

Vježbe III

BCD kod (nastavak)

Oduzimanje u BCD kodu:

1. Umanjilac predstaviti komplementom devetke (jedinični komplement plus $10_{(10)}=1010_{(2)}$ koja se dodaje svakoj cifri pojedinačno, pri čemu se pretek sa cifre na cifru se odbacuje).
2. Umanjenik sabrati sa komplementom devetke umanjioaca (po pravilima sabiranja u BCD kodu).
3. Pretek 1 označava pozitivan rezultat (predznak +, nema uticaja na brojčanu vrijednost) i dobijenom rezultatu se dodaje 1. Ukoliko preteka nema rezultat je negativan, pa je potrebno naći komplement devetke rezultata.

1. Izvršiti oduzimanje u BCD kodu:

- a) 495 i 278, b) 278 i 495.

a)

495		0100	1001	0101
- 278		0010	0111	1000
217	j.k.	1101	1000	0111
	d.k.	1010	1010	1010
		0111	0010	0001
		0100	1001	0101
		0111	0010	0001
		1011	1011	0110
		0110	0110	
	①	0010	0001	0110
1 pretek ⇒ rezultat pozitivan				1
		0010	0001	0111
		2	1	7 ₍₁₀₎

b)

278		0010	0111	1000
- 495		0100	1001	0101
- 217	j.k.	1011	0110	1010
	d.k.	1010	1010	1010
		0101	0000	0100
		0010	0111	1000
		0101	0000	0100
		0111	0111	1100
				0110
	①	0111	1000	0010
0 pretek ⇒ rezultat negativan		1000	0111	1101
		1010	1010	1010
		0010	0001	0111
		-2	1	7

Bulova algebra

$$\begin{array}{ll} A + 0 = A & A \times 0 = 0 \\ A + 1 = 1 & A \times 1 = A \\ A + A = A & A \times A = A \\ A + \bar{A} = 1 & A \times \bar{A} = 0 \end{array} \quad \overline{\overline{A}} = A$$

De Morganova teorema

$$\begin{array}{l} \overline{A+B} = \bar{A} \times \bar{B} \\ \overline{A \times B} = \bar{A} + \bar{B} \end{array}$$

2. Koristeći pravila Bulove agebre uprostiti izraze:

a) $A + \bar{A}B$ b) $A+B+\bar{A}\bar{B}$

a) $A + \bar{A}B = A(1+B) + \bar{A}B = A + AB + \bar{A}B = A + B(A + \bar{A}) = A + B$

b) $A+B+\bar{A}\bar{B} = A+B+\overline{A+B} = 1$

3. Koristeći pravila Bulove agebre uprostiti izraze:

a) $F = \overline{AC} \times \overline{ABC} \times \overline{BC} + ABC$ b) $F = \overline{ABC} + \overline{ABC} + \overline{ABC} + \overline{ABC}$

a)

$$\begin{aligned} F = \overline{AC} \times \overline{ABC} \times \overline{BC} + ABC &= (\bar{A} + \bar{C}) \times (A + \bar{B} + \bar{C}) \times (B + \bar{C}) + ABC = \\ &= (\bar{A}\bar{A} + \bar{A}\bar{B} + \bar{A}\bar{C} + \bar{A}\bar{C} + \bar{C}\bar{B} + \bar{C}\bar{C}) \times (B + \bar{C}) + ABC = \\ &= (\bar{A}\bar{B} + \bar{C}(\bar{A} + A + \bar{C} + \bar{B})) \times (B + \bar{C}) + ABC = \\ &= (\bar{A}\bar{B} + \bar{C}(1 + \bar{C} + \bar{B})) \times (B + \bar{C}) + ABC = \\ &= (\bar{A}\bar{B} + \bar{C}) \times (B + \bar{C}) + ABC = \\ &= \bar{A}\bar{B}B + \bar{A}\bar{B}\bar{C} + \bar{C}B + \bar{C}\bar{C} + ABC = \bar{C}(\bar{A}\bar{B} + B + 1 + AB) = \bar{C} \end{aligned}$$

b)

$$\begin{aligned} F = \overline{ABC} + \overline{ABC} + \overline{ABC} + \overline{ABC} &= \overline{AB}(C + \bar{C}) + \overline{AB}(C + \bar{C}) = \\ &= \bar{A}\bar{B} + \bar{A}\bar{B} = \bar{A}(\bar{B} + \bar{B}) = \bar{A} \end{aligned}$$

4. Koristeći pravila Bulove agebre uprostiti funkciju.

$$\begin{aligned} F &= (A + \bar{C})(\overline{C+EB}) + (\overline{DC+EA})(\overline{B+C}) = (A + \bar{C})\overline{CEB} + (\overline{DC+EA})\bar{B}\bar{C} = \\ &= (A + \bar{C})\bar{C}(\bar{E} + \bar{B}) + \bar{B}\bar{C}\bar{D}\bar{C} + \bar{B}\bar{C}\bar{E}\bar{A} = (\bar{A}\bar{C} + \bar{C}\bar{C})(\bar{E} + \bar{B}) + \bar{B}\bar{C}\bar{E}\bar{A} = \\ &= \bar{C}(A + 1)(\bar{E} + \bar{B}) + \bar{B}\bar{C}\bar{E}\bar{A} = \bar{C}(\bar{E} + \bar{B}) + \bar{B}\bar{C}\bar{E}\bar{A} = \bar{C}\bar{E} + \bar{C}\bar{B} + \bar{B}\bar{C}\bar{E}\bar{A} = \\ &= \bar{C}\bar{E} + \bar{B}\bar{C}(1 + \bar{E}\bar{A}) = \bar{C}\bar{E} + \bar{B}\bar{C} = \bar{C}(\bar{E} + \bar{B}) \end{aligned}$$

5. Koristeći pravila Bulove algebre dokazati identitet $BC+ABD+A\bar{C}=BC+A\bar{C}$

$$\begin{aligned} BC+ABD+A\bar{C} &= BC+ABD(C+\bar{C})+A\bar{C} = BC+ACBD+A\bar{C}BD+A\bar{C} = \\ &= BC(1+AD)+A\bar{C}(BD+1) = BC+A\bar{C} \end{aligned}$$

6. Koristeći pravila Bulove algebre pokazati da važi $X_1 + X_2X_3 = (X_1 + X_2)(X_1 + X_3)$.

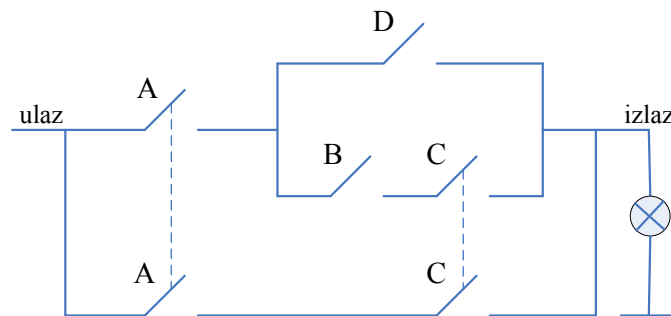
Posmatrajući desnu stranu uočavamo da nam nedostaju članovi X_1X_3 i X_1X_2 .

$$\begin{aligned} X_1 + X_2X_3 &= X_1(X_2 + \bar{X}_2) + X_2X_3 = \\ &= X_1X_2 + X_1\bar{X}_2 + X_2X_3 + X_1X_2 = \\ &= X_1(X_2 + \bar{X}_2) + X_2X_3 + X_1X_2 = \\ &= X_1 + X_2X_3 + X_1X_2 = \\ &= X_1(1 + X_3) + X_2X_3 + X_1X_2 = \\ &= X_1 + X_1X_3 + X_2X_3 + X_1X_2 = \\ &= X_1X_1 + X_1X_3 + X_2X_3 + X_1X_2 = \\ &= X_1(X_1 + X_3) + X_2(X_1 + X_3) = \\ &= (X_1 + X_2)(X_1 + X_3) \end{aligned}$$

7. Na slici je prikazana mreža sa relejima čiji kontakti A, B, C, D zauzimaju otvoren položaj u mirnom stanju.

a) Odrediti funkciju F koju realizuje data mreža uzimajući da je $F=1$ kada je put od ulaza ka izlazu mreže zatvoren.

b) Uprošćavanjem dobijene funkcije pokazati da njena vrijednost ne zavisi od položaja kontakta B.



a) Donji put:

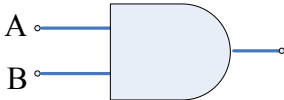
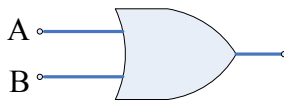
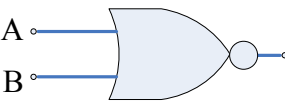
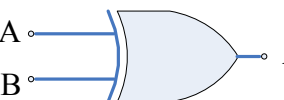
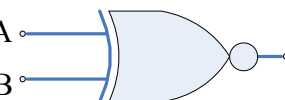
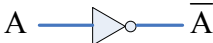
$$F=AC \text{ (i A i C moraju biti zatvoreni)}$$

Gornji put:

$$F=A(D+BC)$$

b) $F=AC+AD+ABC=AC(1+B)+AD=AC+AD=A(C+D) \Rightarrow$ ne zavisi od B

Logička kola i njihova funkcija

I kolo (množač) 	<table><tr><th>A</th><th>B</th><th>AB</th><th>$\overline{AB}$</th></tr><tr><td>0</td><td>0</td><td>0</td><td>1</td></tr><tr><td>0</td><td>1</td><td>0</td><td>1</td></tr><tr><td>1</td><td>0</td><td>0</td><td>1</td></tr><tr><td>1</td><td>1</td><td>1</td><td>0</td></tr></table>	A	B	AB	$\overline{AB}$	0	0	0	1	0	1	0	1	1	0	0	1	1	1	1	0	NI kolo 
A	B	AB	$\overline{AB}$																			
0	0	0	1																			
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1	0	0	1																			
1	1	1	0																			
ILI kolo (sabirač) 	<table><tr><th>A</th><th>B</th><th>A+B</th><th>$\overline{A+B}$</th></tr><tr><td>0</td><td>0</td><td>0</td><td>1</td></tr><tr><td>0</td><td>1</td><td>1</td><td>0</td></tr><tr><td>1</td><td>0</td><td>1</td><td>0</td></tr><tr><td>1</td><td>1</td><td>1</td><td>0</td></tr></table>	A	B	A+B	$\overline{A+B}$	0	0	0	1	0	1	1	0	1	0	1	0	1	1	1	0	NILI kolo 
A	B	A+B	$\overline{A+B}$																			
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EKSKLUZIVNO ILI kolo 	<table><tr><th>A</th><th>B</th><th>$A \oplus B$</th><th>$\overline{A \oplus B}$</th></tr><tr><td>0</td><td>0</td><td>0</td><td>1</td></tr><tr><td>0</td><td>1</td><td>1</td><td>0</td></tr><tr><td>1</td><td>0</td><td>1</td><td>0</td></tr><tr><td>1</td><td>1</td><td>0</td><td>1</td></tr></table>	A	B	$A \oplus B$	$\overline{A \oplus B}$	0	0	0	1	0	1	1	0	1	0	1	0	1	1	0	1	EKSKLUZIVNO NILI kolo 
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0	0	0	1																			
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INVERTOR 	<table><tr><th>A</th><th>$\overline{A}$</th></tr><tr><td>0</td><td>1</td></tr><tr><td>1</td><td>0</td></tr></table>	A	$\overline{A}$	0	1	1	0															
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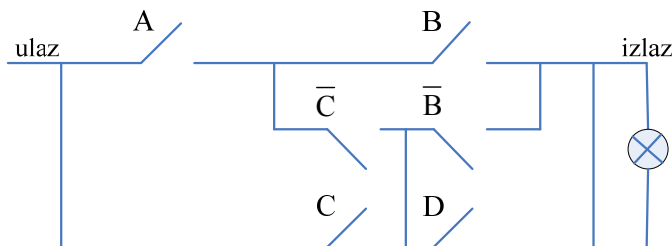
**Napomena:*

Dodavanjem invertora prije ulaska signala u posmatrano kolo invertuje se samo taj signal!

Na primjer:



8. Na slici je prikazana mreža sa položajima kontakta releja u mirnom stanju, pri čemu je kontakt zatvoren ako je vrijednost promjenljive koja njima upravlja jednak logičkoj jedinici. Odrediti funkciju spoljnih puteva ulaz-izlaz, i po mogućnosti uprostiti datu mrežu, a zatim je predstaviti pomoću logičkih kola.



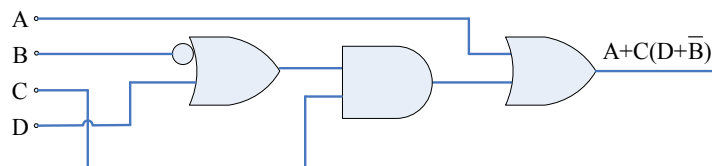
$$F = AB + A\bar{C}\bar{B} + CD + C\bar{B} + C\bar{C}\bar{B} + A\bar{C}D = AB + A\bar{C}(\bar{B} + D) + C(D + \bar{B}) = AB + (\bar{B} + D)(A\bar{C} + C) =$$

pri čemu je:

$$A\bar{C} + C = A\bar{C} + C(A + \bar{A}) = A\bar{C} + AC + \bar{A}C + AC = A(C + \bar{C}) + C(A + \bar{A}) = A + C$$

pa dobijamo da je funkcija:

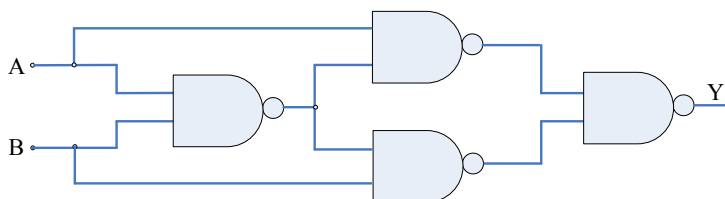
$$AB + (\bar{B} + D)(A + C) = AB + A\bar{B} + AD + C(\bar{B} + D) = A(B + \bar{B}) + AD + C(\bar{B} + D) = A(1 + D) + C(\bar{B} + D) = A + C(D + \bar{B})$$



9. Formirati „ekskluzivno ILI“ kolo koristeći samo NI kola.

$$Y = A \oplus B = \bar{A}B + A\bar{B}$$

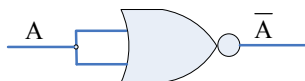
$$Y = \bar{A}B + A\bar{B} + \bar{A}\bar{A} + \bar{B}\bar{B} = A(\bar{A} + \bar{B}) + B(\bar{A} + \bar{B}) = A\bar{A}\bar{B} + B\bar{A}\bar{B} = \bar{A}\bar{B} + \bar{B}A\bar{B} = \bar{A}\bar{B} + \bar{B}A\bar{B} = (\bar{A}\bar{B})(\bar{B}A\bar{B})$$



10 Realizovati funkciju $Y = \bar{A}\bar{B} + AB$ koristeći samo NILI kola.

$$Y = \bar{A}\bar{B} + AB = \bar{A} + B + \bar{A}\bar{B} = \bar{A} + B + \bar{A} + \bar{B} = \bar{A} + B + \bar{A} + \bar{B}$$

Ukoliko na oba ulaza NILI kola dovedemo isti signal dobijamo inverter:



što nam je potrebno za krajnje komplementiranje u posmatranom izrazu. Primjetimo da konstruisanje invertora na isti način moguće učiniti upotrebom NI kola.

