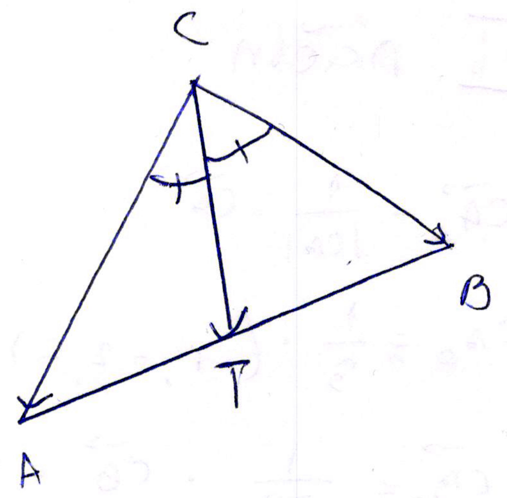
1. $A(3, -4, -1)$

$B(-1, 4, 3)$

$C(2, -2, -3)$

$\measuredangle(\vec{CA}, \vec{CT}) = \measuredangle(\vec{CB}, \vec{CT})$

$\Downarrow$



$\vec{CT}$ - vektor simetrale ugla u tjemenu C

Simetrala ugla trougla dijeli naspramnu stranicu u odnosu druge dvije stranice:

$$\frac{|\vec{AT}|}{|\vec{TB}|} = |\lambda| = \frac{|\vec{CA}|}{|\vec{CB}|} = \frac{3}{9}$$

$\vec{CA} = (1, -2, 2)$ $|\vec{CA}| = \sqrt{9} = 3$

$\vec{CB} = (-3, 6, 6)$ $|\vec{CB}| = \sqrt{81} = 9$

$$\boxed{\lambda = \frac{1}{3}}$$

$T(x, y, z)$

$$x = \frac{x_A + \lambda x_B}{1 + \lambda} = \frac{3 + \frac{1}{3}}{1 + \frac{1}{3}} = \frac{\frac{8}{3}}{\frac{4}{3}} = 2$$

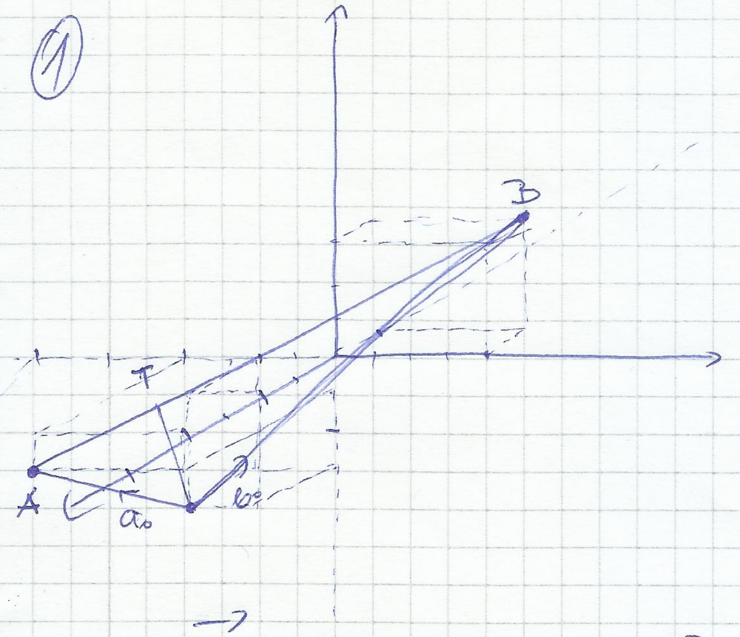
$$y = \frac{y_A + \lambda y_B}{1 + \lambda} = \frac{-\frac{8}{3}}{\frac{4}{3}} = -2$$

$$z = \frac{z_A + \lambda z_B}{1 + \lambda} = \frac{-1 + 1}{\frac{4}{3}} = 0$$

$$\boxed{T(2, -2, 0)}$$

Pavle Džuverović, 17/20, ETR

1



$\triangle ABC$ $\qquad$ $T = ?$

$A(3,-4,-1)$ $\qquad$ $T \in AB$

$B(-1,4,3)$ $\qquad$ $\measuredangle(\vec{CA},\vec{CT}) = \measuredangle(\vec{CB},\vec{CT})$

$C(2,-2,-3)$

$\vec{CA} = (1,-2,2)$ $\qquad$ $\vec{a_0} = \dfrac{\vec{CA}}{|\vec{CA}|} = \dfrac{(1,-2,2)}{3} = \left(\frac{1}{3}, -\frac{2}{3}, \frac{2}{3}\right)$

$\vec{CB} = (-3,6,6)$

$\vec{AB} = (-4,8,4)$ $\qquad$ $\vec{b_0} = \dfrac{\vec{CB}}{|\vec{CB}|} = \dfrac{(-3,6,6)}{9} = \left(-\frac{1}{3}, \frac{2}{3}, \frac{2}{3}\right)$

$\vec{CT} = k(\vec{a_0} + \vec{b_0}) = k\left(0,0,\frac{4}{3}\right) = \left(0,0,\frac{4k}{3}\right)$

$\vec{AT} = m \cdot \vec{AB} = m(-4,8,4) = (-4m, 8m, 4m)$

$\vec{CT} = \vec{CA} + \vec{AT}$

$\left(0,0,\frac{4k}{3}\right) = (1,-2,2) + (-4m, 8m, 4m)$

$\left(0,0,\frac{4k}{3}\right) = (1-4m, 8m-2, 4m+2)$

$m = \dfrac{1}{4}$ $\qquad$ $\dfrac{4k}{3} = 4m+2$

$\dfrac{4k}{3} = \cancel{4} \cdot \dfrac{1}{\cancel{4}} + 2$ $\qquad$ $\vec{CT} = (0,0,3)$

$k = 3 \cdot \dfrac{3}{4} = \dfrac{9}{4}$ $\qquad$ $T = (2,-2,0)$

Мирка Миладиновић 14/20 ЕТР

Aleksandra Šćekić 57/20
Matija Popović 15/20
elektronika, telekomunikacije i računari

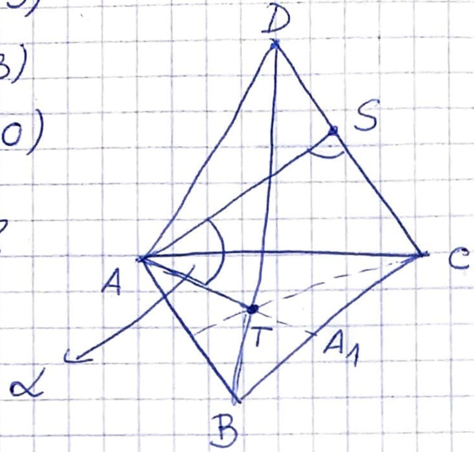
② $A(0,-2,0)$

$B(2,0,-3)$

$C(1,-1,3)$

$D(-2,2,0)$

$\sphericalangle SAT = ?$



$\vec{AT} = \frac{2}{3} AA_1$ $\quad \vec{CD} = (-3,3,-3)$

$\vec{AB} = (2,2,-3)$

$\vec{BC} = (-1,-1,6)$

$\vec{AC} = (1,1,3)$

$|\vec{CD}| = \sqrt{9+9+9} = 3\sqrt{3}$

$|CD|^2 = 27$

$\vec{AA_1} = \vec{AB} + \vec{BA_1}$

$\vec{AT} = \frac{2}{3}\vec{AA_1} \Rightarrow \vec{AT} = \frac{2}{3}\left(\vec{AB} + \frac{1}{2}\vec{BC}\right)$

$\vec{AT} = \frac{2}{3}\left(2-\frac{1}{2}, 2-\frac{1}{2}, -3+3\right) =$

$= \frac{2}{3}\left(\frac{3}{2}, \frac{3}{2}, 0\right) \Rightarrow \boxed{\begin{array}{l}\vec{AT} = (1,1,0)\\ |\vec{AT}| = \sqrt{1+1} = \sqrt{2}\end{array}}$

$\vec{AS} = \vec{AC} + \vec{CS}$

$\vec{AS} = \vec{AC} + \lambda \cdot \vec{CD} \,/\cdot \vec{CD}$

$0 = \vec{AC}\cdot\vec{CD} + \lambda\cdot|\vec{CD}|^2$

$(1,1,3)(-3,3,-3) + 27\cdot\lambda = 0$

$-3+3-9+27\lambda = 0$

$\boxed{\lambda = \frac{1}{3}}$

$\Rightarrow \vec{AS} = (1,1,3) + (-1,1,-3)$

$\boxed{\begin{array}{l}\vec{AS} = (0,2,2)\\ |\vec{AS}| = \sqrt{4+4} = 2\sqrt{2}\end{array}}$

$\vec{AT}\cdot\vec{AS} = |\vec{AT}||\vec{AS}|\cdot\cos\alpha$

$\cos\alpha = \frac{\vec{AT}\cdot\vec{AS}}{|\vec{AT}|\cdot|\vec{AS}|}$

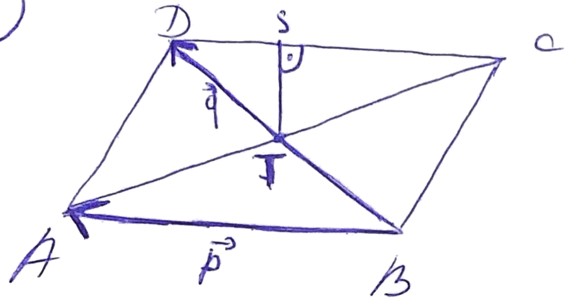
$\cos\alpha = \frac{(1,1,0)\cdot(0,2,2)}{\sqrt{2}\cdot 2\sqrt{2}}$

$= \frac{0+2+0}{4}$

$= \frac{1}{2}$

$\Rightarrow \boxed{\alpha = \frac{\pi}{3}}$

3.



$\vec{ST} = ?$

$$\vec{ST} = \vec{SD} + \vec{DT}$$

$$\vec{ST} = \lambda\vec{p} + \left(-\frac{1}{2}\right)\vec{q} \quad / \cdot \quad \vec{CD} = \vec{p}$$

$$0 = \lambda|\vec{p}|^2 - \frac{1}{2}\vec{p}\vec{q}$$

$$\lambda|\vec{p}|^2 = \frac{1}{2}\vec{p}\vec{q}$$

$$\lambda = \frac{\frac{1}{2}\vec{p}\vec{q}}{|\vec{p}|^2}$$

$$\boxed{\vec{ST} = \frac{1}{2}\frac{\vec{p}\vec{q}}{|\vec{p}|^2}\cdot\vec{p} - \frac{1}{2}\vec{q}}$$

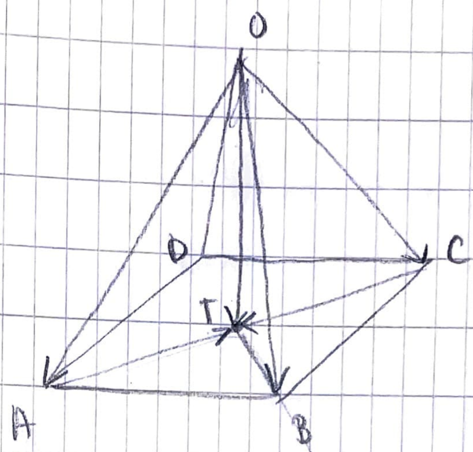
Anja Racković 89/20 ETR

Ksenija Maraš 26/20 ETR

Matija Lukačević 37/20 ETR

4. Vušanović Milica 4/20, ETR

$\vec{OA} = (0, 2, -1)$
$\vec{OC} = (-2, -2, 3)$
$\vec{OT} = ?$
$\vec{OB} = ?$



$$\left.\begin{array}{l}\vec{OT} = \vec{OA} + \vec{AT} \\ \vec{OT} = \vec{OC} + \vec{CT}\end{array}\right\} \text{ „+"}$$

$$\vec{OT} + \vec{OT} = \vec{OA} + \cancel{\vec{AT}} + \vec{OC} + \cancel{\vec{CT}}$$
$$2\vec{OT} = \vec{OA} + \vec{OC}$$
$$\vec{OT} = \frac{1}{2}(\vec{OA} + \vec{OC}) \Rightarrow \vec{OT} = \frac{1}{2}(-2, 0, 2)$$
$$\sim \vec{OT} = (-1, 0, 1) \sim$$

$\vec{OB} = \vec{OT} + \vec{TB}$

$\vec{c} = \vec{TO} \times \vec{TA}$ $\qquad \vec{c} \parallel \vec{TB}$

$\vec{TA} = \frac{1}{2}\vec{CA}$
$\vec{TA} = \frac{1}{2}(\vec{OA} - \vec{OC})$
$\vec{TA} = \frac{1}{2}(2, 4, -4)$
$\vec{TA} = (1, 2, -2)$

$$\vec{c} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 0 & -1 \\ 1 & 2 & -2 \end{vmatrix} = i(0+2) - \vec{j}(-2+1) + \vec{k}(2-0)$$

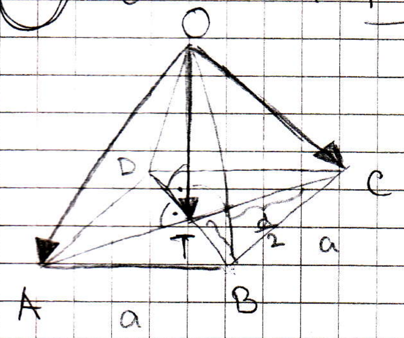
$\vec{c} = 2\vec{i} + \vec{j} + 2\vec{k}$
$\vec{c} = (2, 1, 2) \Rightarrow |\vec{c}| = \sqrt{9} = 3$

$$\vec{r_0} = \frac{1}{|\vec{c}|} \cdot \vec{c} = \frac{1}{3} \cdot (2, 1, 2) = \left(\frac{2}{3}, \frac{1}{3}, \frac{2}{3}\right)$$

$$\vec{TB} = |\vec{TB}| \cdot \vec{r_0} = 3 \cdot \left(\frac{2}{3}, \frac{1}{3}, \frac{2}{3}\right) \Rightarrow \vec{TB}(2, 1, 2)$$
$|\vec{TB}| = |\vec{c}|$

$$\vec{OB} = \vec{OT} + \vec{TB}$$
$$\sim \vec{OB} = (-1, 0, 1) + (2, 1, 2) = (1, 1, 3) \sim$$

4. $\vec{OA} = (0,5,-1)$ $\quad \vec{OC} = (-4,3,1)$ $\quad \vec{OT}, \vec{OB} = ?$



$$\boxed{\vec{AC} = \vec{OC} - \vec{OA} = (-4-0, 3-5, 1+1) = (-4,-2,2)}$$

$$\vec{OT} = \vec{OC} + \underbrace{\vec{CT}}_{-\frac{1}{2}\vec{AC}} = (-4,3,1) - \frac{1}{2}(-4,-2,2) =$$

$$= (-4,3,1) + (2,1,-1) = (-4+2, 3+1, 1-1)$$

$$\boxed{\vec{OT} = (-2,4,0)}$$

$\vec{DB} = (x,y,z)$

$\vec{DB} \cdot \vec{AC} = 0 \qquad \vec{DB} \cdot \vec{OT} = 0$

$-4x - 2y + 2z = 0 \quad | :2 \qquad -2x + 4y = 0 \ (2)$

$-2x - y + z = 0 \ (1)$

$(1) \ominus (2) \Rightarrow -5y + z = 0 \Rightarrow \boxed{z = 5y}$

$-2x \underbrace{-y + 5y}_{4y} = 0 \Rightarrow \cancel{-2}x = \cancel{-4}y \Rightarrow \boxed{x = 2y} \qquad \vec{DB}(2y, y, 5y)$

$|\vec{DB}| = |\vec{AC}| = \sqrt{16+4+4} = \sqrt{24}$

$\sqrt{4y^2 + y^2 + 25y^2} = \sqrt{24} \quad |^2$

$30y^2 = 24$

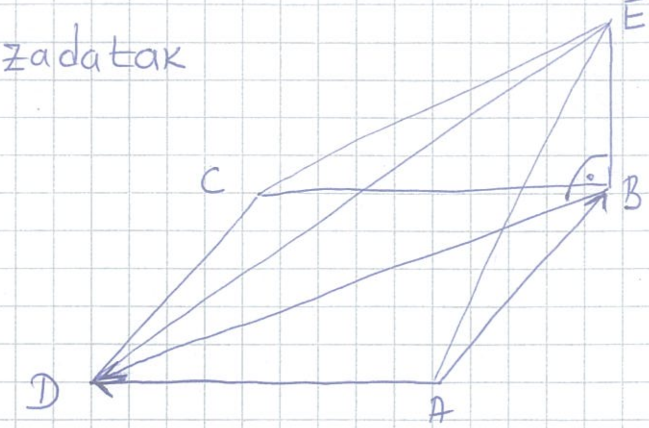
$y^2 = \frac{\cancel{24}}{\cancel{30}} = \frac{4}{5} \Rightarrow \boxed{y = \frac{2}{\sqrt{5}}} \quad \boxed{x = \frac{4}{\sqrt{5}}} \quad \boxed{z = \frac{10}{\sqrt{5}}}$

$\vec{DB} = \left(\frac{4}{\sqrt{5}}, \frac{2}{\sqrt{5}}, \frac{10}{\sqrt{5}}\right)$

$\vec{OB} = \vec{OT} + \frac{1}{2}\vec{DB} = (-2,4,0) + \left(\frac{2}{\sqrt{5}}, \frac{1}{\sqrt{5}}, \frac{5}{\sqrt{5}}\right) = \left(\frac{2-2\sqrt{5}}{\sqrt{5}}, \frac{1+4\sqrt{5}}{\sqrt{5}}, \frac{5}{\sqrt{5}}\right)$

Nada Jovović, EA, 46/20

5. zadatak



$V = 15$

$D(0,0,0)$

B – podnožje visine

$\overrightarrow{BD} = (1,1,0)$

$\overrightarrow{AB} = (3,2,1)$

$E = ?$

Petar Nedić
43/20 ETR

$\overrightarrow{BD} = (1,1,0)$, $D(0,0,0) \Rightarrow B(-1,-1,0)$

$\overrightarrow{AD} = \overrightarrow{AB} + \overrightarrow{BD} = (4,3,1)$

$V_p = \frac{1}{3} B \cdot H$, $H = |\overrightarrow{BE}|$

$45 = |\overrightarrow{AD} \times \overrightarrow{AB}| \cdot |\overrightarrow{BE}|$

$$\boxed{|\overrightarrow{BE}| = \frac{\cancel{45}^{15}\sqrt{3}}{\cancel{3}_1} = 15\sqrt{3}}$$

$$\overrightarrow{AD} \times \overrightarrow{AB} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 4 & 3 & 1 \\ 3 & 2 & 1 \end{vmatrix} = (3-2)\cdot\vec{i} - (4-3)\cdot\vec{j} + (8-9)\cdot\vec{k} =$$

$$= \vec{i} - \vec{j} - \vec{k} = (1,-1,-1)$$

$$|\overrightarrow{AD} \times \overrightarrow{AB}| = \sqrt{1+1+1} = \sqrt{3}$$

$$\left.\begin{array}{l} \overrightarrow{BE} \perp \overrightarrow{AD} \\ \overrightarrow{BE} \perp \overrightarrow{AB} \end{array}\right\} \overrightarrow{BE} = \lambda(\overrightarrow{AD} \times \overrightarrow{AB})$$

Vektori $\overrightarrow{BE}$ i $\overrightarrow{AD} \times \overrightarrow{AB}$ su suprotnog smjera ($\overrightarrow{AD} \times \overrightarrow{AB}$, $\overrightarrow{AD}$, $\overrightarrow{AB}$ čini lijevu trojku vektora) (~~pa su suprotnog smjera~~)

$\vec{h_0}$ – jedinični vektor visine $\overrightarrow{BE}$

$$\vec{h_0} = -\frac{\sqrt{3}}{3} \cdot (\overrightarrow{AD} \times \overrightarrow{AB})$$

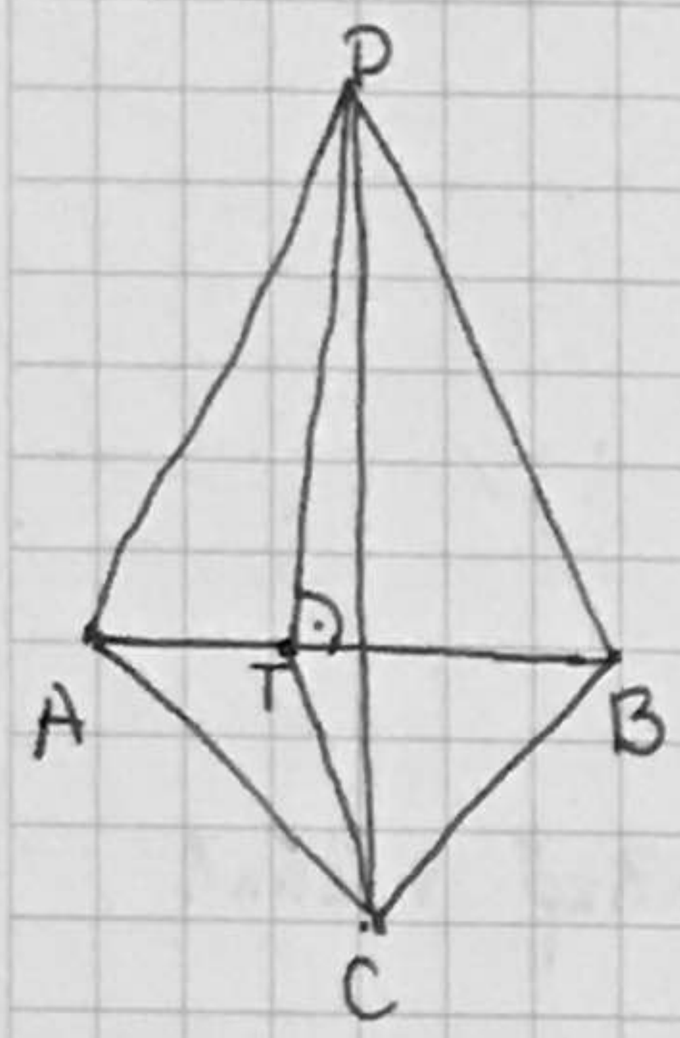
$$\overrightarrow{BE} = |\overrightarrow{BE}| \cdot \vec{h_0} = \cancel{15}^{5}\sqrt{3}\left(-\frac{\sqrt{3}}{\cancel{3}_1}\right) \cdot (\overrightarrow{AD} \times \overrightarrow{AB})$$

$$\overrightarrow{BE} = -15(\overrightarrow{AD} \times \overrightarrow{AB}) = -15(1,-1,-1)$$

$$\boxed{\overrightarrow{BE} = (-15, 15, 15)}, \quad B(-1,-1,0) \Rightarrow E(-16, 14, 15)$$

Albina Berishaj 83/20 ETR

6. Strane ABC i ABD tetraedra ABCD su jednakokraki trouglovi sa zajedničkom osnovicom AB koji leži u međusobno ortogonalnim ravnima. Neka je T podnožje visine tetraedra iz tjemena D. Ako je zapremina tetraedra $V=16$, a $\vec{AT}=(2,3,1)$ i $\vec{BC}=(1,0,2)$ odrediti ~~vektore~~ koordinate vektora $\vec{TD}$.



$V=16$

$\vec{AT}=(2,3,1)$

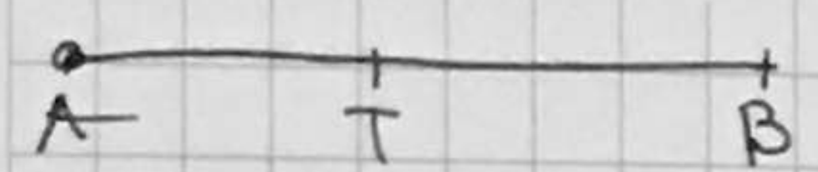
$\vec{BC}=(1,0,2)$

$\vec{TD}=?$

$\frac{1}{6}\left|(\vec{AC}\times\vec{AB})\cdot\vec{AD}\right|=16$

$\left|(\vec{AC}\times\vec{AB})\cdot\vec{AD}\right|=96$

$\Rightarrow |\vec{AC}\times\vec{AB}| = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 5 & 6 & 4 \\ 4 & 6 & 2 \end{vmatrix}\begin{matrix} \vec{i} & \vec{j} \\ 5 & 6 \\ 4 & 6 \end{matrix} = 12\vec{i}+16\vec{j}+30\vec{k}-24\vec{k}-24\vec{i}-10\vec{j} = (-12,6,6)$



$\vec{AB}=2\vec{AT} \Rightarrow \vec{AB}=(4,6,1)$

$\left.\begin{matrix} \vec{TD}\perp\vec{AC} \\ \vec{AB}\perp\vec{TD} \end{matrix}\right\}$ $AC\times AB \parallel \vec{TD} \Rightarrow \vec{TD}=\lambda|\vec{AC}\times\vec{AB}| \Rightarrow \lambda=\frac{\vec{TD}}{|\vec{AC}\times\vec{AB}|}=\frac{96}{216}=\frac{4}{9}$

$|\vec{TD}|=(-12\lambda, 6\lambda, 6\lambda)$

$(\vec{AC}\times\vec{AB})\cdot\vec{AD} = \cancelto{0}{(\vec{AC}\times\vec{AB})\cdot\vec{AT}} + (\vec{AC}\times\vec{AB})\cdot\vec{TD} = \lambda|\vec{AC}\times\vec{AB}|^2$

$|\vec{AC}\times\vec{AB}| = \sqrt{144+36+36} = \sqrt{216}$

$|\vec{AC}\times AB|^2 = 216$

$\vec{TD} \Rightarrow \frac{x_1+\lambda x_2}{1+\lambda}$

$x=-16/3 \quad y=8/3 \quad z=8/3$

$\vec{TD}=\left(-\frac{16}{3}, \frac{8}{3}, \frac{8}{3}\right)$

Vojnović Valentina 99/20 ETR
Stanišić Nikolina 88/20

6. ABC i ABD - jednokokraki

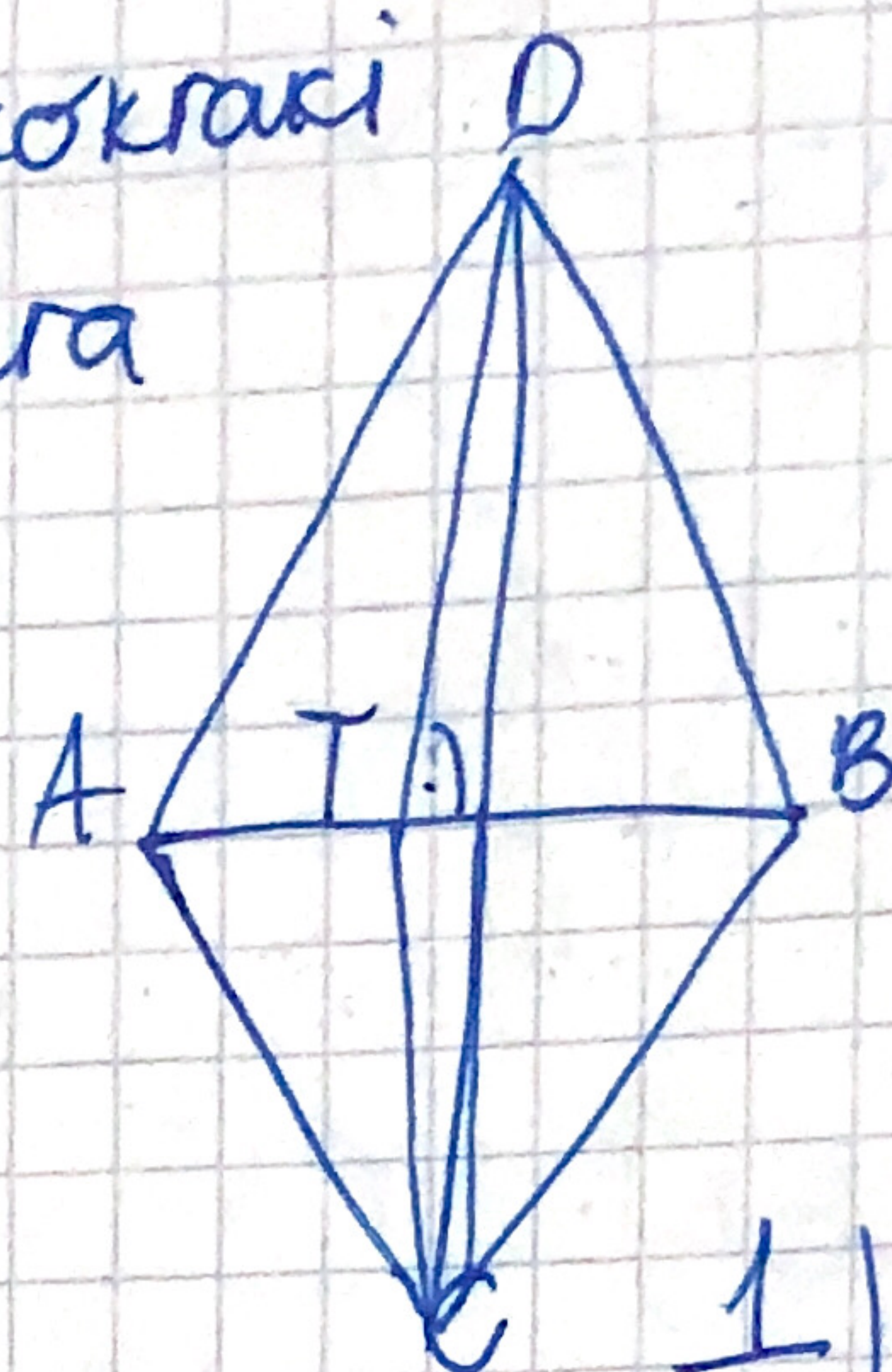
DT - visina tetraedra

$V=16$

$\vec{AT}=(2,3,1)$

$\vec{BC}=(1,0,2)$

$\vec{TD}=?$



$\frac{1}{6}\left|(\vec{AC}\times\vec{AB})\cdot\vec{AD}\right|=16$

$\left|(\vec{AC}\times\vec{AB})\cdot\vec{AD}\right|=36$

$\vec{AB}=2\cdot\vec{AT}=(4,6,2)$

$$\vec{AC}\times\vec{AB}=\left|\begin{array}{ccc|cc} \vec{i} & \vec{j} & \vec{k} & \vec{i} & \vec{j} \\ 5 & 6 & 4 & 5 & 6 \\ 4 & 6 & 2 & 4 & 6 \end{array}\right|$$

$$=12\vec{i}+16\vec{j}+30\vec{k}-24\vec{k}-24\vec{i}-10\vec{j}$$

$$=(-12,6,6)$$

$\vec{TD}\perp\vec{AC}$ i $\vec{AB}\perp\vec{TD}\Rightarrow\vec{TD}=(-12\lambda,6\lambda,6\lambda)$

$$(\vec{AC}\times\vec{AB})\cdot\vec{AD}=(\vec{AC}\times\vec{AB})\cdot\vec{AT}+(\vec{AC}\times\vec{AB})\cdot\vec{TD}=\lambda|\vec{AC}\times\vec{AB}|^2$$

$$|\vec{AC}\times\vec{AB}|=\sqrt{(-12)^2+6^2+6^2}=\sqrt{216}\Rightarrow|\vec{AC}\times\vec{AB}|^2=216$$

$$\lambda=\frac{96}{216}=\frac{4}{9}$$

$$\Rightarrow\vec{TD}=\left(-\frac{16}{3},\frac{8}{3},\frac{8}{3}\right)$$

⑦ $\vec m \perp \vec n \Rightarrow \vec m\cdot\vec n = 0 \Rightarrow \sphericalangle(\vec m,\vec n)=90^\circ$ Ralević Anđa, 71/20, ETR

$(\vec n+\vec p)\perp(\vec n-\vec p)$

$\sphericalangle(\vec m,\vec p) = \sphericalangle(\vec n,\vec p)=\alpha$

$|\vec n| = |\vec m\times\vec n| = (\vec n-4\vec m)\cdot\vec p = 2$

$\boxed{|\vec n|=2}$ $|\vec m\times \vec n| = |\vec m||\vec n|\cdot\sin\sphericalangle(\vec m,\vec n)$

$\cancel{2} = |\vec m|\cdot\cancel{2}\cdot 1$

$\boxed{|\vec m|=1}$

$(\vec n-4\vec m)\vec p=2$

$|\vec n|\cdot|\vec p|\cdot\cos\sphericalangle(\vec n,\vec p) - 4|\vec m||\vec p|\cdot\cos\sphericalangle(\vec m,\vec p)=2$

$\cos\sphericalangle(\vec n,\vec p)=\cos\sphericalangle(\vec m,\vec p)=\cos\alpha$

$\cancel{2}|\vec p|\cos\alpha - \cancel{4}^{2}|\vec p|\cos\alpha=\cancel{2}$

$-|\vec p|\cos\alpha=1$

$\cos\alpha=-\frac{1}{|\vec p|}$ $\boxed{\cos\alpha=-\frac{1}{2}}$

$(\vec n+\vec p)\perp(\vec n-\vec p)\Rightarrow(\vec n+\vec p)(\vec n-\vec p)=0$

$|\vec n|^2-|\vec p|^2=0$

$|\vec n|^2=|\vec p|^2$

$|\vec n|=|\vec p|$

$\boxed{|\vec p|=2}$

$|\cancel{(\vec m\cdot\vec n)}^{0}\vec m + (\vec n\cdot\vec p)\cdot\vec n + (\vec p\cdot\vec m)\cdot\vec p| =$

$=|(|\vec n||\vec p|\cdot\cos\alpha)\cdot\vec n + (|\vec p||\vec m|\cdot\cos\alpha)\vec p| =$

$=|[\cancel{2}\cdot 2\cdot(-\frac{1}{\cancel{2}})]\vec n + [\cancel{2}\cdot 1\cdot(-\frac{1}{\cancel{2}})]\vec p|=$

$|-2\vec n-\vec p| = |-\sqrt{(2\vec n+\vec p)^2}| = |-\sqrt{4|\vec n|^2+4\vec n\cdot\vec p+|\vec p|^2}| =$

$=|-\sqrt{4\cdot4+4\cdot2\cdot\cancel{2}\cdot(-\frac{1}{\cancel{2}})+4}| = |-\sqrt{20-8}| = |\sqrt{12}| = 2\sqrt{3}$