$ $\Rightarrow \sum_{n=2}^{\infty} a_n$ konvergira za $\alpha > \frac{1}{2}$

0. $\sum_{n=2}^{\infty} \frac{\ln n}{n}$ $a_n = \frac{\ln n}{n}$

$\frac{1}{n} < \frac{\ln n}{n}$, $\forall n \geq 3$

$b_n = \frac{1}{n}$ ⟹ Kriterijum upoređivanja

$b_n < a_n$ $\forall n \geq 3$

red $\sum_{n=2}^{\infty} b_n$ divergira kao hiperharmonijski za koji je $p=1$

1° $b_n < a_n$, $\forall n \geq 3$
2° $\sum_{n=2}^{\infty} b_n$ divergira
⟹ po poredbenom kriterijumu $\sum_{n=2}^{\infty} a_n$ divergira

11. $\sum_{n=1}^{\infty} \frac{1}{n\cdot 2^n}$ $a_n = \frac{1}{n\cdot 2^n}$ $\frac{1}{n\cdot 2^n} \leq \frac{1}{2^n}$, $\forall n \in N$

$b_n = \frac{1}{2^n}$ → Kriterijum upoređivanja

Red $\sum_{n=1}^{\infty} b_n$ je geom. red za koji je $q = \frac{1}{2}$, a kako je $|q| < 1$ to ovaj red konvergira

1° $a_n \leq b_n$, $\forall n \in N$
2° $\sum_{n=1}^{\infty} b_n$ konvergira
⟹ po kriterijumu upoređivanja ovaj red $\sum_{n=1}^{\infty} a_n$ konvergira

12. $\sum_{n=1}^{\infty} \frac{1}{3^n+n}$ $a_n = \frac{1}{3^n+n}$ $\frac{1}{3^n+n} \leq \frac{1}{3^n}$, $\forall n \in N$

13. $\sum_{n=1}^{\infty} \frac{\sin 2n}{1+2n}$ $a_n = \frac{\sin 2n}{1+2n} < \frac{1}{1+2n} < \frac{1}{2n}$

14 $\sum_{n=1}^{\infty} \frac{(n!)^2}{2^{n^2}}$

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} \begin{cases} < 1 & \text{konvergira} \\ > 1 & \text{divergira} \\ = 1 & \text{neki drugi krit.} \end{cases}$$ Dalamberov kriterijum

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{\frac{((n+1)!)^2}{2^{(n+1)^2}}}{\frac{(n!)^2}{2^{n^2}}} = \lim_{n\to\infty} \frac{\frac{(n+1)^2 \cancel{(n!)^2}}{\cancel{2^{n^2}} \cdot 2^{2n+1}}}{\frac{\cancel{(n!)^2}}{\cancel{2^{n^2}}}} = \lim_{n\to\infty} \frac{(n+1)^2}{2^{2n+1}} = 0 < 1$$

Red suma $\sum_{n=1}^{\infty} a_n$ konvergira po D.k.

15. $\frac{4}{2} + \frac{4\cdot 7}{2\cdot 6} + \frac{4\cdot 7\cdot 10}{2\cdot 6\cdot 10} + \dots \longrightarrow \sum_{n=1}^{\infty} \frac{4\cdot 7 \cdots (3n+1)}{2\cdot 6 \cdots (4n-2)}$

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{\frac{\cancel{4\cdot 7 \cdots (3n+1)}(3n+4)}{\cancel{2\cdot 6 \cdots (4n-2)}(4n+2)}}{\frac{\cancel{4\cdot 7 \cdots (3n+1)}}{\cancel{2\cdot 6 \cdots (4n-2)}}} = \lim_{n\to\infty} \frac{3n+4}{4n+2} = \frac{3}{4} < 1 \xrightarrow[\text{po D.k.}]{} \sum_{n=1}^{\infty} a_n \text{ konvergira}$$

16. $\sum_{n=1}^{\infty} \frac{n!}{n^n}$ $\quad a_n = \frac{n!}{n^n}$

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{\frac{(n+1)!}{(n+1)^{n+1}}}{\frac{n!}{n^n}} = \lim_{n\to\infty} \frac{n^n}{(n+1)^n} = \lim_{n\to\infty} \left(\frac{n}{n+1}\right)^n = \lim_{n\to\infty} \left(\frac{n+1}{n}\right)^{-n} =$$

$$= \lim_{n\to\infty} \left(\left(1+\frac{1}{n}\right)^n\right)^{-1} = e^{-1} = \frac{1}{e} < 1 \xrightarrow[\text{po D.k.}]{} \sum_{n=1}^{\infty} \frac{n!}{n^n} \text{ konvergira}$$

17. $\sum_{n=1}^{\infty} \left(\frac{n-1}{n+1}\right)^{n(n-1)}$ $\longrightarrow$ Košijev kriterijum

$$\lim_{n\to\infty} \sqrt[n]{a_n} = \lim_{n\to\infty} \left(\frac{n-1}{n+1}\right)^{n-1} = \lim_{n\to\infty} \left(1 + \frac{n-1}{n+1} - 1\right)^{n-1} = \lim_{n\to\infty} \left(1 + \frac{-2}{n+1}\right)^{\frac{n+1}{-2} \cdot \frac{-2(n-1)}{n+1}} =$$

$$= e^{\lim_{n\to\infty} \frac{-2n+2}{n+1}} = e^{-2} = \frac{1}{e^2} < 1 \xrightarrow[\text{po K.k.}]{} \sum_{n=1}^{\infty} a_n \text{ konvergira}$$

18. $\sum_{n=1}^{\infty} \left(\frac{1+\cos n}{2+\cos n}\right)^{2n-\ln n}$

$$\frac{1+\cos n}{2+\cos n} = 1 - \frac{1}{2+\cos n} \leq 1 - \frac{1}{2+1} = \frac{2}{3}$$

$$\lim_{n\to\infty} \sqrt[n]{a_n} \leq \lim_{n\to\infty} \sqrt[n]{\left(\frac{2}{3}\right)^{2n-\ln n}} = \lim_{n\to\infty} \left(\frac{2}{3}\right)^{2-\frac{\ln n}{n}} = \frac{4}{9} < 1$$

($\frac{\ln n}{n} \to 0$ L.p.)

Na osnovu košijevog kriterijuma red $\sum_{n=1}^{\infty} a_n$ konvergira

$\sum_{n=1}^{\infty} \left(\frac{n-1}{n}\right)^{n^2}$ $\quad a_n = \left(\frac{n-1}{n}\right)^{n^2}$ $\quad \lim_{n\to\infty} \sqrt[n]{a_n} = \lim_{n\to\infty} \left(\frac{n-1}{n}\right)^n = 1 \Rightarrow$ D.k. ~~X~~

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \left(\frac{n-1}{n}\right)^n = \lim_{n\to\infty} \left(1 + \frac{-1}{n}\right)^{-n\cdot(-1)} = e^{-1} = \frac{1}{e} \neq 0$$

$$\lim_{n\to\infty} a_n \neq 0 \Rightarrow \sum_{n=1}^{\infty} a_n \text{ divergira}$$

20. $\sum_{n=1}^{\infty} \frac{n!}{(a+1)(a+2)\cdots(a+n)}$; $a > 0$

Dalamb. $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{\frac{(n+1)\,n!}{(a+1)(a+2)\cdots(a+n)(a+n+1)}}{\frac{n!}{(a+1)(a+2)\cdots(a+n)}} = \lim_{n\to\infty} \frac{n+1}{a+n+1} = 1 \Rightarrow$ D.k. ne daje odg.

Primijenimo Rabov test $\rightarrow$ ako je $\lim > 1$ konv, $\lim < 1$ diverg.

$$\lim_{n\to\infty} n\left(\frac{a_n}{a_{n+1}} - 1\right) = \lim_{n\to\infty} n\left(\frac{a+n+1}{n+1} - 1\right) = \lim_{n\to\infty} n\cdot\frac{a}{n+1} = \lim_{n\to\infty} a\cdot\frac{n}{n+1} = a$$

Za $a > 1$ konvergira

Za $0 < a < 1$ divergira

ako je $a = 1$ dobijamo brojni red

$$\sum_{n=1}^{\infty} \frac{n!}{2\cdot 3 \cdots (1+n)} \rightarrow a_n = \frac{1}{n+1}$$

$b_n = \frac{1}{n}$; $\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{n}{n+1} = 1 \Rightarrow \sum a_n$ i $\sum b_n$ ekvikonvergentni

Red $\sum b_n$ divergira kao hiperharm. $\Rightarrow$ i $\sum a_n$ divergira

21. $\sum_{n=1}^{\infty} \frac{(-2)^n + 3n^2}{3^n}$ $\quad \underbrace{\sum_{n=1}^{\infty} \left(-\frac{2}{3}\right)^n}_{a_n} + \underbrace{\sum_{n=1}^{\infty} \frac{3n^2}{3^n}}_{b_n}$

$\sum a_n$ je geom. red $|q| = \left|-\frac{2}{3}\right| < 1 \Rightarrow a_n$ konvergira 1°

$\lim_{n\to\infty} \frac{b_{n+1}}{b_n} = \frac{1}{3} < 1 \Rightarrow$ konvergira 2°

Iz 1° i 2° $\Rightarrow$ red $\sum (a_n + b_n)$ konvergira

22. $\sum_{n=1}^{\infty} n \cdot e^{-n^2}$ $\quad a_n = n \cdot e^{-n^2}$

Neka je $f(x) = x \cdot e^{-x^2}$. Tada je $a_n = f(n)$, $\forall n \in \mathbb{N}$

1° $f(x) = \frac{x}{e^{x^2}}$; f je neprekidna kao kompozicija elementarnih u oblasti definisanosti pa i na $[1, +\infty)$

2° $f(x) = \frac{x}{e^{x^2}} > 0$, $\forall x \in [1, +\infty)$

$x_1 < x_2$; $f(x_1) \geq f(x_2)$

3° $f'(x) = \frac{1-2x^2}{e^{x^2}} < 0 \iff 1-2x^2 < 0 \iff x \in \left(-\infty, -\frac{\sqrt{2}}{2}\right) \cup \left(\frac{\sqrt{2}}{2}, +\infty\right)$

Odavde slijedi da je $f'(x) < 0$ i za $\forall x \in [1, +\infty)$ pa je f-ja na $[1, +\infty)$ opadajuća

Iz 1°, 2° i 3° na osnovu Košijevog integralnog kriterijuma slijedi da red $\sum_{n=1}^{\infty} a_n$ konvergira akko $\int_1^{+\infty} f(x)\,dx$ konvergira.

$$\int_1^{+\infty} f(x)\,dx = \lim_{B \to +\infty} \int_1^B x e^{-x^2} dx = \left[\begin{array}{l} -x^2 = t \\ x\,dx = -\frac{dt}{2} \end{array}\right] = \ldots = \lim_{B \to +\infty} \left(-\frac{1}{2} \cdot \cancel{\frac{1}{e^{B^2}}}^{0} + \frac{1}{2} \cdot \frac{1}{e} \right) = \frac{1}{2e}$$

$\Rightarrow$ integral $\int_1^{+\infty} f(x)\,dx$ konv., pa i red $\sum a_n$ konv.

23. $\sum_{n=2}^{\infty} \frac{1}{n \cdot \ln^2 n} \rightarrow a_n = \frac{1}{n \cdot \ln^2 n}$

Neka je $f(x) = \frac{1}{x \cdot \ln^2 x}$. Tada je $f(n) = a_n$, $\forall n \geq 2$

1° neprekidna

$f(x) = \frac{1}{x \ln^2 x} \Rightarrow$ neprekidna na int. $[2, +\infty)$

2° znak

$f(x) = \frac{1}{x \ln^2 x} > 0$, $\forall x \in [2, +\infty)$

3° $f'(x) = -\frac{\ln x + 2}{x^2 \ln^3 x} < 0$, $\forall x \in [2, +\infty)$ $\quad f \searrow$ na $[2, +\infty)$

Iz 1°, 2° i 3° $\Rightarrow$ red $\sum_{n=2}^{\infty} a_n$ konv. akko $\int_2^{+\infty} f(x)\,dx$ konv.

$$\int_2^{+\infty} f(x)\,dx = \lim_{B \to +\infty} \int_2^B \frac{1}{x \ln^2 x} dx = \left[\begin{array}{l} \ln x = t \\ \frac{dx}{x} = dt \end{array}\right] = \ldots = \lim_{B \to +\infty} \left(-\frac{1}{\ln B} + \frac{1}{\ln 2} \right) = \frac{1}{\ln 2}$$

$\Rightarrow$ ovaj integral konvergira $\Rightarrow$ red $\sum_{n=2}^{\infty} a_n$ konvergira

$1 - \frac{3}{2} + \frac{5}{4} - \frac{7}{8}$ * Samo Lajbnicov kriterijum

$$\sum_{n=1}^{\infty} (-1)^{n-1} \cdot \frac{2n-1}{2^{n-1}}$$

1° posmatramo niz $b_n = |a_n| = \frac{2n-1}{2^{n-1}}$

$$b_n - b_{n+1} = \frac{2n-1}{2^{n-1}} - \frac{2n+1}{2^n} = \frac{2(2n-1) - 2n - 1}{2^n} = \frac{2n-3}{2^n} > 0, \ \forall n \geq 2$$

(izbacujemo b_1, odnosno a_1, ali to ne utiče na konvergenciju)

$b_n > b_{n+1}$, $\forall n \geq 2 \Rightarrow$ Niz (b_n) je opadajući

2° $\lim\limits_{n\to\infty} b_n = \lim\limits_{n\to\infty} \frac{2n-1}{2^{n-1}} = 0$

Iz 1° i 2° na osnovu Lajbnicovog krit. $\Rightarrow$ red $\sum\limits_{n=1}^{\infty} a_n$ konvergira

25. $\sum\limits_{n=1}^{\infty} \frac{(-1)^n n!}{(2n)!}$ $\quad a_n = \frac{(-1)^n \cdot n!}{(2n)!}$ $\quad b_n = |a_n| = \frac{n!}{(2n)!}$

1° $$\frac{b_n}{b_{n+1}} = \frac{\frac{\cancel{n!}}{\cancel{(2n)!}}}{\frac{(n+1)\cancel{n!}}{(2n+2)(2n+1)\cancel{(2n)!}}} = \frac{(2n+2)(2n+1)}{n+1} = \frac{4n^2+6n+2}{n+1} > 1 \ / (n+1) > 0, \ \forall n$$

$4n^2 + 6n + 2 > n + 1$

$4n^2 + 5n + 1 > 0$ $\qquad 4n^2 + 5n + 1 = 4(n+1)(n+\frac{1}{4})$

$n_{1,2} = \frac{-5 \pm 3}{8} \Rightarrow n_1 = -1, \ n_2 = -\frac{1}{4}$ $\qquad 4n^2 + 5n + 1 > 0, \ \forall n \in N$

$-1 \quad -\frac{1}{4}$

$\frac{b_n}{b_{n+1}} > 1 \Rightarrow b_n > b_{n+1} \Rightarrow$

$\Rightarrow b_n$ opadajući

2° $$\lim_{n\to\infty} b_n = \lim_{n\to\infty} \frac{n!}{(2n)!} = \lim \frac{n!}{1 \cdot 2 \cdot \ldots \cdot n(n+1) \ldots (n+n)!} =$$

$$= \lim_{n\to\infty} \frac{1}{n(n+1)\cdots(2n)} = 0$$

Iz 1° i 2° $\Rightarrow$ red a_n konv. po Lajbnicovom kriterijumu

26. $\sum\limits_{n=1}^{\infty} (-1)^n \cdot \frac{2}{\sqrt{n^2+2}+n}$ $\quad b_n = |a_n| = \frac{2}{\sqrt{n^2+2}+n}$

$n^2 < (n+1)^2 \Rightarrow n^2 + 2 < (n+1)^2 + 2$

2° $\lim\limits_{n\to\infty} b_n = 0$ $\qquad \sqrt{n^2+2} + n < \sqrt{(n+1)^2+2} + n$

$\frac{2}{\sqrt{n^2+2}+n} > \frac{2}{\sqrt{(n+1)^2+2}+n} \Rightarrow$ red op

$b_n > b_{n+1}$

$\forall n \in$

Iz 1° i 2° $\Rightarrow$ red $\sum a_n$ konvergira

$n=3$ $n+4$

1° $f(x) = \frac{\sqrt{x}}{x+4}$; $f'(x) = \frac{4-x}{2\sqrt{x}(x+4)^2} < 0 \iff 4-x < 0 \iff x > 4$

$f'(x) < 0$, $\forall x \in [5, +\infty)$ f je opadajuća na $[5, +\infty)$ — $n < n+1$

$f(n) > f(n+1)$

$b_n > b_{n+1}$, $\forall n \geq 5$

2° $\lim\limits_{n\to\infty} b_n = 0$ Iz 1° i 2° $\Rightarrow$ red $\sum a_n$ konv

28. $\sum\limits_{n=2}^{\infty} \frac{(-1)^{n-1}}{\ln^2 n} \cdot \cos\frac{\pi}{n+1}$ $a_n = \frac{(-1)^{n-1}}{\ln^2 n}$, $b_n = \cos\frac{\pi}{n+1}$

red konv. niz b_n monoton i ograničen

I $\sum\limits_{n=2}^{\infty} a_n$ je alternativni - Lajbnic $\Rightarrow$ red $a_n b_n$ konverg.

$c_n = |a_n| = \frac{1}{\ln^2 n}$

1° $\ln^2 n < \ln^2(n+1)$, $\forall n \geq 2$ 2° $\lim\limits_{n\to\infty} c_n = 0$

$\frac{1}{\ln^2 n} > \frac{1}{\ln^2(n+1)}$, $\forall n \geq 2$

Iz 1° i 2° $\Rightarrow a_n$ konvergira

$c_n > c_{n+1}$, $\forall n \geq 2$; c_n je opadajući

II 1° $|b_n| = |\cos\frac{\pi}{n+1}| \leq 1$ $\Rightarrow$ niz je ograničen

2° monoton : $n+1 < n+2$

$\frac{\pi}{n+1} > \frac{\pi}{n+2}$, $\forall n \geq 2$

$\frac{\pi}{n+1} \in (0, \frac{\pi}{2})$, $\forall n \geq 2$ Na $(0, \frac{\pi}{2})$ $\cos x$ je opadajuća f-ja

$\cos\frac{\pi}{n+1} < \cos\frac{\pi}{n+2}$

$b_n < b_{n+1}$, $\forall n \geq 2$ $\Rightarrow$ b_n je monotono rastući

I $\sum\limits_{n=2}^{\infty} a_n$ konv. II b_n monot. i ogr. $\Rightarrow \sum\limits_{n=2}^{\infty} a_n b_n$ konvergira (Abelov kriterijum)

29. Dirihleov kriterijum

$\sum\limits_{n=1}^{\infty} \left(-\frac{1}{2}\right)^n \cdot \frac{1}{n}$ $a_n = \left(-\frac{1}{2}\right)^n$, $b_n = \frac{1}{n}$

1° pokazati da je (S_n) niz parcijalnih suma ogranič

2° da je b_n opadajući, teži nuli

I 1° $\frac{1}{n} > \frac{1}{n+1}$ $\forall n$

$b_n > b_{n+1}$, $\forall n \Rightarrow b_n$ je opadajući

2° $\lim\limits_{n\to\infty} b_n = 0$

II $\sum_{n=1}^{\infty} a_n \to$ niz parcijalnih suma ograničen

$$S_n = a_1 + a_2 + \dots + a_n$$

$$S_n = -\frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \dots + \left(-\frac{1}{2}\right)^n = -\frac{1}{2}\,\frac{1-\left(-\frac{1}{2}\right)^n}{1-\left(-\frac{1}{2}\right)} = -\frac{1}{3}\left(1-\left(-\frac{1}{2}\right)^n\right)$$

$$|S_n| = \frac{1}{3}\underbrace{\left|1-\left(-\frac{1}{2}\right)^n\right|}_{\text{najviše } 3/2} \le \frac{1}{3}\cdot\frac{3}{2} \le \frac{1}{2} \Rightarrow \text{niz } (S_n) \text{ je ograničen}$$

Iz I i II na osnovu Dirihleovog krit. $\sum_{n=1}^{\infty} a_n b_n$ konvergira

— Furijeovi redovi —

1. F-ju $f(x) = \begin{cases} \pi & -\pi \le x < 0 \\ x, & 0 \le x \le \pi \end{cases}$ razviti u Furijeov red na $(-\pi, \pi)$

$\to$ ni parna ni neparna $\quad a = -\pi, \; b = \pi$

$$\frac{a_0}{2} + \sum_{n=1}^{\infty}\left(a_n \cos\frac{2n\pi}{2\pi}x + b_n \sin\frac{2n\pi}{2\pi}x\right) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left(a_n\cos nx + b_n \sin nx\right)$$

$$a_0 = \frac{2}{2\pi}\int_{-\pi}^{\pi} f(x)\,dx = \frac{1}{\pi}\left(\int_{-\pi}^{0}\pi\,dx + \int_{0}^{\pi} x\,dx\right) = \frac{1}{\pi}\left(\pi^2 + \frac{\pi^2}{2}\right) = \frac{3\pi}{2}$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\,dx = \int_{-\pi}^{0}\cos nx\,dx + \frac{1}{\pi}\int_{0}^{\pi} x\cos nx\,dx = \underbrace{\frac{1}{n}\sin nx\Big|_{-\pi}^{0}}_{0} + \frac{1}{\pi}\int_{0}^{\pi} x\cos nx\,dx$$

$$= \begin{bmatrix} u = x \to du = dx \\ v = \int\cos nx\,dx = \frac{1}{n}\sin nx \end{bmatrix} = \frac{1}{\pi}\cdot\frac{x\sin nx}{n}\Big|_{0}^{\pi} - \frac{1}{\pi}\cdot\frac{1}{n}\int_{0}^{\pi}\sin nx\,dx = \frac{1}{\pi n^2}\cos nx\Big|_{0}^{\pi} =$$

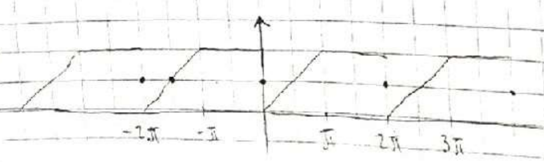
$$= \frac{1}{n^2\pi}\left(\cos n\pi - 1\right) = \begin{cases} 0, & n - \text{parno} \\ \frac{2}{n^2\pi}, & n - \text{neparno} \end{cases}$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\,dx = \frac{1}{\pi}\int_{-\pi}^{0}\pi\sin nx\,dx + \frac{1}{\pi}\int_{0}^{\pi} x\sin nx\,dx = -\frac{1}{n}\cos nx\Big|_{-\pi}^{0} +$$

$$+\frac{1}{\pi}\left(-\frac{x\cos nx}{n}\Big|_{0}^{\pi} + \frac{1}{n}\int_{0}^{\pi}\cos nx\,dx\right) = -\frac{1}{n}\left(1-\cos(-n\pi)\right) + \frac{1}{\pi}\left(-\frac{\pi\cos n\pi}{n} + \cancel{\frac{1}{n^2}\sin}\right)$$

$$= -\frac{1}{n} + \frac{\cos n\pi}{n} - \frac{\cos n\pi}{n} = -\frac{1}{n}$$

$$f(x) = \frac{3\pi}{4} + \sum_{n=1}^{\infty}\left(\frac{-2}{(2n-1)^2\pi}\cos(2n-1)x\right) + \sum_{n=1}^{\infty}\left((-1)\cdot\frac{1}{n}\sin nx\right) \quad x \in (-\pi, \pi)$$



Razviti u Furijeov red f-ju $f(x) = 5x^2$ na intervalu $(-\pi, \pi)$

$f(x) = 5x^2 \rightarrow$ f-ja je parna $\forall n \in \mathbb{N}$, $b_n = 0$

razvijamo samo po kosinusima

$$a_0 = \frac{2}{2\pi}\int_{-\pi}^{\pi} f(x)\,dx = \frac{1}{\pi}\int_{-\pi}^{\pi} 5x^2\,dx = \frac{5}{\pi}\,\frac{x^3}{3}\Big|_{-\pi}^{\pi} = \frac{5}{\pi}\left(\frac{\pi^3}{3} + \frac{\pi^3}{3}\right) = \frac{10\pi^2}{3}$$

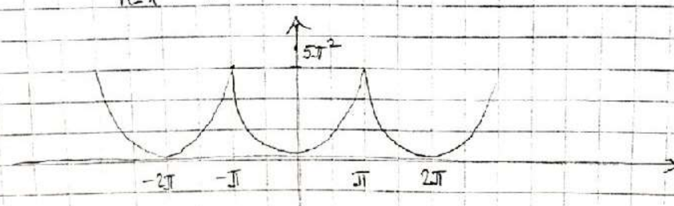
$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos\frac{2n\pi}{2\pi}x\,dx = \frac{1}{\pi}\int_{-\pi}^{\pi} 5x^2\cos nx\,dx = \begin{bmatrix} u = x^2 & du = 2x\,dx \\ v = \int \cos nx\,dx = \frac{1}{n}\sin nx & \end{bmatrix} =$$

$$= \frac{5}{\pi}\left(\frac{x^2 \sin nx}{n}\Big|_{-\pi}^{\pi} - \frac{2}{n}\int_{-\pi}^{\pi} x\sin nx\,dx\right) = \frac{5}{\pi}\left(-\frac{2}{n}\int_{-\pi}^{\pi} x\sin nx\,dx\right) = \begin{bmatrix} u = x & du = dx \\ v = -\frac{1}{n}\cos nx & \end{bmatrix}$$

$$= -\frac{10}{n\pi}\left(-\frac{x\cos nx}{n}\Big|_{-\pi}^{\pi} + \frac{1}{n}\int_{-\pi}^{\pi}\cos nx\,dx\right) = -\frac{10}{n\pi}\left(-\left(\frac{\pi\cos n\pi}{n} - \frac{-\pi\cos n\pi}{n}\right) +\right.$$

$$\left. + \cancel{\frac{1}{n^2}\sin nx\Big|_{-\pi}^{\pi}}_{0}\right) = -\frac{10}{n\pi}\left(-\frac{2\pi\cos n\pi}{n}\right) = \frac{20\cos n\pi}{n^2} = \frac{20}{n^2}(-1)^n = \begin{cases} \frac{20}{n^2}, & n\text{-par} \\ -\frac{20}{n^2}, & n\text{-nepar} \end{cases}$$

$$f(x) = \frac{5\pi^2}{3} + 20\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2}\cdot\cos nx, \quad x \in (-\pi, \pi)$$



3. F-ju $f(x) = 2x - x^2$ razviti u Furijeov red na $(0, 3)$ $\Rightarrow a = 0,\ b = 3$

$f(x) = x(2-x) \rightarrow 0, 2$ Tjeme $T\left(-\frac{b}{2a}, \frac{D}{4a}\right) \Rightarrow T(1, 1)$

$x = 3 \Rightarrow y = -3$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left(a_n\cos\frac{2n\pi}{3}x + b_n\sin\frac{2n\pi}{3}\right)$$

$$a_0 = \frac{2}{3}\int_0^3 f(x)\,dx = \frac{2}{3}\int_0^3 (2x - x^2)\,dx = \frac{2}{3}\left(x^2\Big|_0^3 - \frac{x^3}{3}\Big|_0^3\right) = \frac{2}{3}\left(9 - \frac{27}{3}\right) = 0$$

$$a_n = \frac{2}{3}\int_0^3 f(x)\cos\frac{2n\pi}{3}x\,dx = \frac{2}{3}\int_0^3 (2x - x^2)\cos\frac{2n\pi}{3}x\,dx = \frac{4}{3}\int_0^3 x\cos\frac{2n\pi}{3}x\,dx -$$

$$-\frac{2}{3}\int_0^3 x^2\cos\frac{2n\pi}{3}x\,dx = \begin{bmatrix} u = x & du = dx \\ v = \int\cos\frac{2n\pi}{3}x\,dx = \frac{3}{2n\pi}\sin\frac{2n\pi}{3}x & \end{bmatrix} \quad \begin{bmatrix} u = x^2 & du = 2x\,dx \\ v = \frac{3}{2n\pi}\sin\frac{2n\pi}{3}x & \end{bmatrix}$$

$$\frac{4}{3}\left(\cancel{\frac{3x}{2n\pi}\sin\frac{2n\pi}{3}x\Big|_0^3} - \int_0^3 \frac{3}{2n\pi}\sin\frac{2n\pi}{3}x\,dx\right) - \frac{2}{3}\left(\cancel{x^2\cdot\frac{3}{2n\pi}\sin\frac{2n\pi}{3}x\Big|_0^3} - 2\int_0^3\frac{3x}{2n\pi}\sin\frac{2n\pi}{3}x\,dx\right.$$

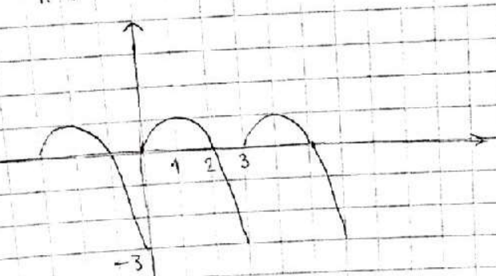
$$= -\frac{4}{3}\cdot\frac{3}{2n\pi}\int_0^3 \sin\frac{2n\pi}{3}x\,dx + \frac{4}{3}\cdot\frac{3}{2n\pi}\int_0^3 x\sin\frac{2n\pi}{3}x\,dx = \frac{2}{n\pi}\cdot\frac{3}{2n\pi}\cos\frac{2n\pi}{3}x\Big|_0^3 +$$

$$+\frac{2}{n\pi}\left(-\frac{3x\cos\frac{2n\pi}{3}x}{2n\pi}\Big|_0^3 + \int_0^3\frac{3}{2n\pi}\cos\frac{2n\pi}{3}x\,dx\right) = \frac{3}{n^2\pi^2}\underbrace{(\cos 2n\pi - \cos 0)}_{0} - \frac{2}{n\pi}\left(\frac{9\cos 2n\pi}{2n\pi}\right. +$$

$$\left. + \frac{3}{2n\pi}\cdot\frac{3}{2n\pi}\sin\frac{2n\pi}{3}x\Big|_0^3\right) = -\frac{9}{n^2\pi^2}$$

$$b_n = \frac{2}{3}\int_0^3 f(x)\sin\frac{2n\pi}{3}x\,dx = \frac{2}{3}\int_0^3 (2x-x^2)\sin\frac{2n\pi}{3}x\,dx = \dots = \frac{3}{n\pi}$$

$$f(x) = \sum_{n=1}^{\infty}\left(-\frac{9}{n^2\pi^2}\cos\frac{2n\pi}{3}x + \frac{3}{n\pi}\sin\frac{2n\pi}{3}x\right), \quad x\in(0,3)$$

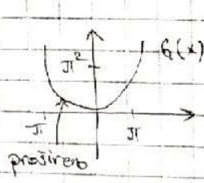


4. F-ju $f(x)=x^2$ razviti u kosinusni Furijeov red na $[0,\pi]$

$\rightarrow$ f parno proširujemo

$$G(x) = \begin{cases} f(x), & x\in[0,\pi] \\ f(-x), & x\in[-\pi,0]\end{cases} = \begin{cases} x^2, & x\in[0,\pi] \\ (-x)^2, & x\in[-\pi,0]\end{cases} = \left\{ x^2,\ x\in[-\pi,\pi]\right.$$

G(x) razvijamo u F. red na $[-\pi,\pi]$



G → parna ⇒ razvija nju proširenja

$b_n = 0$

$$a_0 = \frac{1}{\pi}\cdot 2\int_0^{\pi} f(x)\,dx = \frac{2}{\pi}\int_0^{\pi}x^2\,dx = \frac{2}{\pi}\frac{x^3}{3}\Big|_0^{\pi} = \frac{2\pi^2}{3}$$

$$a_n = \frac{2}{2\pi}\int_{-\pi}^{\pi} f(x)\cos\frac{2n\pi}{2\pi}x\,dx = \frac{1}{\pi}\int_{-\pi}^{\pi}\underbrace{x^2\cos nx}_{\text{parna}}\,dx = \frac{2}{\pi}\int_0^{\pi}x^2\cos nx\,dx = \begin{bmatrix} x^2=u & du= \\ v=\frac{1}{n}\sin nx & \end{bmatrix}$$

$$= \frac{2}{\pi}\left(\underbrace{\frac{x^2\sin nx}{n}\Big|_0^{\pi}}_{0} - \frac{2}{n}\int_0^{\pi}x\sin nx\,dx\right) = -\frac{4}{n\pi}\int_0^{\pi}x\sin x\,dx = \begin{bmatrix} u=x & du=dx \\ v=-\frac{1}{n}\cos nx & \end{bmatrix}$$

$$= -\frac{4}{n\pi}\left(-\frac{x\cos nx}{n}\Big|_0^{\pi} + \frac{1}{n}\int_0^{\pi}\cos nx\,dx\right) = -\frac{4}{n\pi}\left(-\frac{\pi\cos n\pi}{n} + \frac{1}{n^2}\sin nx\Big|_0^{\pi}\right) =$$

$$= \frac{4\pi}{n^2\pi}\cos n\pi = \frac{4}{n^2}(-1)^n$$

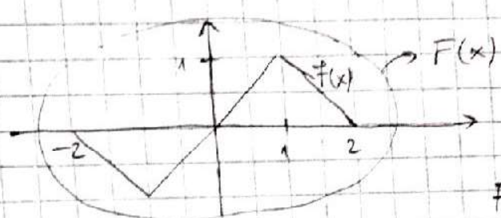
$$G(x)=\frac{\pi^2}{3}+\sum_{n=1}^{\infty}\left(\frac{4}{n^2}(-1)^n\cos n\pi\right),\quad x\in[-\pi,\pi]$$

$$f(x)=\frac{\pi^2}{3}+4\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2}\cos nx,\quad x\in[0,\pi]$$

5. F-ju $f(x)=\begin{cases}x, & x\in[0,1)\\ 2-x, & x\in[1,2)\end{cases}$ razviti u sinusni Furijeov red.

$\Downarrow$ neparno proširujemo f-ju

$$F(x)=\begin{cases}f(x), & x\in[0,2)\\ -f(-x), & x\in[-2,0)\end{cases}=\begin{cases}x, & x\in(0,1)\\ 2-x, & x\in(1,2)\\ -(-x)=x, & x\in(-1,0)\\ -(2-(-x))=-2-x, & x\in[-2,-1]\end{cases}$$



F razvijamo u Furijeov red na $[-2,2]$

$a_n=0,\ \forall n\in\mathbb{N}\ \rightarrow$ jer je F neparna

$$a_0=\frac{2}{4}\int_{-2}^{2}F(x)\,dx=\frac{1}{2}\int_{-2}^{-1}f(x)\,dx+\frac{1}{2}\int_{-1}^{0}f(x)\,dx+\frac{1}{2}\int_{0}^{1}f(x)\,dx+\frac{1}{2}\int_{1}^{2}f(x)\,dx=$$

$$=\frac{1}{2}\int_{-2}^{-1}(-2-x)\,dx+\frac{1}{2}\int_{-1}^{0}x\,dx+\frac{1}{2}\int_{0}^{1}x\,dx+\frac{1}{2}\int_{1}^{2}(2-x)\,dx=$$

$$=-x\Big|_{-2}^{-1}-\frac{x^2}{4}\Big|_{-2}^{-1}+\frac{1}{4}x^2\Big|_{-1}^{0}+\frac{x^2}{4}\Big|_{0}^{1}+x\Big|_{1}^{2}-\frac{1}{4}x^2\Big|_{1}^{2}=-1+\frac{3}{4}-\frac{1}{4}+\frac{1}{4}+1-1+\frac{1}{4}=0$$

$$b_n=\frac{2}{4}\int_{-2}^{2}\underbrace{\underbrace{F(x)}_{N}\,\underbrace{\sin\frac{n\pi}{2}x}_{N}}_{P}\,dx=\frac{1}{2}\cdot 2\int_{0}^{2}F(x)\sin\frac{n\pi}{2}x\,dx=\int_{0}^{1}x\sin\frac{n\pi}{2}x\,dx+$$

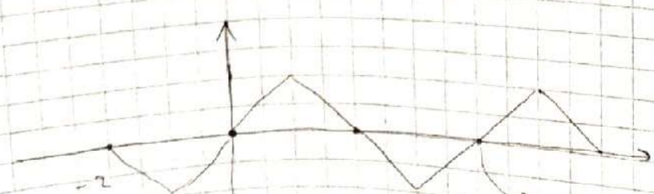
$$+\int_{1}^{2}(2-x)\sin\frac{n\pi}{2}x\,dx=\ \dots\ =\frac{8}{n^2\pi^2}\sin\frac{n\pi}{2}=\begin{cases}0, & n\text{-parno}\\ (-1)^k\dfrac{8}{(2k+1)^2\pi^2}, & n=2k+1\end{cases}$$

↳ mora ovako zbog nule da se piše

$$F(x)=\frac{8}{\pi^2}\sum_{k=0}^{\infty}\frac{(-1)^k}{(2k+1)^2}\sin\frac{(2k+1)\pi}{2},\quad x\in[-2,2]$$

$\downarrow$

$f(x)=\qquad\qquad\qquad,\quad x\in[0,2]$



granične tačke

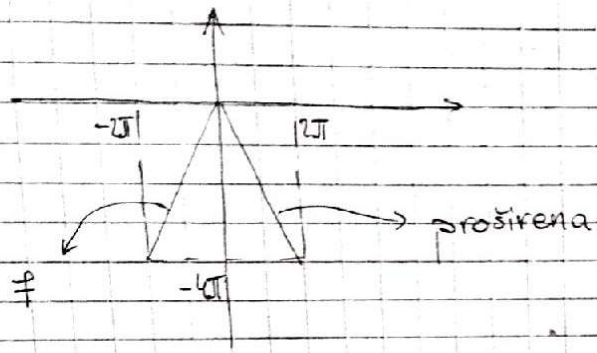
Razviti u red po kosinusima f-ju $f(x) = 2x$ na $(-2\pi, 0)$

$\Downarrow$

proširujemo parno

$$G(x) = \begin{cases} f(x), & x \in [-2\pi, 0) \\ f(-x), & x \in (0, 2\pi] \end{cases} = \begin{cases} 2x, & x \in [-2\pi, 0) \\ -2x, & x \in (0, 2\pi] \end{cases}$$

G razviti na $[-2\pi, 2\pi]$



7. F-ju $f(x) = -\left|\frac{3\pi}{8} - \frac{x}{2}\right|$ razviti u red po sinusima na $[0, \pi]$

$\rightarrow$ f neparno proširujemo

$$F(x) = \begin{cases} f(x), & x \in [0, \pi] \\ -f(-x), & x \in [-\pi, 0) \end{cases} = \begin{cases} -\left|\frac{3\pi}{8} - \frac{x}{2}\right|, & x \in [0, \pi] \\ \left|\frac{3\pi}{8} + \frac{x}{2}\right|, & x \in [-\pi, 0) \end{cases}$$

$$-\left|\frac{3\pi}{8} - \frac{x}{2}\right| = \begin{cases} \frac{x}{2} - \frac{3\pi}{8}, & x \leq \frac{3\pi}{4} \\ \frac{3\pi}{8} - \frac{x}{2}, & x > \frac{3\pi}{4} \end{cases}$$

$$\left|\frac{3\pi}{8} + \frac{x}{2}\right| = \begin{cases} \frac{x}{2} + \frac{3\pi}{8}, & x \geq -\frac{3\pi}{4} \\ -\frac{x}{2} - \frac{3\pi}{8}, & x < -\frac{3\pi}{4} \end{cases}$$

F se razvija na $(\underset{a}{-\pi}, \underset{b}{\pi})$

3. (a) Skicirati grafik funkcije $f(x) = \dfrac{x}{2} - \left|\dfrac{x}{2} - 1\right|$, a zatim je razviti u Furijeov red na intervalu $[1, 3]$.

a)

3. $f(x)=\begin{cases} \frac{x}{2}-\left(\frac{x}{2}-1\right), & \frac{x}{2}-1\geq 0 \\ \frac{x}{2}-\left(1-\frac{x}{2}\right), & \frac{x}{2}-1<0 \end{cases} = \begin{cases} 1, & 2\leq x\leq 3 \\ x-1, & 1<x<2 \end{cases}$

$$f(x)=\frac{a_0}{2}+\sum_{n=1}^{\infty}(a_n\cos n\pi x+b_n\sin n\pi x), \quad 1<x<3$$

$$a_0=\int_1^3 f(x)\,dx=\int_1^2(x-1)dx+\int_2^3 1\,dx=\frac{x^2}{2}\Big|_1^2-x\Big|_1^2+x\Big|_2^3=2-\frac{1}{2}=\frac{3}{2}$$

$$a_n=\int_1^3 f(x)\cos n\pi x\,dx=\int_1^2(x-1)\cos n\pi x\,dx+\int_2^3\cos n\pi x\,dx=$$

$$=\left[\begin{array}{l} u=x-1 \Rightarrow du=dx \\ v=\int\cos n\pi x\,dx=\frac{1}{n\pi}\sin n\pi x \end{array}\right]=$$

$$=\underbrace{\frac{x-1}{n\pi}\sin n\pi x\Big|_1^2}_{0}-\frac{1}{n\pi}\int_1^2\sin n\pi x\,dx+\underbrace{\frac{1}{n\pi}\sin n\pi x\Big|_2^3}_{0}=$$

$$=-\frac{1}{n\pi}\cdot\frac{-1}{n\pi}\cos n\pi x\Big|_1^2=\frac{1}{n^2\pi^2}(\cos 2n\pi-\cos n\pi)=$$

$$=\frac{1}{n^2\pi^2}\left(1-(-1)^n\right)=\begin{cases} 0, & n-\text{parno} \\ \frac{2}{n^2\pi^2}, & n-\text{neparno} \end{cases}$$

$$b_n=\int_1^3 f(x)\sin n\pi x\,dx=\int_1^2(x-1)\sin n\pi x\,dx+\int_2^3 1\cdot\sin n\pi x\,dx=$$

$$=\left[\begin{array}{l} u=x-1 \Rightarrow du=dx \\ v=\int\sin n\pi x\,dx=-\frac{1}{n\pi}\cos n\pi x \end{array}\right]=$$

$$=-\frac{x-1}{n\pi}\cdot\cos n\pi x\Big|_1^2+\frac{1}{n\pi}\int_1^2\cos n\pi x\,dx-\frac{1}{n\pi}\cos n\pi x\Big|_2^3=$$

$$=-\frac{1}{n\pi}\cdot\cos 2n\pi+\frac{1}{n\pi}\cdot\frac{1}{n\pi}\underbrace{\sin n\pi x\Big|_1^2}_{0}-\frac{1}{n\pi}\underbrace{(\cos 3n\pi-\cos 2n\pi)}_{-1}$$

$$= -\frac{1}{n\pi}\cdot(+1) - \frac{1}{n\pi}\cdot\left(\cos 3n\pi - 1\right) = \left\lceil -\cancel{\frac{1}{n\pi}} - \frac{1}{n\pi}\cos 3n\pi + \cancel{\frac{1}{n\pi}} = \right\rfloor$$

$$\cos 3n\pi = (-1)^n = \begin{cases} 1 & n\text{-par.} \\ -1 & n\text{-nepar.} \end{cases}$$

$$= -\frac{1}{n\pi} - \frac{1}{n\pi}\left((-1)^n - 1\right) = \begin{cases} -\frac{1}{n\pi}, & n-\text{parno} \\ \frac{1}{n\pi}, & n-\text{neparno} \end{cases} = (-1)^{n-1}\frac{1}{n\pi}$$

$$f(x) = \frac{3}{4} + \sum_{n=1}^{\infty} \frac{2}{(2n-1)^2\pi}\cos n\pi x + \sum_{n=1}^{\infty} (-1)^{n-1}\cdot\frac{1}{n\pi}\cdot\sin n\pi x$$

M2, 1/3/4. KOL. (AVG.)

1. a) $f(x) = e^{-\frac{x}{\pi}}$ $\quad [-\pi, 0]$

Funkciju $f(x)$ bi trebalo neparno proširiti

$$F(x) = \begin{cases} f(x), & x \in [-\pi, 0] \\ -f(-x), & x \in (0, \pi] \end{cases}$$

$$F(x) = \begin{cases} e^{-\frac{x}{\pi}}, & x \in [-\pi, 0] \\ -e^{\frac{x}{\pi}}, & x \in (0, \pi] \end{cases}$$

Funkciju $F(x)$ razvijamo u F. red na $(-\pi, \pi)$

Kako je F neparna to su $a_n = 0$, $\forall n \in \mathbb{N}$.

$$F(x) = \frac{1}{2} a_0 + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$$

$$F(x) = \sum_{n=1}^{\infty} b_n \sin nx$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} \underbrace{\underbrace{F(x)}_{\text{nep.}} \underbrace{\sin nx}_{\text{nep.}}}_{\text{parna}} dx = \frac{1}{\pi} \cdot 2 \int_0^{\pi} F(x) \sin nx \, dx =$$

$$= \frac{2}{\pi} \int_0^{\pi} -e^{\frac{x}{\pi}} \sin nx \, dx = -\frac{2}{\pi} \int_0^{\pi} e^{\frac{x}{\pi}} \cdot \sin nx \, dx$$

$$I = \int_0^{\pi} e^{\frac{x}{\pi}} \cdot \sin nx \, dx = \begin{bmatrix} u = e^{\frac{x}{\pi}} \Rightarrow du = \frac{1}{\pi} e^{\frac{x}{\pi}} dx \\ v = -\frac{1}{n} \cos nx \end{bmatrix}$$

$$= -\frac{1}{n} e^{\frac{x}{\pi}} \cdot \cos nx \Big|_0^{\pi} + \frac{1}{n} \cdot \frac{1}{\pi} \int_0^{\pi} e^{\frac{x}{\pi}} \cdot \cos nx \, dx =$$

$$= -\frac{1}{n} (e \cdot \cos n\pi - \cos 0) + \begin{bmatrix} u = e^{\frac{x}{\pi}} \Rightarrow du = \frac{1}{\pi} \cdot e^{\frac{x}{\pi}} dx \\ v = \frac{1}{n} \sin nx \end{bmatrix}$$

$$= -\frac{1}{n} \cdot (e \cdot (-1)^n - 1) + \frac{1}{n\pi} \left[\frac{1}{n} \cdot e^{\frac{x}{\pi}} \underbrace{\sin nx}_{0} \Big|_0^{\pi} - \frac{1}{n} \cdot \frac{1}{\pi} \int_0^{\pi} e^{\frac{x}{\pi}} \cdot \sin nx \, dx \right]$$

$$= -\frac{1}{n}\left(e\cdot(-1)^n-1\right) - \frac{1}{n^2\pi^2}\cdot\int_0^{\pi} e^{\frac{x}{\pi}} \sin nx \, dx =$$

$$I = -\frac{1}{n}\left(e\cdot(-1)^n-1\right) - \frac{1}{n^2\pi^2}\cdot I$$

$$\left(1+\frac{1}{n^2\pi^2}\right) I = \frac{1}{n}\cdot\left(1-e\cdot(-1)^n\right)$$

$$\frac{n^2\pi^2+1}{n^2\pi^2}\cdot I = \frac{1}{n}\cdot\left(1-e\cdot(-1)^n\right)$$

$$(n^2\pi^2+1)\cdot I = n\pi^2\cdot\left(1-e(-1)^n\right)$$

$$I = \frac{n\pi^2}{n^2\pi^2+1}\cdot\left(1-e\cdot(-1)^n\right)$$

$$b_n = -\frac{2}{\pi}\cdot I = -\frac{2}{\pi}\cdot\frac{n\pi^2}{n^2\pi^2+1}\cdot\left(1-e\cdot(-1)^n\right)$$

$$b_n = -\frac{2\cdot n\pi}{n^2\pi^2+1}\cdot\left(1-e\cdot(-1)^n\right) = \frac{2n\pi}{n^2\pi^2+1}\cdot\left(e(-1)^n-1\right)$$

$$f(x) = \sum_{n=1}^{\infty} \frac{2n\pi}{n^2\pi^2+1}\cdot\left(e\cdot(-1)^n-1\right)\cdot \sin nx \qquad x\in[-\pi,0]$$

tom.

(a) Funkciju $f(x) = |2x+1| - |2x+3|$ razviti u red po sinusima na intervalu $[-2, 0]$. Napisati izraz za proširenu funkciju i skicirati njen grafik.

∞ 1

3 $f(x)=|2x+1|-|2x+3|$

$$|2x+1| = \begin{cases} 2x+1, & x \geq -\frac{1}{2} \\ -(2x+1), & x < -\frac{1}{2} \end{cases}$$

$$|2x+3| = \begin{cases} 2x+3, & x \geq -\frac{3}{2} \\ -(2x+3), & x < -\frac{3}{2} \end{cases}$$

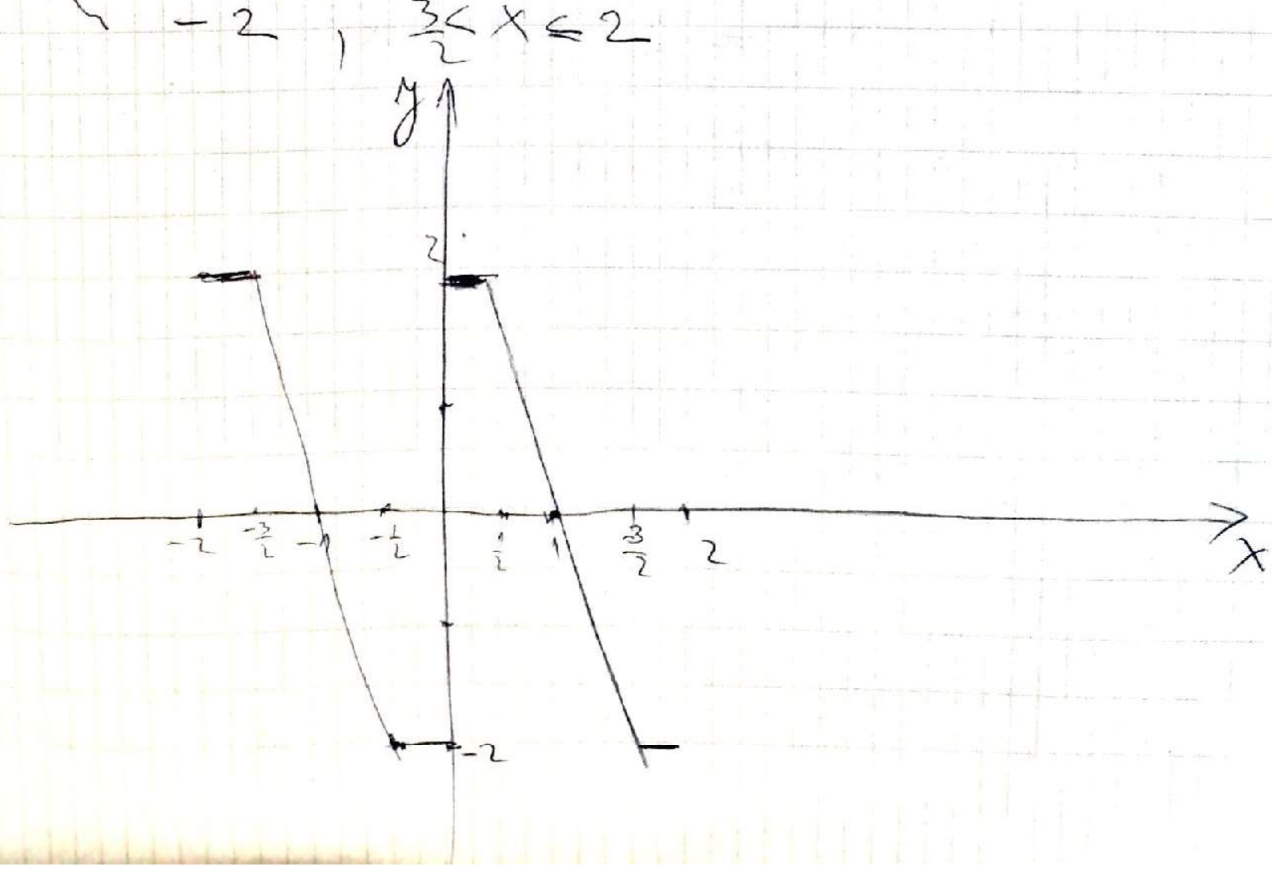
$$f(x)=|2x+1|-|2x+3| = \begin{cases} 2x+1-(2x+3), & -\frac{1}{2} \leq x \leq 0 \\ -(2x+1)-(2x+3), & -\frac{3}{2} \leq x < -\frac{1}{2} \\ -(2x+1)-(-(2x+3)), & -2 < x < -\frac{3}{2} \end{cases}$$

$$= \begin{cases} -2, & -\frac{1}{2} \leq x \leq 0 \\ -4x-4, & -\frac{3}{2} < x < -\frac{1}{2} \\ 2, & -2 < x < -\frac{3}{2} \end{cases}$$

$$F(x) = \begin{cases} f(x), & x \in [-2, 0] \\ -f(-x), & x \in (0, 2] \end{cases}$$

$$F(x) = \begin{cases} -2, & -\frac{1}{2} \leq x \leq 0 \\ -4x-4, & -\frac{3}{2} \leq x < -\frac{1}{2} \\ 2, & -2 < x < -\frac{3}{2} \\ 2, & 0 < x \leq \frac{1}{2} \\ -4x+4, & \frac{1}{2} < x \leq \frac{3}{2} \\ -2, & \frac{3}{2} < x \leq 2 \end{cases}$$

$$f(-x) = -4(-x) - 4$$
$$= 4x - 4$$
$$-f(-x) = -4x + 4$$



$[-2, 2]$

F $-\underset{2}{\text{nep.}}$ koef. $a_n = 0, \forall n \in \mathbb{N}_0$

$$b_n = \frac{2}{4}\int_{-2} F(x) \sin \frac{2n\pi}{4} x \, dx =$$

$$= \frac{1}{2}\int_{-2}^{2} \underbrace{\underbrace{F(x)}_{nep.} \underbrace{\sin \frac{n\pi}{2} x}_{nep.}}_{p.} dx = \frac{1}{2} \cdot 2 \cdot \int_0^2 F(x) \sin \frac{n\pi}{2} x \, dx$$

$$= \int_0^2 F(x) \sin \frac{n\pi}{2} x \, dx =$$

$$= \int_0^{1/2} 2 \cdot \sin \frac{n\pi}{2} x \, dx + \int_{\frac{1}{2}}^{3/2} (4-4x) \sin \frac{n\pi}{2} x \, dx + \int_{\frac{3}{2}}^{2} (-2) \sin \frac{n\pi}{2} x \, dx$$

$1/2$ ... $\frac{1}{2}$

3.

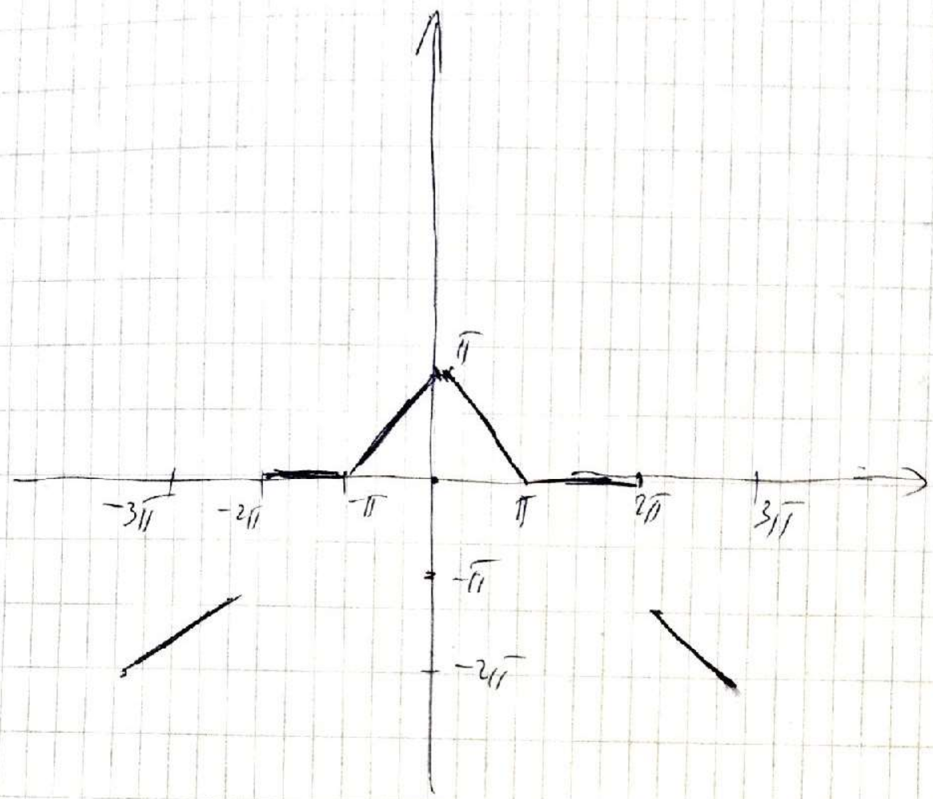
odgovarajuću skicu.

2. (15) Razviti u red po kosinusima na intervalu $(-3\pi, 0)$ funkciju

$$f(x) = \begin{cases} 0, & x \in [-2\pi, -\pi] \\ \pi + x, & x \notin [-2\pi, -\pi] \end{cases}.$$

Napomena: Napisati izraz za proširenu funkciju i skicirati njen grafik.

K2: $f(x) = \begin{cases} 0, & x \in [-2\pi, -\pi] \\ \pi + x, & x \notin [-2\pi, -\pi] \end{cases}$



$$F(x) = \begin{cases} f(x), & x \in (-3\pi, 0) \\ f(-x), & x \in (0, 3\pi) \end{cases}$$

$$F(x) = \begin{cases} 0, & x \in (-2\pi, -\pi) \\ \pi + x, & x \in (-3\pi, -2\pi) \cup (-\pi, 0) \\ 0, & x \in (\pi, 2\pi) \\ \pi - x, & x \in (0, \pi) \cup (2\pi, 3\pi) \end{cases}$$

$F(x)$ - razvijamo u Furijeov red na $(-3\pi\ 3\pi)$

F je parna pa je $b_n = 0$, $\forall n$.

$$a_n = \frac{2}{b-a} \int_a F(x) \cos \frac{2n\pi}{b-a} x\, dx, \quad \begin{matrix} a = -3\pi \\ b = 3\pi \end{matrix}$$

$$a_n = \frac{2}{6\pi} \underbrace{\int_{-3\pi}^{3\pi} \underbrace{F(x)}_{p.} \cdot \underbrace{\cos \frac{2n\pi}{6\pi} x}_{p.}\, dx}_{p.} =$$

$$= \frac{1}{3\pi} \cdot 2 \cdot \int_0^{3\pi} F(x) \cdot \cos \frac{nx}{3}\, dx =$$

$$= \frac{2}{3}\pi \int_0^{\pi} (\pi - x) \cdot \cos \frac{nx}{3}\, dx + \frac{2}{3}\pi \cdot \int_{\pi}^{2\pi} 0 \cdot \cos \frac{nx}{3}\, dx +$$

$$+ \frac{2}{3}\pi \cdot \int_{2\pi}^{3\pi} (\pi - x) \cdot \cos \frac{nx}{3}\, dx = \ldots$$