1. Vušanović Milica 4/2020
Džuverović Pavle 17/2020

$z = ?$

$|\bar{z}| = |z+1|$, $\arg(\bar{z}^2) = \arg\left(\frac{(1+i)^7}{(1-i)^3}\right)$, $z = x+yi$

$\sim \quad |x-yi| = |x+yi+1|$

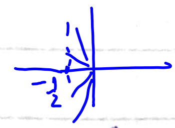
$\sqrt{x^2+y^2} = \sqrt{(x+1)^2+y^2} \;/^2$

$x^2+y^2 = (x+1)^2+y^2$

$\cancel{x^2}+\cancel{y^2} = \cancel{x^2}+2x+1+\cancel{y^2}$

$2x = -1 \Rightarrow x = -\frac{1}{2}$

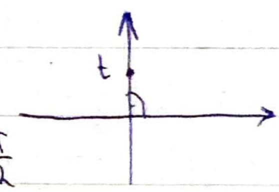
$\arg z \in \left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$



$\sim \frac{(1+i)^7}{(1-i)^3} = \frac{(1+i)((1+i)^2)^3}{(1-i)(1-i)^2} = \frac{(1+i)(\cancel{1}+2i+\cancel{i^2})^3}{(1-i)(\cancel{1}-2i+\cancel{i^2})} = \frac{(1+i)\,8i^3}{-2i(1-i)} = \frac{(1+i)\cdot 8\cdot(-i)}{-2i-2} =$

$= \frac{-8i(1+i)}{2(-i-1)} = \frac{-4i(1+i)}{-i-1} = \frac{-4i-4i^2}{-i-1} = \frac{4-4i}{-1-i} = \frac{4(1-i)}{-1-i}\cdot\frac{-1+i}{-1+i} = \frac{4(1-i)(i-1)}{1-i^2} =$

$= \frac{4(i-1-i^2+i)}{2} = 2(2i) = 4i$



$t = 4i$, $t = |t|\cdot(\cos\varphi + i\sin\varphi)$; $|t| = \sqrt{4^2} = 4$, $tg\varphi = +\frac{b}{a} = 0 \Rightarrow \varphi = \frac{\pi}{2}$

$\arg(\bar{z}^2) = \frac{\pi}{2}$

$(\bar{z}^2) = |\bar{z}|^2 \cdot \cos(2\varphi + i\sin 2\varphi)$

$2\varphi = \frac{\pi}{2} + 2k\pi \;/:2$

$\varphi = \frac{\pi}{4} + k\pi \quad , k = 0,1$

za $k=0 \Rightarrow \varphi = \frac{\pi}{4}$

za $k=1 \Rightarrow \varphi = \frac{5\pi}{4}$

~~1)~~ $\arg\bar{z} = \frac{\pi}{4}$

$\arg z + \arg\bar{z} = 2\pi$

$\arg z = 2\pi - \frac{\pi}{4} = \frac{7\pi}{4} \notin \left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$

$tg\varphi = \frac{y}{x}$

$tg\left(\frac{7\pi}{4}\right) = \frac{y}{x} \Rightarrow tg\left(\pi + \frac{3\pi}{4}\right) = \frac{y}{x}$

$tg\left(\pi - \frac{\pi}{4}\right) = \frac{y}{x} \Rightarrow -tg\frac{\pi}{4} = \frac{y}{x}$

$-1 = \frac{y}{x} \Rightarrow y = -1\cdot x = -1\cdot\left(-\frac{1}{2}\right) = \frac{1}{2}$

$\Downarrow$

rešenje $z = -\frac{1}{2} + \frac{1}{2}i$

2) $\arg\bar{z} = \frac{5\pi}{4} \Rightarrow \arg z = \frac{3\pi}{4}$

$tg\left(\pi - \frac{\pi}{4}\right) = \frac{y}{x}$

$\Downarrow$ dobijemo isti rezultat!

$z = -\frac{1}{2} + \frac{1}{2}i$

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2. Zadatak

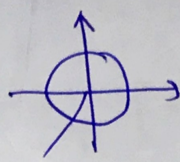
$z = ?$

$w = -\sqrt{3} - 3i$

$Im(-\sqrt{3}|z|w + i|\bar{z}|\bar{w}) = \sqrt{3}$, $arg(z^4) = arg(w^{11})$

$|w| = \sqrt{3+9} = \sqrt{12} = \sqrt{4 \cdot 3} = 2\sqrt{3}$

$tg\varphi = \frac{-3}{-\sqrt{3}} = \frac{3\sqrt{3}}{3} = \sqrt{3}$



$\Rightarrow \boxed{arg\, w = \frac{4\pi}{3}}$

$-\sqrt{3}|z|(-\sqrt{3} - 3i) + i|\bar{z}|(-\sqrt{3} + 3i) =$

$= (3 + 3\sqrt{3}i)|z| + (-\sqrt{3}i - 3)|\bar{z}| =$

$|z| = |\bar{z}|$

$= \cancel{3|z|} + 3\sqrt{3}\, i|z| - \sqrt{3}i|\bar{z}| - \cancel{3|\bar{z}|} \Rightarrow$

$\Rightarrow 2\sqrt{3}|z| = \sqrt{3} \; / : \sqrt{3}$

$2|z| = 1$

$\boxed{|z| = \frac{1}{2}}$

$w^{11} = |w|^{11}(\cos 11\varphi + i \sin 11\varphi)$

$w^{11} = |w|^{11}\left(\cos \frac{44\pi}{3} + i \sin \frac{44\pi}{3}\right)$

$w^{11} = |w|^{11}\left(\cos\left(14\pi + \frac{2\pi}{3}\right) + i \sin\left(14\pi + \frac{2\pi}{3}\right)\right)$

$w^{11} = |w|^{11}\left(\cos \frac{2\pi}{3} + i \sin \frac{2\pi}{3}\right)$

$\boxed{arg\, w^{11} = \frac{2\pi}{3}} = arg(z^4)$

$z^4 = |z|^4(\cos 4\varphi + i \sin 4\varphi)$

$4\varphi = \frac{2\pi}{3} + 2k\pi$

$\boxed{\text{Za } k = 0}$

$4\varphi = \frac{2\pi}{3} \Rightarrow \boxed{\varphi = \frac{\pi}{6}}$

$\boxed{\text{Za } k = 1}$

$4\varphi = \frac{2\pi}{3} + 2k\pi$

$4\varphi = \frac{2\pi}{3} + 2\pi$

$\boxed{\varphi = \frac{2\pi}{3}}$

Za $k = 2$

$4\varphi = \frac{2\pi}{3} + 4\pi$

$\boxed{\varphi = \frac{7\pi}{6}}$

$\boxed{Za\ k=3}$

$$4\varphi = \frac{2\pi}{3} + 2k\pi$$

$$4\varphi = \frac{2\pi}{3} + 6\pi$$

$$\boxed{\varphi = \frac{5\pi}{3}}$$

$\boxed{Za\ k=4}$

$$4\varphi = \frac{2\pi}{3} + 2k\pi$$

$$4\varphi = \frac{2\pi}{3} + 8\pi$$

$$\cancel{\boxed{\varphi = \frac{13\pi}{6} > 2\pi}}$$

$$z_0 = \frac{1}{2}\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$$

$$z_1 = \frac{1}{2}\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$$

$$z_2 = \frac{1}{2}\left(\cos\frac{7\pi}{6} + i\sin\frac{7\pi}{6}\right)$$

$$z_3 = \frac{1}{2}\left(\cos\frac{5\pi}{3} + i\sin\frac{5\pi}{3}\right)$$

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③ $\sqrt[3]{z} = ?$

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- $\arg\left(z(\sqrt{11} - i\sqrt{11})^{11}\right) = \pi$

$t = \sqrt{11} - i\sqrt{11} \quad |t| = \sqrt{22} \quad tg\alpha = -1 \quad \arg t = \frac{7\pi}{4}$

$\arg t^{11} = 11 \cdot \frac{7\pi}{4} = \frac{77\pi}{4} = 18\pi + \frac{5\pi}{4} = \underline{\underline{\frac{5\pi}{4}}}$

$\arg z = \varphi$

$\varphi + \frac{5\pi}{4} = \pi + 2k\pi$

$\varphi = -\frac{\pi}{4} + 2k\pi$

$\varphi = -\frac{\pi}{4} = \frac{7\pi}{4}$

- $\operatorname{Im}\left(5 + i + \sqrt[5]{|52 - i|} + 5i \operatorname{Re}\left(\frac{|z|}{i-2}\right)\right) = \frac{3}{4}$

$\frac{|z|}{i-2} \cdot \frac{i+2}{i+2} = -\frac{|z|}{5} i - \frac{2|z|}{5}$

$\operatorname{Im}\left(5 + i + \sqrt[5]{|52 - i|} + \cancel{5} i \cdot \left(-\frac{2|z|}{\cancel{5}}\right)\right) = \frac{3}{4}$

$1 - 2|z| = \frac{3}{4}$

$|z| = \frac{1}{8}$

$\sqrt[3]{z} = \sqrt[3]{|z|}\left(\cos\frac{\varphi + 2k\pi}{n} + i \sin\frac{\varphi + 2k\pi}{n}\right)$

$\sqrt[3]{z} = \frac{1}{2}\left(\cos\frac{\frac{7\pi}{4} + 2k\pi}{3} + i\sin\frac{\frac{7\pi}{4} + 2k\pi}{3}\right)$

$k = 0 \quad z_0 = \frac{1}{2}\left(\cos\frac{7\pi}{12} + i\sin\frac{7\pi}{12}\right)$

$k = 1 \quad z_1 = \frac{1}{2}\left(\cos\frac{15\pi}{12} + i\sin\frac{15\pi}{12}\right)$

$k = 2 \quad z_2 = \frac{1}{2}\left(\cos\frac{23\pi}{12} + i\sin\frac{23\pi}{12}\right)$

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4. $|z| = |w| = 1$ $\quad \arg(-iz^2) = 0$

$\frac{z}{w} = n \; , n<0 , n \in R$

w - pripada donjoj poluravni

$-i = (\cos\frac{3\pi}{2} + i\sin\frac{3\pi}{2})$

$-iz^2 = |z|^2 (\cos 2\varphi + \frac{3\pi}{2} + i\sin 2\varphi + \frac{3\pi}{2})$

$2\varphi + \frac{3\pi}{2} = 0 + 2k\pi$

$2\varphi = -\frac{3\pi}{2} + 2k\pi$

$\varphi = -\frac{3\pi}{4} + k\pi \; ; \; \varphi = \frac{5\pi}{4} + k\pi$

$k=0 , \varphi = \frac{5\pi}{4}$

$k=1 , \varphi = \frac{\pi}{4}$

$\frac{z}{w} = \frac{|z|}{|w|} (\cos(\varphi - \alpha) + i\sin(\varphi - \alpha))$

$\cos(\varphi - \alpha) < 0$

$\sin(\varphi - \alpha) = 0$

za $\sin\alpha < 0$ $\quad w$ pripada donjoj poluravni

$\sin(\varphi - \alpha) = 0$

$\frac{5\pi}{4} - \alpha = 0 + k\pi \quad \lor \quad \frac{\pi}{4} - \alpha = 0 + k\pi$

$k=0 , \alpha = \frac{5\pi}{4} \; ; \; k=1 , \alpha = \frac{\pi}{4} \quad \lor \quad \alpha = \frac{\pi}{4} - k\pi$

$\alpha = \frac{\pi}{4} \; ; \; \alpha = \frac{5\pi}{4}$ — $\sin\frac{5\pi}{4} < 0$

$\varphi = \frac{\pi}{4}$

$z = \cos\frac{\pi}{4} + i\sin\frac{\pi}{4} = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} i$

$w = \cos\frac{5\pi}{4} + i\sin\frac{5\pi}{4} = -\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} i$

5.

$$\begin{cases} ax + 2y - az = 1 \\ -y + az = 0 \\ ax + y = a \end{cases}$$

$$\det A = \begin{vmatrix} a & 2 & -a \\ 0 & -1 & a \\ a & 1 & 0 \end{vmatrix} = 2a^2 - a^2 - a^2 = 0$$

$D = 0$

$$D_1 = \begin{vmatrix} 1 & 2 & -a \\ 0 & -1 & a \\ a & 1 & 0 \end{vmatrix} = 2a^2 - a^2 - a = a(a-1)$$

$$D_2 = \begin{vmatrix} a & 1 & -a \\ 0 & 0 & a \\ a & a & 0 \end{vmatrix} = a^2 - a^3 = a^2(1-a)$$

$$D_3 = \begin{vmatrix} a & 2 & 1 \\ 0 & -1 & 0 \\ a & 1 & a \end{vmatrix} = -a^2 + a = a(1-a)$$

1° za $a=0$ $D = D_1 = D_2 = D_3 = 0 \Rightarrow$ rešenje se traži korišćenjem druge metode

$$\begin{cases} 2y = 1 \\ -y = 0 \\ y = 0 \end{cases} \Rightarrow \begin{cases} y = \frac{1}{2} \\ y = 0 \\ y = 0 \end{cases}$$ 1

Sistem je nesaglasan

2° $a = 1$ $D = D_1 = D_2 = D_3 = 0 \rightarrow$ koristi se druga metoda

$$\begin{cases} x + 2y - z = 1 \\ -y + z = 0 \\ x + y = 1 \end{cases} \Rightarrow \begin{matrix} y = z \\ x = 1 - z \end{matrix}$$

$1 - z + 2z - z = 1$

$0z = 0$

$$\begin{cases} y = z \\ x = 1 - z \\ z \end{cases}$$

Sistem je saglasan, neodređen

Opšte rešenje:

$(1-z, z, z)$, z - slobodna nepoznata

3° $a \neq 0 \wedge a \neq 1$ $D = 0 \wedge D_1, D_2, D_3 \neq 0$

$\Rightarrow$ Sistem je nesaglasan

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6.
$$\begin{aligned} x_1+2x_2+x_3&=1\\ 2x_1+4x_2+(a^2+1)x_3&=a+1\\ x_1+2x_2+(2-a^2)x_3&=2-a \end{aligned}$$

$$P=\left(\begin{array}{ccc|c} 1 & 2 & 1 & 1\\ 2 & 4 & a^2+1 & a+1\\ 1 & 2 & 2-a^2 & 2-a \end{array}\right) \begin{matrix}(-2)\,|\,(-1)\\ +\\ +\end{matrix} \sim \left(\begin{array}{ccc|c} 1 & 2 & 1 & 1\\ 0 & 0 & a^2-1 & a-1\\ 0 & 0 & 1-a^2 & 1-a \end{array}\right) \begin{matrix}(-1)\\ +\end{matrix} \sim \left(\begin{array}{ccc|c} 1 & 2 & 1 & 1\\ 0 & 0 & a^2-1 & a-1\\ 0 & 0 & 0 & 0 \end{array}\right)$$

1) $a^2-1=0$

$a=1 \lor a=-1$

a) $a=1$

$$P\sim\left(\begin{array}{ccc|c} 1 & 2 & 1 & 1\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 \end{array}\right)$$

rang A = rang P = 1 < 3

систем је сагласан и неодређен

$x_1+2x_2+x_3=1$

$x_1=1-2x_2-x_3$

$(x_1,x_2,x_3)=(1-2x_2-x_3,x_2,x_3)$

Опште решење

б) $a=-1$

$$P\sim\left(\begin{array}{ccc|c} 1 & 2 & 1 & 1\\ 0 & 0 & 0 & -2\\ 0 & 0 & 0 & 0 \end{array}\right)$$

rang A = 1

rang P = 2

систем је несагласан

2) $a^2-1\neq 0 \quad a\neq 1 \land a\neq -1$

$$P\sim\left(\begin{array}{ccc|c} 1 & 2 & 1 & 1\\ 0 & 0 & a^2-1 & a-1\\ 0 & 0 & 0 & 0 \end{array}\right)$$

rang A = rang P = 2 < 3

Систем је сагласан и неодређен

$$\begin{cases} x_1+2x_2+x_3=1\\ (a^2-1)x_3=a-1 \end{cases} \qquad \begin{cases} x_2=\frac{1}{2}(1-x_1-x_3)\\ (a+1)(a-1)x_3=a-1 \end{cases}$$

$x_2=\frac{1}{2}\left(1-x_1-\frac{1}{a+1}\right)$

$x_3=\frac{1}{a+1}$

Опште решење: $(x_1,x_2,x_3)=\left(x_1,\ \frac{1}{2}\left(1-x_1-\frac{1}{a+1}\right),\ \frac{1}{a+1}\right)$

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7. $((X\cdot A)^{-1}+B^{-1})^{-1}=X$

$$\left[((X\cdot A)^{-1}+B^{-1})^{-1}\right]^{-1}=X^{-1}$$

$$X^{-1}=(X\cdot A)^{-1}+B^{-1}$$

$$X^{-1}=A^{-1}\cdot X^{-1}+B^{-1}$$

$$X^{-1}-A^{-1}\cdot X^{-1}=B^{-1}$$

$$(E-A^{-1})^{-1}\,/\,(E-A^{-1})X^{-1}=B^{-1}$$

$$\underbrace{(E-A^{-1})^{-1}(E-A^{-1})}_{E}X^{-1}=(E-A^{-1})^{-1}\cdot B^{-1}$$

$$X^{-1}=(E-A^{-1})^{-1}\cdot B^{-1}$$

$$(X^{-1})^{-1}=\left[(E-A^{-1})^{-1}\cdot B^{-1}\right]^{-1}$$

$$X=(E-A^{-1})\cdot B$$

$$A=\begin{bmatrix}2&0&0\\0&-1&0\\1&0&1\end{bmatrix}\quad B=\begin{bmatrix}1&1&0\\0&-1&1\\2&0&0\end{bmatrix}$$

$$\det A=2\cdot\begin{vmatrix}-1&0\\0&1\end{vmatrix}-0\begin{vmatrix}0&0\\1&1\end{vmatrix}+0\begin{vmatrix}0&-1\\1&0\end{vmatrix}=-2$$

$$A_{11}=(-1)^{1+1}\begin{vmatrix}-1&0\\0&1\end{vmatrix}=-1\qquad A_{21}=(-1)^{2+1}\begin{vmatrix}0&0\\0&1\end{vmatrix}=0\qquad A_{31}=(-1)^{3+1}\begin{vmatrix}0&0\\-1&0\end{vmatrix}=0$$

$$A_{12}=(-1)^{1+2}\begin{vmatrix}0&0\\1&1\end{vmatrix}=0\qquad A_{22}=(-1)^{2+2}\begin{vmatrix}2&0\\1&1\end{vmatrix}=2\qquad A_{32}=(-1)^{3+2}\begin{vmatrix}2&0\\0&0\end{vmatrix}=0$$

$$A_{13}=(-1)^{1+3}\begin{vmatrix}0&-1\\1&0\end{vmatrix}=-1\qquad A_{23}=(-1)^{2+3}\begin{vmatrix}2&0\\1&0\end{vmatrix}=0\qquad A_{33}=(-1)^{3+3}\begin{vmatrix}2&0\\0&-1\end{vmatrix}=-2$$

$$A^{-1}=\frac{1}{-2}\begin{bmatrix}-1&0&-1\\0&2&0\\0&0&-2\end{bmatrix}^T=-\frac{1}{2}\begin{bmatrix}-1&0&0\\0&2&0\\-1&0&-2\end{bmatrix}=\begin{bmatrix}\frac{1}{2}&0&0\\0&-1&0\\\frac{1}{2}&0&1\end{bmatrix}$$

$$E-A^{-1}=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}-\begin{bmatrix}\frac{1}{2}&0&0\\0&-1&0\\\frac{1}{2}&0&1\end{bmatrix}=\begin{bmatrix}1/2&0&0\\0&2&0\\-1/2&0&0\end{bmatrix}$$

$$(E-A^{-1})\cdot B=\begin{bmatrix}\frac{1}{2}&0&0\\0&2&0\\-\frac{1}{2}&0&0\end{bmatrix}\cdot\begin{bmatrix}1&1&0\\0&-1&1\\2&0&0\end{bmatrix}=\begin{bmatrix}\frac{1}{2}\cdot1+0\cdot0+0\cdot2&\frac{1}{2}\cdot1+0\cdot(-1)+0\cdot0&\frac{1}{2}\cdot0+0\cdot1+0\cdot0\\0\cdot1+2\cdot0+0\cdot2&0\cdot1+2\cdot(-1)+0\cdot0&0\cdot0+2\cdot1+0\cdot0\\-\frac{1}{2}\cdot1+0\cdot0+0\cdot2&-\frac{1}{2}\cdot1+0\cdot(-1)+0\cdot0&-\frac{1}{2}\cdot0+0\cdot1+0\cdot0\end{bmatrix}=$$

$$=\begin{bmatrix}1/2&1/2&0\\0&-2&2\\-1/2&-1/2&0\end{bmatrix}$$

8. a) $(2E - X \cdot A \cdot A^T)(B^{-1} - B^{-1}XB)^{-1} = B \quad / \cdot (B^{-1} - B^{-1}XB)$

$(2E - X \cdot A \cdot A^T) \cdot E = B(B^{-1} - B^{-1}XB)$

$2E - X \cdot A \cdot A^T = E - E \cdot X \cdot B$

$2E - X \cdot A \cdot A^T = E - XB$

$XB - XAA^T = E - 2E$

$X(B - A \cdot A^T) = -E \quad / (B - A \cdot A^T)^{-1}$

$X \cdot E = -E \cdot (B - A \cdot A^T)^{-1}$

$X = -E(B - A \cdot A^T)^{-1}$

b) $A^{-1}(X^{-1} + 3C^T + E) \cdot B^{-1} = E + A^{-1}B^{-1} \quad / \cdot B \cdot A$

$X^{-1} + 3C^T + E = AB + E$

$X^{-1} = AB - 3C^T \quad / \cdot X$

$E = XAB - X3C^T$

$X(AB - 3C^T) = E \quad / \cdot (AB - 3C^T)^{-1}$

$X = (AB - 3C^T)^{-1}$

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