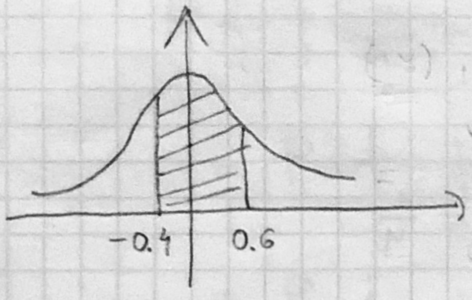
(6) Uspjeh – pojava grba. $p = \frac{1}{2}$, $q = \frac{1}{2}$, $n = 400$.

$$S_{400} : B\left(400, \frac{1}{2}\right)$$

$$P\left\{196 < S_{400} < 206\right\} =$$

$$= P\left\{ \frac{196 - 400 \cdot \frac{1}{2}}{\sqrt{400 \cdot \frac{1}{2} \cdot \frac{1}{2}}} < \frac{S_{400} - 400 \cdot \frac{1}{2}}{\sqrt{400 \cdot \frac{1}{2} \cdot \frac{1}{2}}} < \frac{206 - 400 \cdot \frac{1}{2}}{\sqrt{400 \cdot \frac{1}{2} \cdot \frac{1}{2}}} \right\}$$

$$= P\left\{ -0.4 < X^{*} < 0.6 \right\}$$



$$\overset{(\approx)}{=} \Phi^{*}(0.6) + \Phi^{*}(0.4)$$

$$\overset{(\approx)}{=} 0.2257 + 0.1554$$

$$= 0.3811.$$

pišemo "=" umjesto "≈"

(7) X_i – količina vode koja je dospjela u bazen u toku i-tog sata

$$X_i : \begin{pmatrix} 1 & 2 \\ 0.6 & 0.4 \end{pmatrix} \quad ; \quad EX_i = 1.4, \quad DX_i = 0.24, \quad i = 1, 2, \dots$$

Količina vode u bazenu poslije n sati je: $S_n = \sum_{i=1}^{n} X_i$

Neka je T broj sati potrebnih da se bazen napuni.

T je sluč. vel. i važi

$$100 \le T \le 200.$$

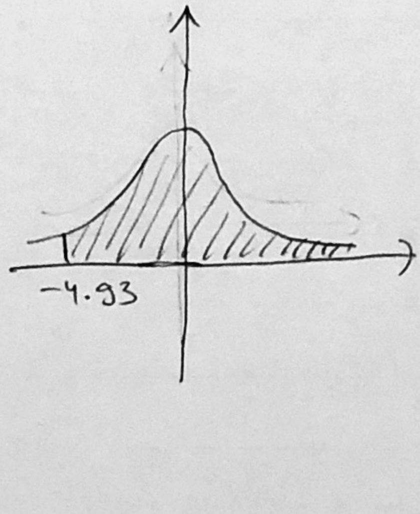
Uočimo da važi:

$$\{T \le n\} = \{S_n \ge 200\}.$$

Dakle,

$$\mathbb{P}\{T < 165\} = \mathbb{P}\{S_{165} > 200\}$$

$$= \mathbb{P}\left\{\frac{S_{165} - 165 \cdot 1.4}{\sqrt{165 \cdot 0.24}} > \frac{200 - 165 \cdot 1.4}{\sqrt{165 \cdot 0.24}}\right\}$$



$$= \mathbb{P}\{X^* > -4.93\}$$

$$= \frac{1}{2} + \Phi^*(4.93)$$

$$\approx 1$$

8

$n = 10000$

S_n - broj osiguranika koji će tražiti odštetu

$S_n : B(n, 0.006)$

X - dobitak osiguravajućeg društva

$X = 700 \cdot 10000 - 10000 \cdot S_n = 10000(700 - S_n)$

$$\mathbb{P}\{X > 0\} = \mathbb{P}\{10000(700 - S_n) > 0\}$$

$$= \mathbb{P}\{S_n < 700\}$$

$$= \mathbb{P}\left\{\frac{S_n - 60}{\sqrt{59.64}} < \frac{700 - 60}{\sqrt{59.64}}\right\}$$

$$= \mathbb{P}\{X^* < 82.87\}$$

$$\approx 1.$$

(9) X_i – broj automobila koji prođu raskrsnicu u toku i-tog minuta $i=\overline{1,120}$

$X_i : \mathcal{P}(6)$, nezavisne,

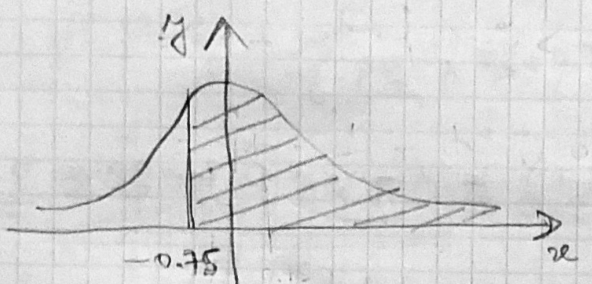
$EX_i = \lambda = 6 \qquad DX_i = \lambda = 6, \quad i=\overline{1,120},$

$S_{120} = \sum_{i=1}^{120} X_i$ – broj automobila koji kroz raskrsnicu prođu za 2 sata

$$P\{S_{120} \geq 700\} = P\left\{ \frac{S_{120} - 120EX_i}{\sqrt{120DX_i}} \geq \frac{700-6\cdot 120}{\sqrt{120\cdot 6}} \right\}$$

$$= P\{X^* \geq -0.75\} \overset{(\approx)}{=} \frac{1}{2} + \Phi^*(0.75)$$

$$= 0.5 + 0.2734 = 0.7734$$



14.

n – broj sijalica u rezervi

X_i – vrijeme trajanja i-te sijalice

$$X_i : \mathcal{E}(0.005)$$

$$EX_i = \frac{1}{0.005} = 200$$

$$DX_i = \frac{1}{(0.005)^2} = 40000 \,, \quad i = \overline{1,n}$$

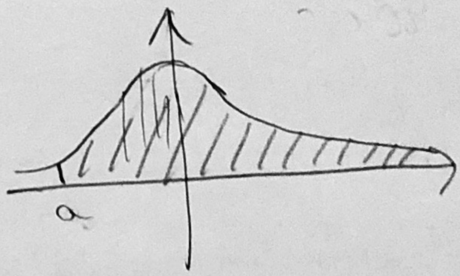
Treba naći n t.d. važi

$$\mathbb{P}\{S_n \geq 10000\} = 0.98.$$

$$0.98 = \mathbb{P}\{S_n \geq 10000\}$$

$$= \mathbb{P}\left\{\frac{S_n - 200n}{\sqrt{40000n}} \geq \frac{10000 - 200n}{\sqrt{40000\,n}}\right\}$$

$$= \mathbb{P}\left\{X^* \geq \underbrace{\frac{50-n}{\sqrt{n}}}_{a}\right\}$$



$$\Rightarrow \Phi^*(-a) = \Phi^*\left(\frac{n-50}{\sqrt{n}}\right) = 0.48$$

$$\Rightarrow \frac{n-50}{\sqrt{n}} = 2.06$$

$$n - 2.06\sqrt{n} - 50 = 0 \,, \quad t = \sqrt{n}$$

$$t^2 - 2.06t - 50 = 0$$

$$t_{1,2} = \frac{2.06 \pm \sqrt{104.24}}{2} = \begin{cases} -6.1157 < 0 \\ 8.1757 \end{cases} \Rightarrow \sqrt{n} = 8.1757$$

$$\Rightarrow \underline{\underline{n \approx 67}}$$

(16) X – broj posjetilaca koji su ušli na prvi ulaz

$X : B(1000, \frac{1}{2})$

n – broj mjesta po garderobi

Da bi bilo mjesta treba da važi:

$$X \leq n \ \wedge \ 1000 - X \leq n$$

odnosno

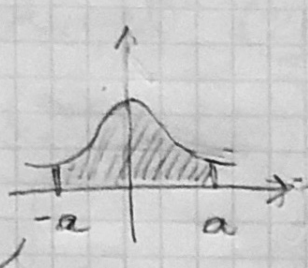
$$1000 - n \leq X \leq n.$$

$$0.99 = \mathbb{P}\{1000 - n \leq X \leq n\}$$

$$= \mathbb{P}\left\{ \frac{1000 - n - 1000 \cdot \frac{1}{2}}{\sqrt{1000 \cdot \frac{1}{2} \cdot \frac{1}{2}}} \leq \frac{X - 1000 \cdot \frac{1}{2}}{\sqrt{1000 \cdot \frac{1}{2} \cdot \frac{1}{2}}} \leq \frac{n - 1000 \cdot \frac{1}{2}}{\sqrt{1000 \cdot \frac{1}{2} \cdot \frac{1}{2}}} \right\}$$

$$= \mathbb{P}\left\{ \frac{500 - n}{5\sqrt{10}} \leq X^* \leq \frac{n - 500}{5\sqrt{10}} \right\}$$

$$= \mathbb{P}\left\{ -\frac{n - 500}{5\sqrt{10}} \leq X^* \leq \frac{n - 500}{5\sqrt{10}} \right\}$$



$$\overset{(\approx)}{=} 2\Phi^*\left(\frac{n - 500}{5\sqrt{10}}\right)$$

$$\Rightarrow \Phi^*\left(\frac{n - 500}{5\sqrt{10}}\right) = 0.495 \Rightarrow \frac{n - 500}{5\sqrt{10}} = 2.58$$

$$\Rightarrow \boxed{n \approx 541}$$