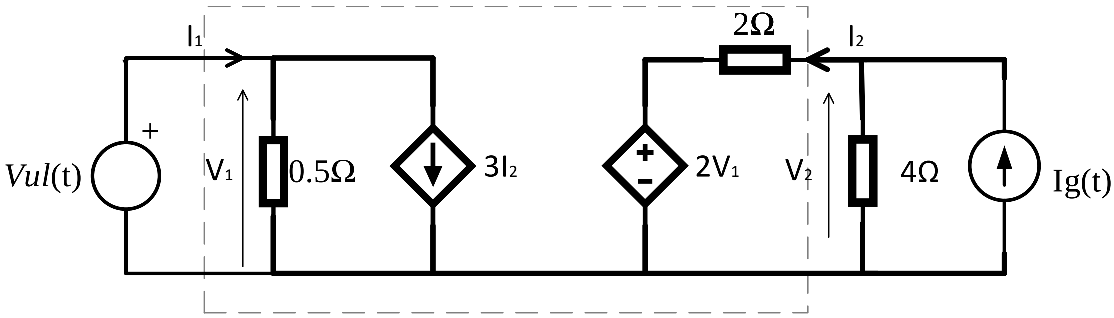
1. Odrediti:
a) *Y* parametre kola sa dva para krajeva (označenog isprekidanom linijom)
b) Ulaznu admitansu kola sa slike.



Rešenje:

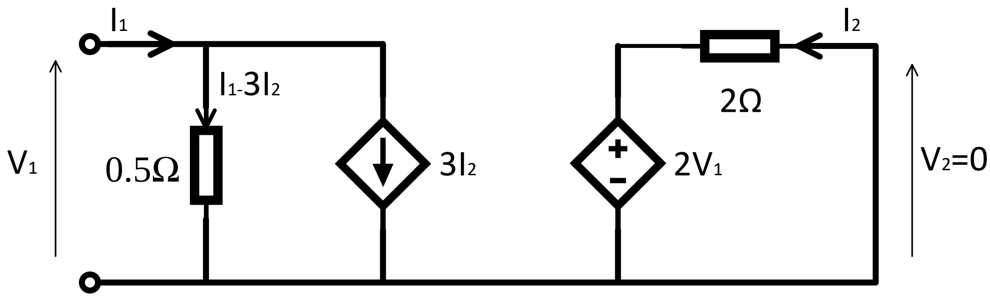
a)

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \cdot \begin{bmatrix} U_1 \\ U_2 \end{bmatrix}$$

$$I_1 = y_{11} \cdot U_1 + y_{12} \cdot U_2$$

$$I_2 = y_{21} \cdot U_1 + y_{22} \cdot U_2$$

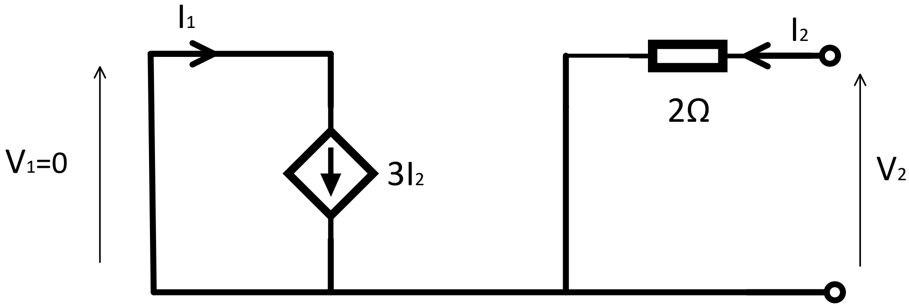
Ekvivalentno kolo za $U_2 = 0$:



$$2U_1 + 2I_2 = 0 \;\Rightarrow\; y_{21} = \frac{I_2}{V_1} = -1$$

$$U_1 = 0.5(I_1 - 3I_2) = \frac{1}{2} I_1 + \frac{3}{2} U_1 \;\Rightarrow\; y_{11} = \frac{I_1}{U_1} = -1$$

Ekvivalentno kolo za $U_1 = 0$:



$$V_2 - 2I_2 = 0 \Rightarrow y_{22} = \frac{I_2}{U_2} = \frac{1}{2}$$

$$3I_2 = I_1 \quad \Rightarrow \quad U_2 = \frac{2}{3} I_1 \quad \Rightarrow \quad y_{12} = \frac{I_1}{U_2} = \frac{3}{2}$$

$$\underset{\sim}{y} = \begin{bmatrix} -1 & \frac{3}{2} \\ -1 & \frac{1}{2} \end{bmatrix}$$

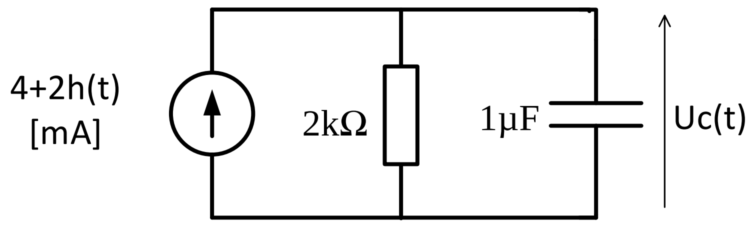
b)

$$U_2 = -4I_2$$

$$6I_2 + 2U_1 = 0 \Rightarrow I_2 = -\frac{1}{3} U_1$$

$$Y_{ul} = \frac{I_1}{U_1} = \frac{-U_1 + \frac{3}{2}U_2}{U_1} = \frac{-U_1 - 6I_2}{U_1} = 1S$$

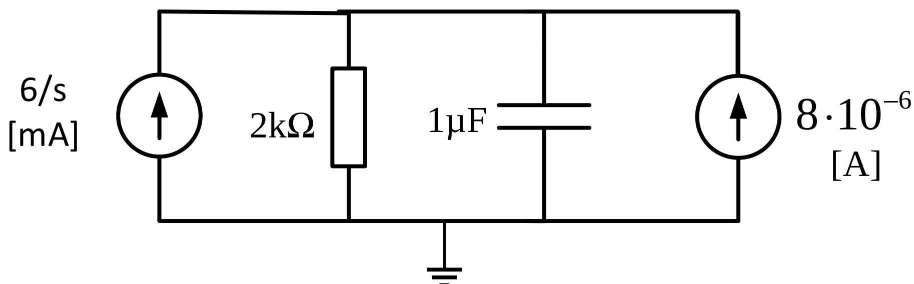
2. Primjenom Laplasove transformacije odrediti napon na kondezatoru.



Rešenje:

$$u_c(0^-) = 8V$$

Za t>0 primjenom Laplasove transformacije i metoda potencijala čvorova dobijamo:

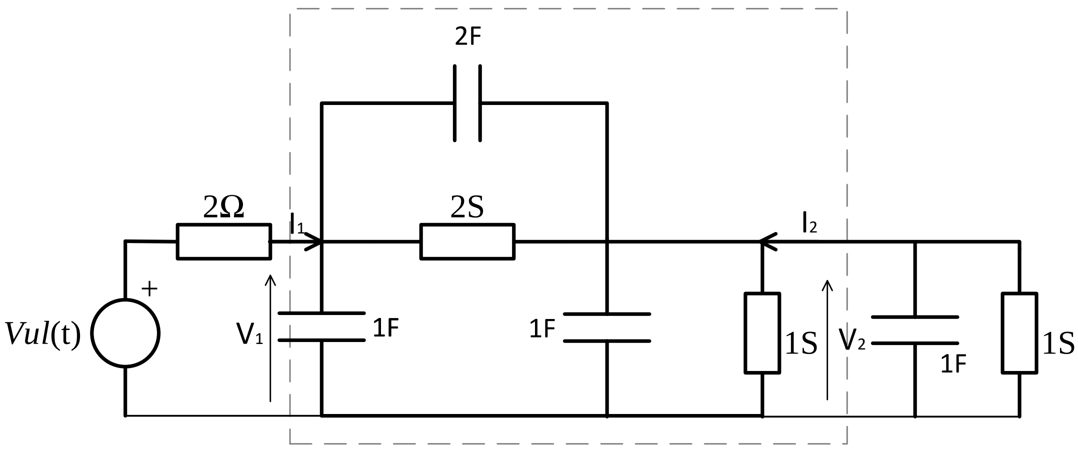


$$u_c(s) \cdot (0.5 \cdot 10^{-3} + 10^{-6} s) = \frac{6}{s} \cdot 10^{-3} + 8 \cdot 10^{-6}$$

$$u_c(s) = \frac{12}{s} - \frac{4}{500 + s}$$

$$u_c(t) = 12h(t) + 4e^{-500t} h(t)$$

3. Odrediti:
   a) Y parametre označenog kola sa dva para krajeva
   b) Ulaznu admitansu kola sa slike.

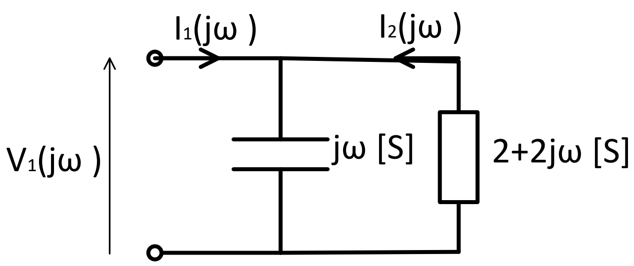


Rešenje:

a) $$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \cdot \begin{bmatrix} U_1 \\ U_2 \end{bmatrix}$$

$$I_1 = y_{11} \cdot U_1 + y_{12} \cdot U_2$$
$$I_2 = y_{21} \cdot U_1 + y_{22} \cdot U_2$$

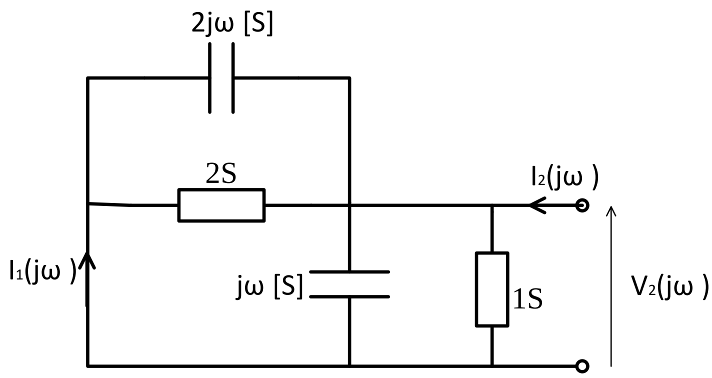
Ekvivalentno kolo za $V_2 = 0$:



$$y_{11} = \frac{I_1(j\omega)}{V_1(j\omega)} = 2 + 3j\omega$$

$$V_1(j\omega) = \frac{-I_2(j\omega)}{2+2j\omega} \Rightarrow y_{21} = \frac{I_2(j\omega)}{V_1(j\omega)} = -2 - 2j\omega$$

Ekvivalentno kolo za $V_1 = 0$:



$$y_{22} = \frac{I_2(j\omega)}{V_2(j\omega)} = 3 + 3j\omega$$

$$y_{12} = \frac{I_1(j\omega)}{V_2(j\omega)} = -2 - 2j\omega$$

b)

$$Y_{ul} = \frac{I_1(\omega)}{V_1(j\omega)} = \frac{y_{11}V_1(j\omega) + y_{12}V_2(j\omega)}{V_1(j\omega)}$$

$$I_2(j\omega) = -V_2(j\omega)\cdot(1 + j\omega) \Rightarrow$$

$$-V_2(j\omega)\cdot(1 + j\omega) = y_{21}V_1(j\omega) + y_{22}V_2(j\omega)$$

$$V_2(j\omega) = \frac{y_{21}V_1(j\omega)}{(1 + j\omega + y_{22})}$$

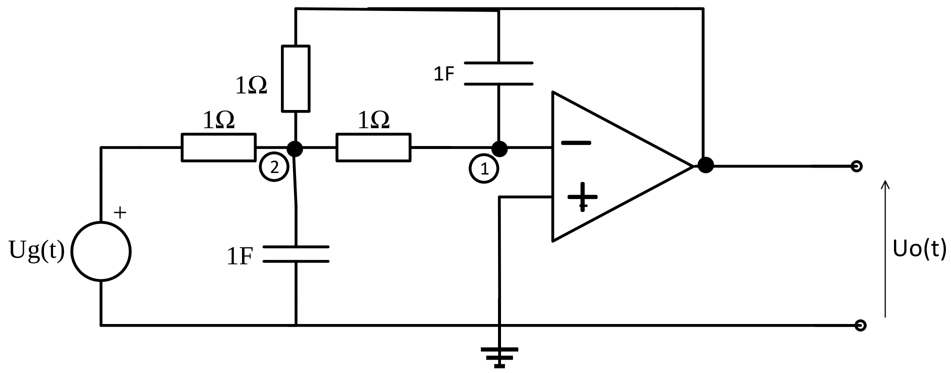
Uvršavanjem poslednjeg izraza u polaznu jednačinu dobijamo:

$$Y_{ul} = \frac{I_1(\omega)}{V_1(j\omega)} = \frac{y_{11}V_1(j\omega) + y_{12}V_2(j\omega)}{V_1(j\omega)} = y_{11} + \frac{y_{12}y_{21}}{(y_{22} + 1 + j\omega)} = 3 + 4j\omega$$

Kada uzmemo u obzir i otpornost naponskog generatora, ukupna admitasa kola je:

$$Y_{ul}* = \frac{3 + 4j\omega}{7 + 8j\omega}$$

4. Odrediti funkciju prenosa $W(s)=\dfrac{U_o(s)}{U_g(s)}$ kola sa slike.



Rešenje:

Jednačine metod potencijala čvorova:

Čvor 1: $(1+1+1+s)\cdot V_2(s)-V_1(s)-U_o(s)=U_g(s)$

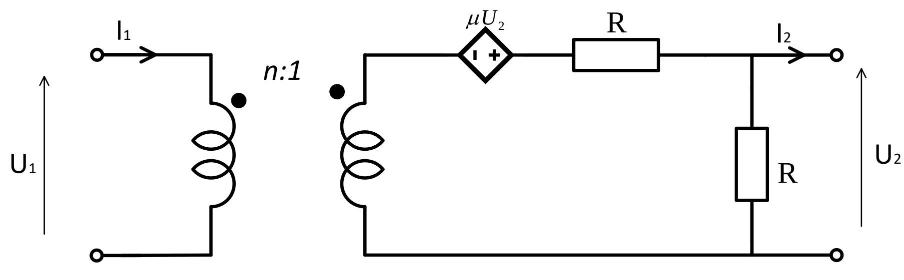
Čvor 2: $(1+s)\cdot V_1(s)-V_2(s)-sU_o(s)=0$

Kako je $V_1(s)=0$, dobija se:

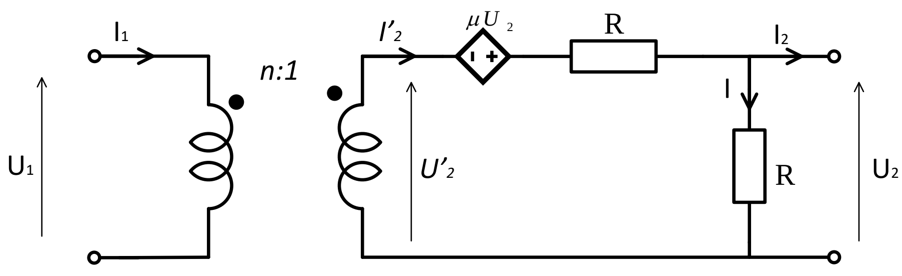
$$V_2(s)=-sU_o(s)$$

$$W(s)=\frac{U_o(s)}{U_g(s)}=\frac{-1}{s(3+s)+1}$$

5. Odrediti *y* parametre kola sa slike. Pod kojim uslovom je dato kolo simetrično?



Rešenje:



$$U_1 = z_{11} \cdot I_1 - z_{12} \cdot I_2$$
$$U_2 = z_{21} \cdot I_1 - z_{22} \cdot I_2$$

$$\frac{U_1}{U'_2} = n \Rightarrow U'_2 = \frac{1}{n} U_1$$
$$\frac{I_1}{I'_2} = \frac{1}{n} \Rightarrow I'_2 = nI_1$$
$$I'_2 = I + I_2 \Rightarrow I = I'_2 - I_2 = nI_1 - I_2$$
$$U_2 = RI$$
$$U_2 = nRI_1 - RI_2 \Rightarrow \quad z_{21} = nR \quad z_{22} = R$$

$$U'_2 - RI - RI'_2 + \mu U_2 = 0$$
$$\frac{U_1}{n} = RnI_1 - RI_2 + RnI_1 - \mu(nRI_1 - RI_2)$$
$$U_1 = (2n^2R - \mu n^2 R)I_1 - n(R - \mu R)I_2$$
$$\Rightarrow \quad z_{11} = 2n^2R - \mu n^2 R \quad z_{12} = (R - \mu R)$$

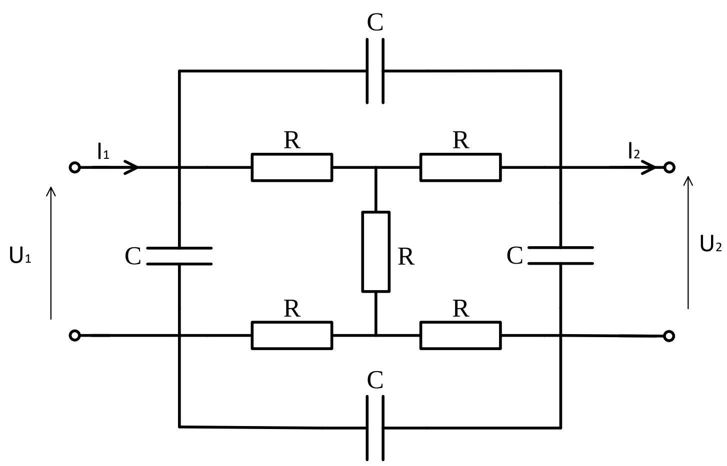
Na osnovu *z* parametara lako se izvode *y* parametri kola:

$$\underset{\sim}{y} = \begin{bmatrix} \frac{1}{n^2R} & -\frac{1-\mu}{nR} \\ -\frac{1}{nR} & \frac{2-\mu}{R} \end{bmatrix}$$

Uslov simetričnosti je:

$$y_{11} = y_{22}$$
$$\frac{1}{n^2R} = \frac{2-\mu}{R}$$
$$n = \frac{1}{\sqrt{2-\mu}}$$

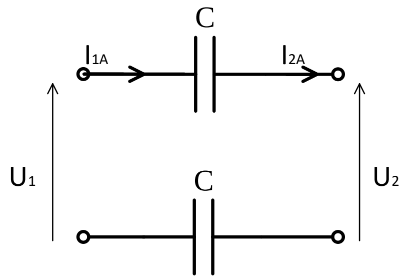
6. Odrediti *y* parametre mreže sa dva para krajeva. Poznato je: R=1/3Ω a C=3/8F.



Rešenje:

Rastavićemo mrežu na dvije podmreže A i B, a zatim odrediti *y* parametre za svaku od njih.

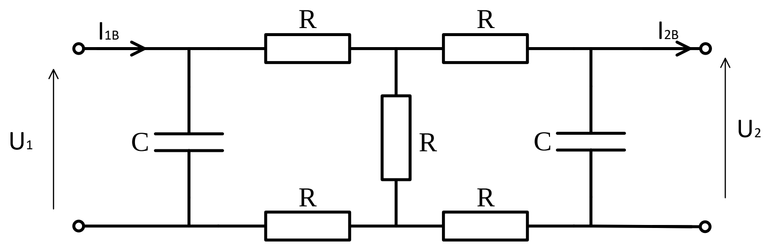
Podmreža A je data na slici ispod.



Primjenom drugog Kirhohovog zakona jednostavno dobijamo *y* parametre kola A:

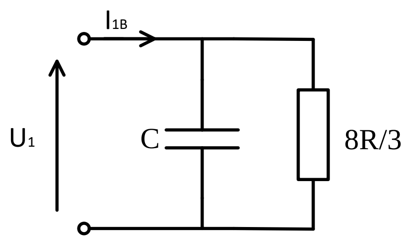
$$\underset{\sim}{y}_A = \begin{bmatrix} j\omega C/2 & -j\omega C/2 \\ -j\omega C/2 & j\omega C/2 \end{bmatrix} = \begin{bmatrix} 3j\omega/16 & -3j\omega/16 \\ -3j\omega/16 & 3j\omega/16 \end{bmatrix}$$

Podmrežu B čini ostatak kola:



Y parametre kola B dobijamo u dva koraka. U prvom koraku napon U_2 postavljamo na 0, i računamo parametre y_{11} i y_{21}. U drugom koraku pretpostavljamo da je napon U_1 jednak 0, i računamo parametre y_{12} i y_{22}.

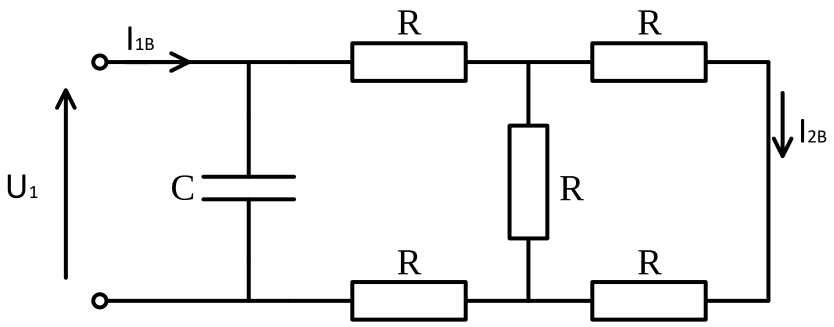
Pri uslovu da je U_2=0, kolo se može uprostiti na sledeći način:



Stoga, dobijamo:

$$y_{11}^{(B)} = \frac{I_{1B}}{U_1} = j\omega C + \frac{3}{8R} = \frac{3j\omega}{8} + \frac{9}{8}$$

Imajuću u vidu polaznu šemu kola za U_2=0:



Primjenom Kirhohovih zakona dobijamo i y_{21} parametar:

$$I_{2B} = \frac{1}{3+8j\omega RC} \cdot I_{1B}$$

$$I_{2B} = \frac{1}{3+8j\omega RC} \cdot y_{11}^{(B)} U_1 = \frac{3}{8} U_1$$

$$y_{21}^{(B)} = \frac{I_{2B}}{U_1} = \frac{3}{8}$$

Pri uslovu da je U_1=0, dobijamo:

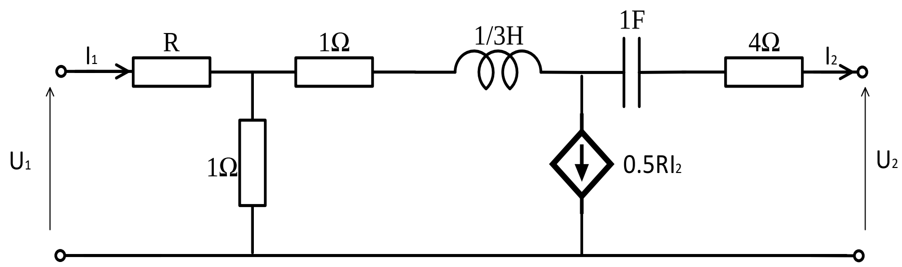
$$y_{12}^{(B)} = \frac{I_{1B}}{U_2} = \frac{3}{8}$$

$$y_{22}^{(B)} = \frac{-I_{2B}}{U_2} = \frac{3j\omega}{8} + \frac{9}{8}$$

Zadato kolo dobijeno je paralelnom vezom kola A i B, pa je konačno rešenje:

$$\underset{\sim}{y} = \underset{\sim}{y_A} + \underset{\sim}{y_B} = \begin{bmatrix} \frac{9j\omega}{16} + \frac{9}{8} & -\frac{3j\omega}{16} + \frac{3}{8} \\ -\frac{3j\omega}{16} + \frac{3}{8} & \frac{9j\omega}{16} + \frac{9}{8} \end{bmatrix}$$

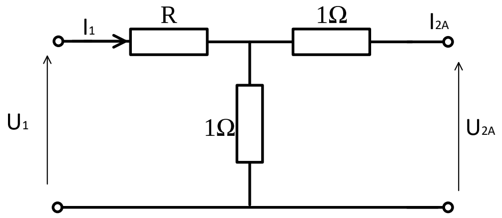
7. Odrediti *a* parametre kola sa dva para krajeva, a zatim odrediti otpornost *R* pri kojoj je mreža recipročna.



Rešenje:

Predstavićemo mrežu u formi dvije kaskadno vezane podmreže (A i B) i naći *a* parametre za svaku od njih nezavisno.

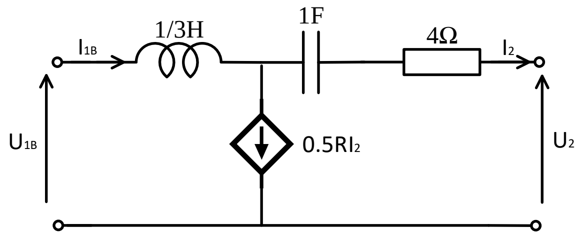
Prva podmreža (A):



Primjenom Kirhohovih zakona dobija se:

$$I_1 = U_{2A} + 2I_{2A}$$
$$U_1 = (1+R)U_{2A} + (1+2R)I_{2A}$$
$$\underset{\sim}{a}^{(A)} = \begin{bmatrix} 1+R & 1+2R \\ 1 & 2 \end{bmatrix}$$

Druga podmreža (B):



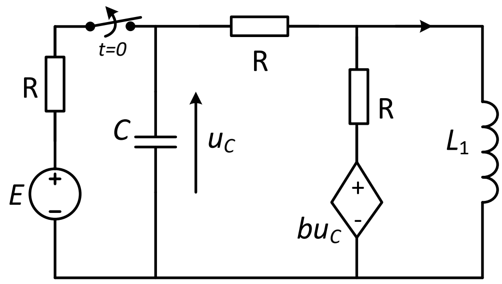
$$I_{1B} = I_2 + 0.5RI_2 = (1+0.5R)I_2$$
$$U_{1B} - U_2 - 4I_2 - \frac{1}{j\omega}I_2 - \frac{j\omega}{3}I_{1B} = 0$$
$$\Rightarrow U_{1B} = U_2 + I_2(4 + \frac{1}{j\omega} + \frac{j\omega}{3}(1+\frac{R}{2}))$$
$$\underset{\sim}{a}^{(B)} = \begin{bmatrix} 1 & 4+\frac{1}{j\omega}+\frac{j\omega}{3}(1+\frac{R}{2}) \\ 0 & 1+\frac{R}{2} \end{bmatrix}$$

Konačno rešenje:

$$\underset{\sim}{a} = \underset{\sim}{a}^{(A)} \cdot \underset{\sim}{a}^{(B)} = \begin{bmatrix} 1+R & (1+R)\cdot(4+\frac{1}{j\omega}+\frac{j\omega}{3}(1+\frac{R}{2})) + (1+2R)(1+\frac{R}{2}) \\ 1 & 6+\frac{1}{j\omega}+\frac{j\omega}{3}(1+\frac{R}{2}) + R \end{bmatrix}$$

Uslov recipročnosti je $\det\left\{\underset{\sim}{a}\right\} = 1$. Iz ovog uslova dabija se da je R=0.

8. Odrediti napon na kondenzatoru nakon otvaranja prekidača rešavajući kolo u s-domenu. Pretpostaviti da je kolo prije otvaranja prekidača bilo u stacionarnom stanju. Poznato je: E=12V, R=10 Ω, C=2F, L=1H, b=2.

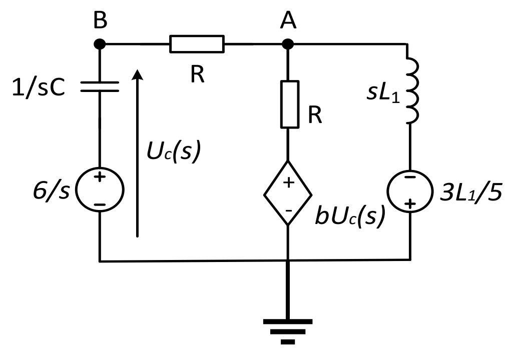


Rešenje:

Početni uslovi za t<0:

$$i_l(0^-) = \frac{3}{5}A$$
$$u_c(0^-) = 6V$$

Ekvivalentno kolo za t>0:



$$V_B(s) = U_c(s)$$
$$(\frac{1}{R}+\frac{1}{R}+\frac{1}{sL_1})V_A(s) - \frac{1}{R}U_c(s) = \frac{bU_c(s)}{R} - \frac{3}{5s}$$
$$(sC+\frac{1}{R})U_c(s) - \frac{1}{R}V_A(s) = 6C$$

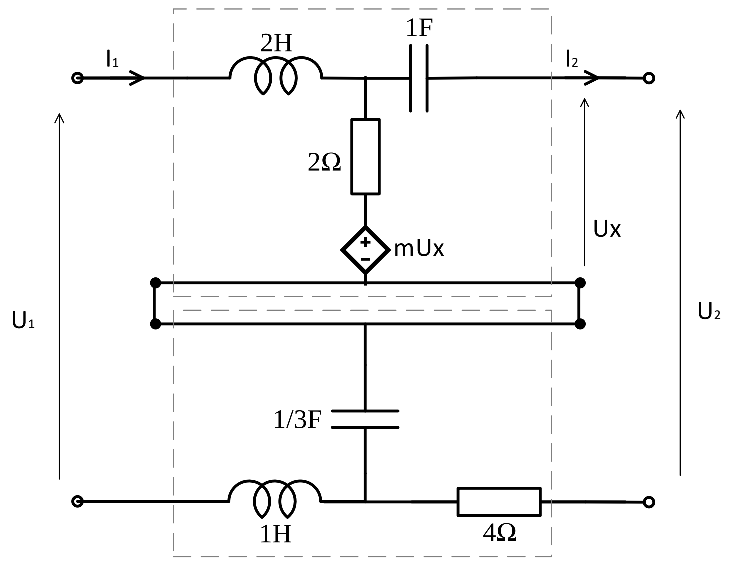
Rešavanjem sistema jednačina dobija se:

$$U_c(s) = \frac{24s+119.4}{4s^2+19.8s+1}$$
$$u_c(t) = 6\mathrm{e}^{-2.475t}\cosh(2.424t)+6\mathrm{e}^{-2.475t}\sinh(2.424t)$$

9. Za mrežu sa dva para krajeva koja je prikazana na slici odrediti:

a) „z“ parametre;

b) vrijednost koeficijenta *m* tako da je parametar z_{12} konstanta (ne zavisi od s).

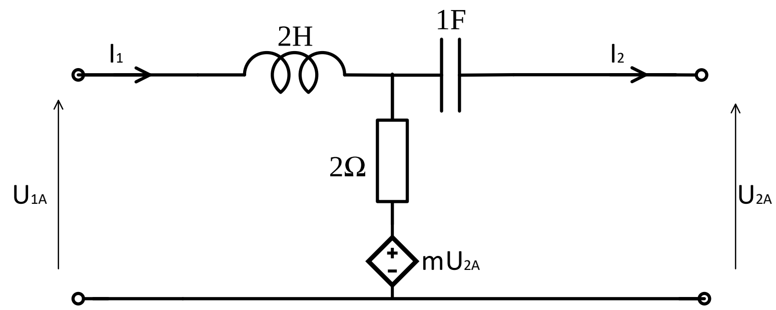


<u>Rešenje:</u>

a)

Posmatraćemo kolo kao rednu vezu dva kola: A i B.

Šema kola A:



Rešavanjem kola u s-domenu dobijamo sledeće jednačine:

$$\underline{U_{2A}} - m\underline{U_{2A}} - 2(\underline{I_1} - \underline{I_2}) + \frac{1}{s}\underline{I_2} = 0$$

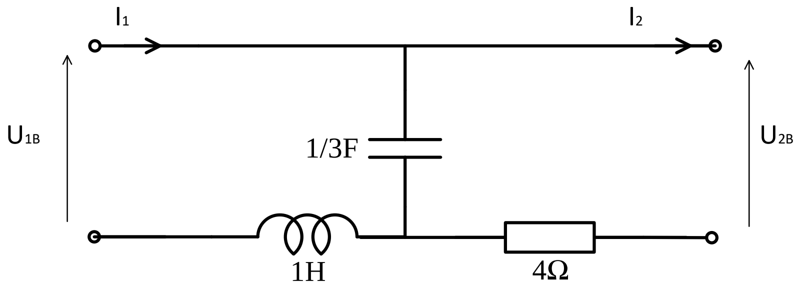
$$\Rightarrow \underline{U_{2A}} = \frac{2}{1-m}\underline{I_1} - \frac{2s+1}{s-sm}\underline{I_2}$$

$$\underline{U_{1A}} - m\underline{U_{2A}} - 2(\underline{I_1} - \underline{I_2}) - 2s\underline{I_1} = 0$$

$$\Rightarrow \underline{U_{1A}} = \underline{I_1}(\frac{2m}{1-m} + 2 + 2s) - \underline{I_2}(\frac{2ms+m}{s-sm} + 2)$$

$$\underset{\sim}{z}^{(A)} = \begin{bmatrix} \frac{2m}{1-m} + 2 + 2s & \frac{2ms+m}{s-sm} + 2 \\ \frac{2}{1-m} & \frac{2s+1}{s-sm} \end{bmatrix}$$

Šema kola B:



$$\underline{U}_{1B} = \frac{3}{s}\underline{I}_1 - \frac{3}{s}\underline{I}_2 + s\underline{I}_1 = (s+\frac{3}{s})\underline{I}_1 - \frac{3}{s}\underline{I}_2$$

$$\underline{U}_{2B} + 4\underline{I}_2 - \frac{3}{s}(\underline{I}_1 - \underline{I}_2) = 0$$

$$\underline{U}_{2B} = \frac{3}{s}\underline{I}_1 - \frac{3}{s}\underline{I}_2 - 4\underline{I}_2 = \frac{3}{s}\underline{I}_1 - (\frac{3}{s}+4)\underline{I}_2$$

$$\underset{\sim}{z}^{(B)} = \begin{bmatrix} s+\frac{3}{s} & \frac{3}{s} \\ \frac{3}{s} & 4+\frac{3}{s} \end{bmatrix}$$

Parametri redne veze:

$$\underset{\sim}{z} = \underset{\sim}{z}^{(A)} + \underset{\sim}{z}^{(B)} = \begin{bmatrix} \frac{2m}{1-m}+2+3s+\frac{3}{s} & \frac{2ms+m}{s-sm}+2+\frac{3}{s} \\ \frac{2}{1-m}+\frac{3}{s} & \frac{2s+1}{s-sm}+4+\frac{3}{s} \end{bmatrix}$$

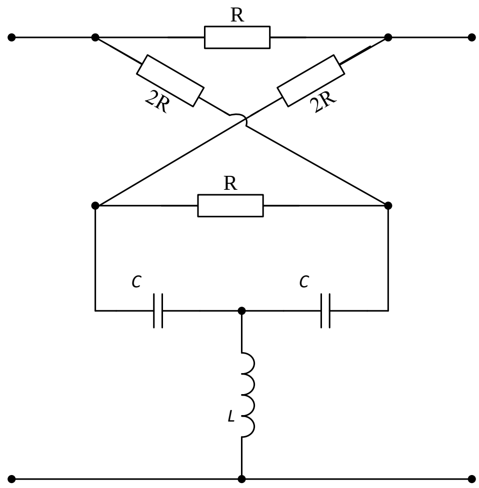
b)

$$z_{12} = \frac{2ms+m}{s-sm}+2+\frac{3}{s} = \frac{2m}{1-m}+2+\frac{1}{s}(3+\frac{m}{1-m})$$

Pri uslovu $3+\frac{m}{1-m}=0$, parameter ne zavisi od *s.*

Iz jednakosti slijedi: $m=\frac{3}{2}$

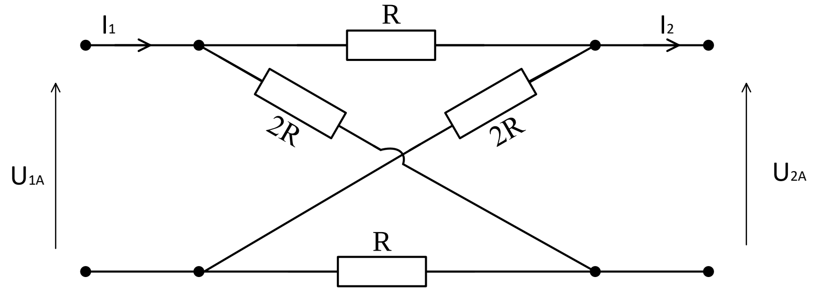
10. Odrediti **„z“** parametre kola sa dva para krajeva sa slike. Poznato je R=2Ω, C=1F i L=1H.



<u>Rešenje:</u>

Zadato kolo posmatraćemo kao rednu vezu dva kola A i B koja će biti nezavisno analizirana.

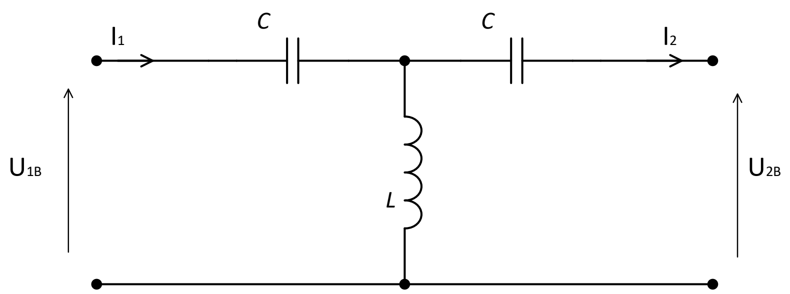
Šema kola A:



“z” parametri kola A su:

$$\underset{\sim}{z}^{(A)} = \begin{bmatrix} \frac{3R}{2} & \frac{R}{2} \\ \frac{R}{2} & \frac{3R}{2} \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix}$$

Šema kola B:



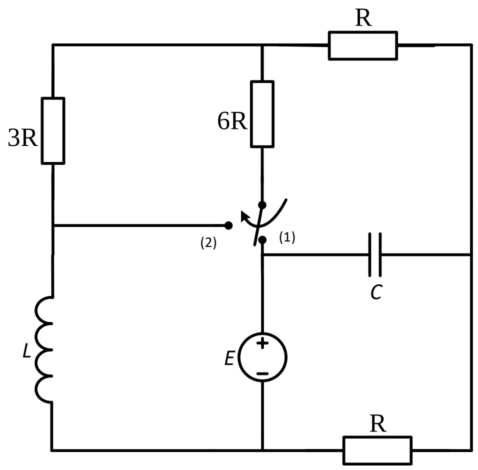
“z” parametri kola B su:

$$z^{(B)} = \begin{bmatrix} \frac{1}{sC} + sL & sL \\ sL & \frac{1}{sC} + sL \end{bmatrix} = \begin{bmatrix} \frac{1}{s} + s & s \\ s & \frac{1}{s} + s \end{bmatrix}$$

Zadato kolo se može konstruisati rednom vezom kola A i B, pa su njegovi “z” parametri:

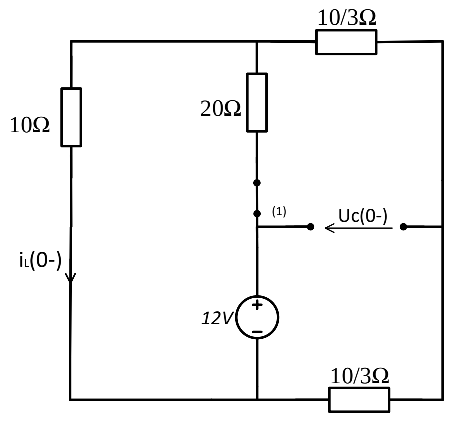
$$z = z^{(A)} + z^{(B)} = \begin{bmatrix} 3 + \frac{1}{s} + s & 1 + s \\ 1 + s & 3 + \frac{1}{s} + s \end{bmatrix}$$

11. Odrediti napon na kondenzatoru nakon komutacije prekidača. Parametri kola su: E=12V, R=10/3Ω, L=10H i C=0.1F.



<u>Rešenje:</u>

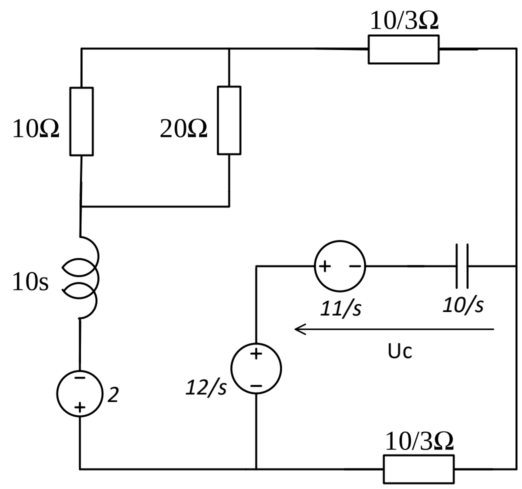
Pretpostavimo da je prekidač u položaju (1) i da se kolo nalazi u stacionarnom stanju:



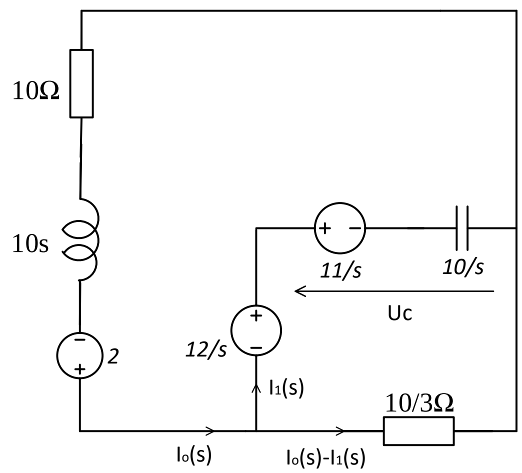
Pod ovim uslovima određujemo napon na kondezatoru i struju kroz kalem:

$$u_c(0^-) = 11V$$
$$i_L(0^-) = 0.2A$$

Sada analiziramo scenario kada je prekidač u položaju (2) i primjenjujemo Laplasovu transformaciju:



Kolo je moguće dodatno uprostiti:



Primjenom II Kirhohovog zakona dolazimo do sledećih jednačina:

$$10I_o(s)+10sI_o(s)-2+\frac{10}{3}(I_o(s)-I_1(s))=0$$

$$-\frac{12}{s}+\frac{10}{s}I_1(s)+\frac{11}{s}-\frac{10}{3}(I_o(s)-I_1(s))=0$$

Rešavanjem sistema dobijamo:

$$I_1(s)=\frac{5s+4}{10(s^2+4s+2)}$$

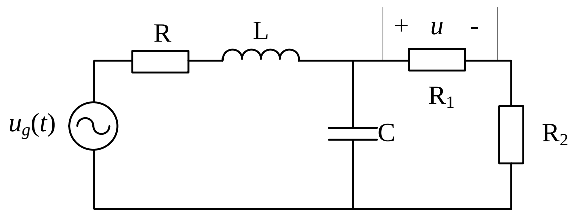
$$U_c(s)=\frac{11}{s}+\frac{10}{s}I_1(s)=\frac{11}{s}+\frac{5s+4}{s(s^2+4s+2)}=\frac{13}{s}-\frac{1.35}{s+3.41}-\frac{0.64}{s+0.58}$$

$$\Rightarrow u_c(t)=13h(t)-1.35e^{-3.41t}h(t)-0.64e^{-0.58t}h(t)$$

12. Za kolo na slici poznat je napon $u(t)=4e^{-5t}h(t-3)$.

a) Odrediti udio ukupne energije koji se disipira na otporniku R_1 koji otpada na opseg učestanosti $\sqrt{3}\le\omega\le 3\sqrt{3}$ rad/s.

b) Odrediti funkciju prenosa definisanu kao $H(s)=U(s)/U_g(s)$, ako su parametri kola: R=9Ω, L=2H, C=2/7F, R_1=2Ω, R_2=5Ω.



Rešenje:

a)

$$u(t)=4e^{-5t}h(t-3)=4e^{-15}e^{-5(t-3)}h(t-3)$$

$$U(j\omega)=\frac{4e^{-15}}{5+j\omega}e^{-3j\omega}$$

$$|U(j\omega)|=\frac{4e^{-15}}{\sqrt{25+\omega^2}}$$

Ukupnu disipiranu energiju računamo primjenom Parsevalove teoreme:

$$E_{uk}=\frac{1}{R_1\pi}\int_0^{+\infty}|U(j\omega)|^2\,d\omega=\frac{16e^{-30}}{10R_1}$$

Energija koja se disipira na otporniku R_1 a otpada na opseg učestanosti $\sqrt{3}\le\omega\le 3\sqrt{3}$ rad/s:

$$E_o=\frac{1}{R_1\pi}\int_{\sqrt{3}}^{3\sqrt{3}}|U(j\omega)|^2\,d\omega=\frac{16e^{-30}}{5R_1\pi}(\arctan\frac{3\sqrt{3}}{5}-\arctan\frac{\sqrt{3}}{5})$$

Procentualno, udio ove energije je:

$$\nu_{\%}=\frac{\arctan\frac{3\sqrt{3}}{5}-\arctan\frac{\sqrt{3}}{5}}{\pi/2}$$

b)

$$H(s)=\frac{U(s)}{U_g(s)}$$

$$Z_{C\,II\,(R_1+R_2)}=\frac{(R_1+R_2)\cdot\frac{1}{sC}}{(R_1+R_2)+\frac{1}{sC}}=\frac{7}{2s+1}$$

$$\Rightarrow U_c(s)=\frac{\frac{7}{2s+1}}{\frac{7}{2s+1}+9+2s}U_g(s)=\frac{7}{4s^2+20s+16}U_g(s)$$

$$U(s)=\frac{R_1}{R_1+R_2}U_c(s)=\frac{2}{4s^2+20s+16}U_g(s)$$

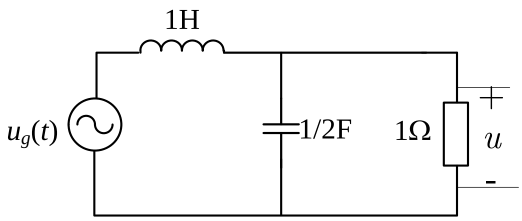
$$\Rightarrow H(s)=\frac{2}{4s^2+20s+16}$$

13. Ako je kolo bilo bez početnih uslova za t<0:

a) Odrediti funkciju prenosa kola ako je ulazna veličina napon generatora, a izlazna veličina napon u.

b) Za tako određenu funkciju prenosa odrediti odziv kola na pobudu u vidu pravougaonog impulsa $u_g(t)=h(t)-h(t-3)$.

c) Ako je za t<0 kolo bilo u stacionarnom stanju, a napon generatora dat izrazom $u_g(t)=3h(-t)+2e^{-3t}h(t)$, nacrtati odgovarajuću šemu kola u s domenu i odrediti struju kroz kondenzator.



Rešenje:

a)

$$Z_{C\,II\,R}=\frac{\frac{2}{s}\cdot 1}{\frac{2}{s}+1}=\frac{2}{2+s}$$

$$U(s)=\frac{\frac{2}{2+s}}{\frac{2}{2+s}+s}U_g(s)=\frac{2}{s^2+2s+2}U_g(s)$$

$$\Rightarrow H(s)=\frac{2}{s^2+2s+2}$$

b)

$$U_g(s)=\frac{1}{s}-\frac{1}{s}e^{-3s}$$

$$U(s)=H(s)\cdot U_g(s)=\frac{1}{s}\cdot\frac{2}{s^2+2s+2}-\frac{1}{s}e^{-3s}\cdot\frac{2}{s^2+2s+2}=\frac{1}{s}-\frac{s+2}{s^2+2s+2}-e^{-3s}(\frac{1}{s}-\frac{s+2}{s^2+2s+2})$$

$$U(s)=\frac{1}{s}-\frac{s+1}{(s+1)^2+1}-\frac{1}{(s+1)^2+1}-e^{-3s}(\frac{1}{s}-\frac{s+1}{(s+1)^2+1}-\frac{1}{(s+1)^2+1})$$

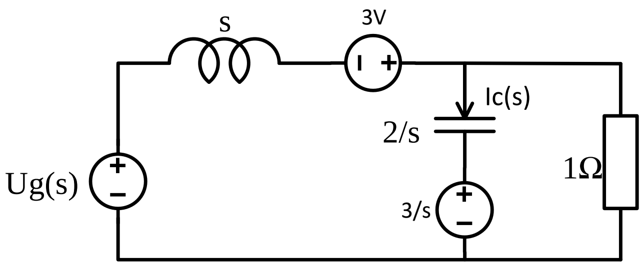
$$u(t)=h(t)-e^{-t}\cos t-e^{-t}\sin t-h(t-3)+e^{-(t-3)}\cos(t-3)+e^{-(t-3)}\sin(t-3)$$

c) Ako pretpostavimo da se za t<0 kolo nalazilo u stacionarom stanju, očigledno je da su početni uslovi kola:

$$i_l(0^-)=\frac{u_g(0^-)}{R}=3A$$

$$u_c(0^-)=u_g(0^-)=3V$$

Laplasova šema kola za t>0:



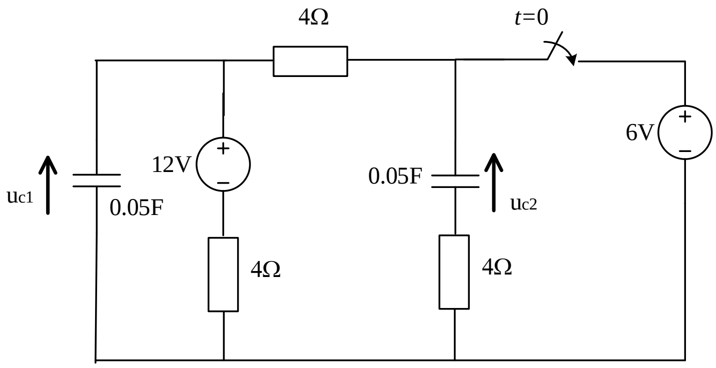
$$U_c(s)=\frac{6s+22}{(s+3)(s^2+2s+2)}+\frac{3s}{s^2+2s+2}=\frac{1}{(s+3)}+\frac{2s-7}{(s^2+2s+2)}$$

$$I_c(s)=\frac{U_c(s)-\frac{3}{s}}{\frac{2}{s}}=-1.5+\frac{s}{2(s+3)}+\frac{(2s-7)s}{2(s^2+2s+2)}$$

Preko inverzne Laplasove transformacije dobijamo izraz za struju u vremenskom domenu:

$$i_c(t)=-1.5e^{-3t}h(t)+1.5e^{-t}\cos t-3.5e^{-t}\sin t$$

14. Za kolo sa slike odrediti napone na oba kondenzatora ako je kolo prije t=0 bilo u stacionarnom stanju.

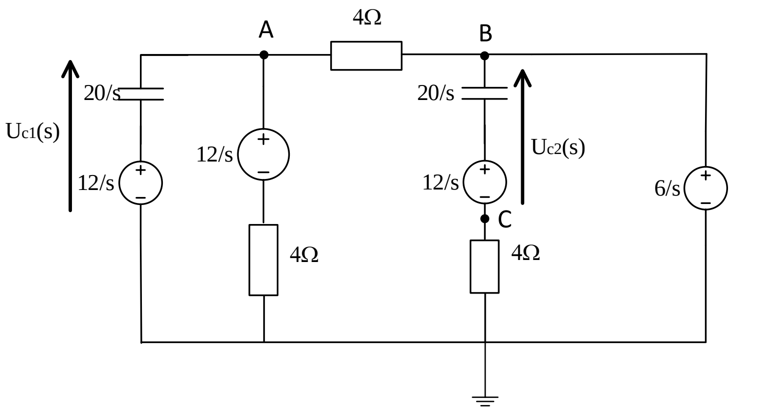


Rešenje:

Analizom kola za t<0 dolazi se do sledećih početnih uslova:

$$u_{c1}(0^-) = u_{c2}(0^-) = 12V$$

Ekvivalentna šema kola u s-domenu za t>0 (prilagođena primjeni metoda potencijala čvorova):



$$V_B(s) = \frac{6}{s}$$

$$(\frac{s}{20} + \frac{1}{4} + \frac{1}{4})V_A(s) - \frac{1}{4}V_B(s) = \frac{3}{5} + \frac{3}{s}$$

$$(\frac{s+10}{20})V_A(s) - \frac{3}{2s} = \frac{3}{5} + \frac{3}{s}$$

$$(\frac{s+10}{20})V_A(s) = \frac{3}{5} + \frac{9}{2s}$$

$$U_{c1}(s) = V_A(s) = \frac{12}{s+10} + \frac{90}{s(s+10)}$$

$$\Rightarrow u_{c1}(t) = 12e^{-10t}h(t) + 9(1-e^{-10t})h(t) = 9h(t) - 3e^{-10t}h(t)$$

$$\left(\frac{1}{4}+\frac{s}{20}\right)V_C(s)-\frac{s}{20}V_B(s)=-\frac{\frac{12}{s}}{\frac{20}{s}}$$

$$\left(\frac{5+s}{20}\right)V_C(s)-\frac{3}{10}=-\frac{3}{5}$$

$$V_C(s)=-\frac{6}{5+s}$$

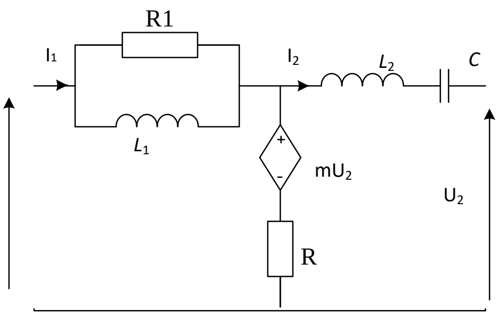
$$U_{c2}(s)=V_B(s)-V_C(s)=\frac{6}{s}+\frac{6}{5+s}$$

$$\Rightarrow u_{c2}(t)=6h(t)+6e^{-5t}h(t)$$

15. Za kolo sa slike poznato je: R_1=2Ω, R=10 Ω, L_1=1H, L_2=2H, C=1/2F, m=0.5. Odrediti:

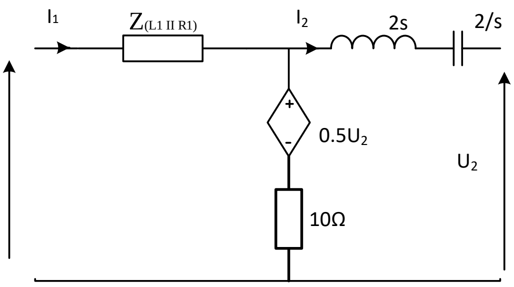
a) z parametre kola.

b) funkciju prenosa H(s)=I_2(s)/I_1(s) ako se kolo zatvori otpornikom R=1Ω, a potom i step odziv kola.



Rešenje:

Kolo se u s-domenu može predstaviti na sledeći način:



a)

$$Z_{L1\,II\,R1}=\frac{2\cdot s}{2+s}$$

Pri uslovu I_1=0 određujemo parametre z_{12} i z_{22}:

$$\underline{U}_2+10\underline{I}_2-0.5\underline{U}_2+(2s+\frac{2}{s})\underline{I}_2=0$$

$$-0.5\underline{U}_2=(10+2s+\frac{2}{s})\underline{I}_2$$

$$\underline{z}_{22}=-\frac{\underline{U}_2}{\underline{I}_2}=20+4s+\frac{4}{s}$$

$$\underline{U}_1=0.5\underline{U}_2-10\underline{I}_2=-0.5\underline{z}_{22}\underline{I}_2-10\underline{I}_2$$

$$\Rightarrow \underline{z}_{12}=-\frac{\underline{U}_1}{\underline{I}_2}=20+2s+\frac{2}{s}$$

Pri uslovu $I_2=0$ određujemo parametre z_{11} i z_{21}:

$$\underline{U}_1=\frac{2s}{s+2}\underline{I}_1+\underline{U}_2$$

$$\underline{U}_2=0.5\underline{U}_2+10\underline{I}_1$$

$$\Rightarrow \underline{z}_{21}=\frac{\underline{U}_2}{\underline{I}_1}=20$$

$$\underline{U}_1=(\frac{2s}{s+2}+20)\underline{I}_1 \Rightarrow \underline{z}_{11}=\frac{\underline{U}_1}{\underline{I}_1}=\frac{22s+40}{s+2}$$

b)

$$\underline{U}_2=1\cdot\underline{I}_2$$

$$\underline{U}_2=\underline{z}_{21}\underline{I}_1-\underline{z}_{22}\underline{I}_2 \Rightarrow \underline{I}_2+\underline{z}_{22}\underline{I}_2=\underline{z}_{21}\underline{I}_1$$

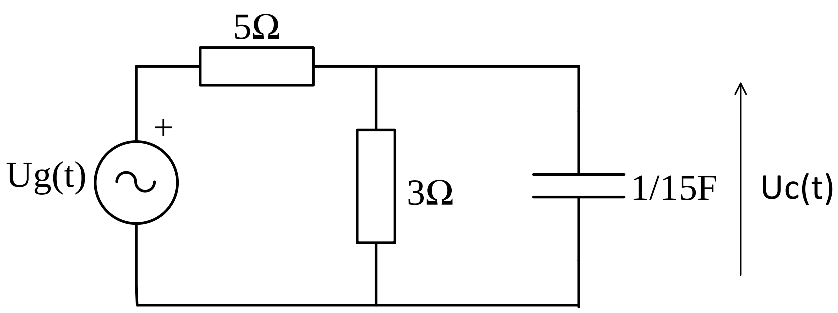
$$\Rightarrow H(s)=\frac{\underline{I}_2}{\underline{I}_1}=\frac{\underline{z}_{21}}{1+\underline{z}_{22}}=\frac{20s}{4s^2+21s+4}$$

c)

$$\underline{I}_2=H(s)\cdot\underline{I}_1=\frac{H(s)}{s}=\frac{20}{4s^2+21s+4)}$$

$$\Rightarrow i_2(t)=(1.03e^{-5.05t}-1.03e^{-0.198})h(t)$$

16. U kolo na slici poznat je napon generatora $u_g(t)=e^{-t}\left(h(t)-h(t-2)\right)$. Odrediti napon na kondezatoru primjenom konvolucionog integrala ako je poznato da je $u_c(0^-)=0V$.



<u>Rešenje:</u>

Funkciju prenosa kola naći ćemo rešavanjem kola u s-domenu.

Uočimo da se kolo može ekvivalentirati rednom vezom otpornika od 5Ω i impedanse:

$$Z_{3\Omega\, II\, C} = \frac{3 \cdot \frac{15}{s}}{3 + \frac{15}{s}} = \frac{15}{s+5}$$

Prekom naponskog razdjelnika određujemo napon na kondezatoru:

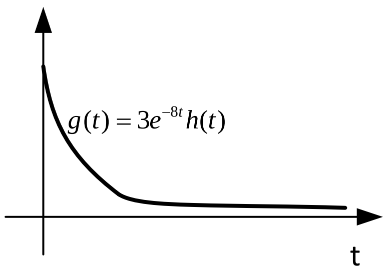
$$U_c(s) = \frac{Z_{3\Omega\, II\, C}}{Z_{3\Omega\, II\, C} + 3} U_g(s) = \frac{3}{s+8} U_g(s)$$

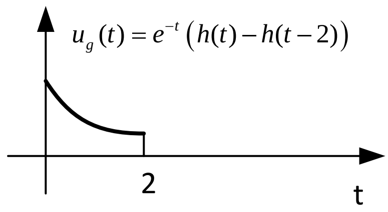
$$\Rightarrow H(s) = \frac{U_c(s)}{U_g(s)} = \frac{3}{s+8}$$

Inverznom Laplasovom transformacijom funkcije prenosa dobijamo impulsni odziv sistema:

$$g(t) = 3e^{-8t} h(t)$$

Napon na kondezatoru (u vremenskom domenu) nalazimo kao konvoluciju impulsnog odziva i pobude sistema: $u_c(t) = g(t) * u_g(t) = \int_{-\infty}^{+\infty} g(t-\tau) u_g(\tau) d\tau$.





Primjenom grafičke metode možemo uočiti 3 intervala u kojima konvuluciona funkcija mijenja oblik:

I interval: $t < 0$

$$u_c(t) = 0$$

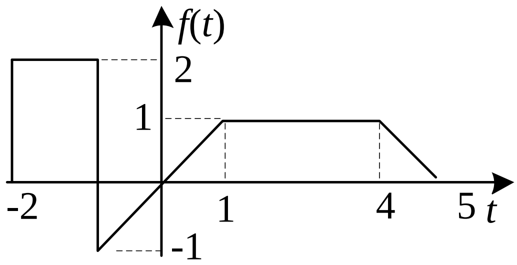
II intereval: $0 \le t \le 2$

$$u_c(t) = 3\int_0^t e^{-\tau} e^{-8(t-\tau)} d\tau = \frac{3}{7}(e^{-t} - e^{-8t})$$

III interval: $t > 2$

$$u_c(t) = \int_0^2 3e^{-\tau} 8e^{-8(t-\tau)} d\tau = \frac{3}{7}(e^{14} - 1)e^{-8t}$$

17. Grafičkim metodom odrediti konvoluciju funkcije *f*(*t*) i funkcije g(*t*)=*h*(*t*+4).

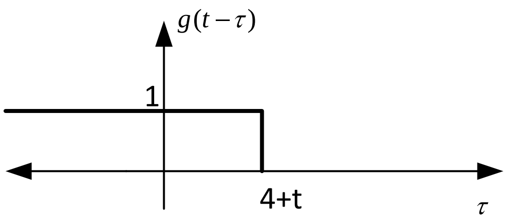


<u>Rešenje:</u>

Za određivanje konvolucije koristimo sledeći integral:

$$y(t)=\int_{-\infty}^{+\infty} f(\tau)g(t-\tau)d\tau$$

Kako funkcija $g(t-\tau)$ ima oblik:



uočavamo da se konvoluciona fukcija mijenja u sledećim intervalima:

Interval I: $4+t<-2 \Leftrightarrow t<-6$

$$y(t)=0$$

Interval II: $-2\le 4+t\le -1 \Leftrightarrow -6\le t\le -5$

$$y(t)=\int_{-2}^{t+4} 2d\tau = 2t+12$$

Interval III: $-1\le 4+t\le 1 \Leftrightarrow -5\le t\le -3$

$$y(t)=\int_{-2}^{-1} 2d\tau + \int_{-1}^{t+4} \tau d\tau = \frac{t^2}{2}+4t+9.5$$

Interval IV: $1\le 4+t\le 4 \Leftrightarrow -3\le t\le 0$

$$y(t)=\int_{-2}^{-1} 2d\tau + \int_{-1}^{1} \tau d\tau + \int_{1}^{t+4} d\tau = t+5$$

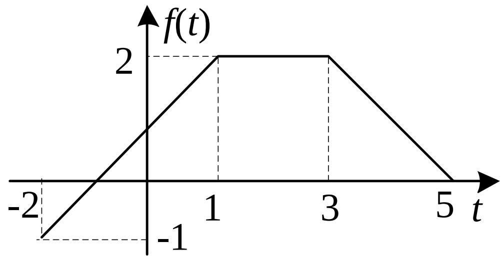
Interval V: $4\le 4+t\le 5 \Leftrightarrow 0\le t\le 1$

$$y(t)=\int_{-2}^{-1} 2d\tau + \int_{-1}^{1} \tau d\tau + \int_{1}^{4} d\tau + \int_{4}^{t+4} (5-\tau)d\tau = -\frac{t^2}{2}+t+5$$

Interval VI: $4+t \ge 5 \Leftrightarrow t \ge 1$

$$y(t)=\int_{-2}^{-1}2d\tau+\int_{-1}^{1}\tau d\tau+\int_{1}^{4}d\tau+\int_{4}^{5}(5-\tau)d\tau=5.5$$

18. Grafičkim metodom odrediti konvoluciju funkcije *f*(*t*) i funkcije g(*t*)=*h*(*t+2).*

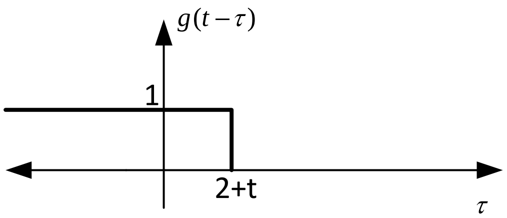


Rešenje:

Za određivanje konvolucije koristimo sledeći integral:

$$y(t)=\int_{-\infty}^{+\infty}f(\tau)g(t-\tau)d\tau$$

Kako funkcija $g(t-\tau)$ ima oblik:



uočavamo da se konvoluciona fukcija mijenja u sledećim intervalima:

Interval I: $t+2 \le -2 \Leftrightarrow t \le -4$

$$y(t)=0$$

Interval II: $-2 \le t+2 \le 1 \Leftrightarrow -4 \le t \le -1$

$$y(t)=\int_{-2}^{t+2}(\tau+1)d\tau=\frac{t^2}{2}+3t+4$$

Interval III: $1 \le t+2 \le 3 \Leftrightarrow -1 \le t \le 1$

$$y(t)=\int_{-2}^{1}(\tau+1)d\tau+\int_{1}^{t+2}2d\tau=2t+3.5$$

Interval IV: $3 \le t+2 \le 5 \Leftrightarrow 1 \le t \le 3$

$$y(t)=\int_{-2}^{1}(\tau+1)d\tau+\int_{1}^{3}2d\tau+\int_{3}^{t+2}(5-\tau)d\tau=-\frac{t^2}{2}+3t+3$$

Interval V: $t+2\geq 5 \Leftrightarrow t\geq 3$

$$y(t)=\int_{-2}^{1}(\tau+1)d\tau+\int_{1}^{3}2d\tau+\int_{3}^{5}(5-\tau)d\tau=7.5$$