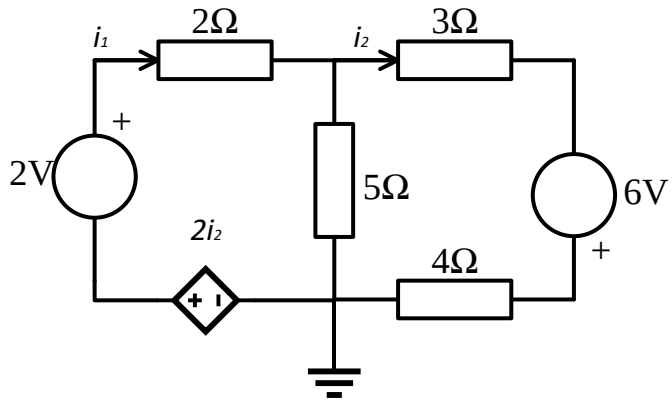
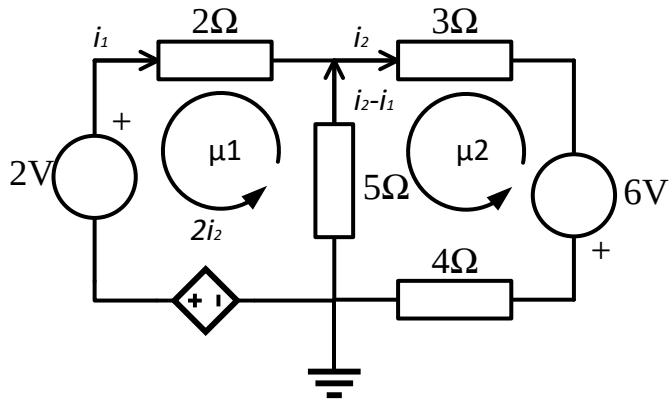
1. Za kolo sa slike izračunati struju $i_1(t)$.



Rešenje:



Primjernom drugog Kirhohovog zakona nad konturama μ_1 i μ_2 dobijamo sistem jednačina:

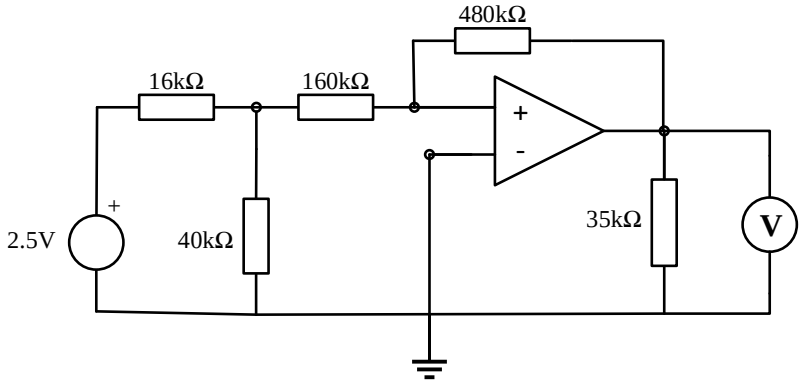
$$2 + 2i_2 + 5(i_2 - i_1) - 2i_1 = 0$$

$$6 - 3i_2 - 5(i_2 - i_1) - 4i_2 = 0$$

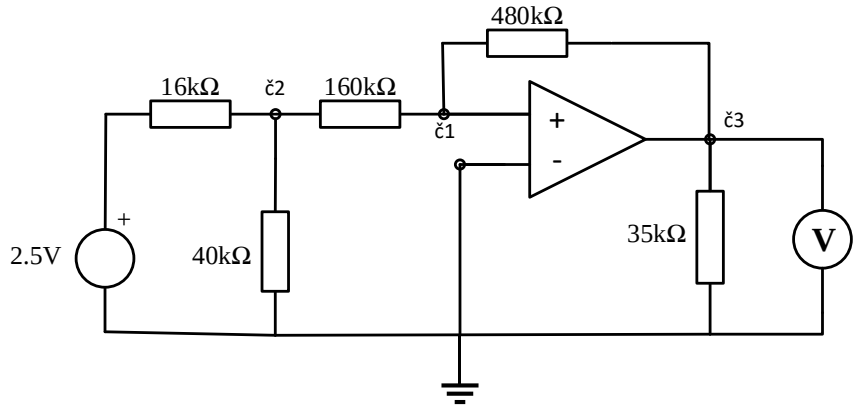
Rešavanjem sistema dolazimo do rešenja:

$$i_1 = 1.347A$$
$$i_2 = 1.061A$$

2. Odrediti pokazivanje voltmetra u kolu sa slike.



Rešenje:



$V_+ = V_- = V_1 = 0$

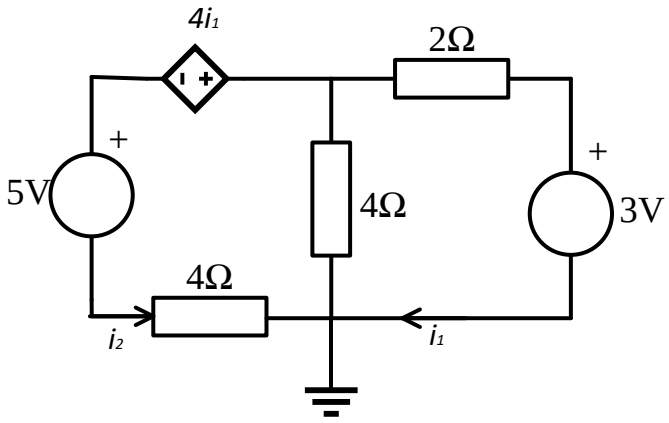
Primjenom metoda potencijala čvorova nad čvorovima č1 I č2 dobijamo sistem jednačina:

$$(\frac{1}{160k\Omega}+\frac{1}{480k\Omega})V_1 - \frac{1}{480k\Omega}V_3 - \frac{1}{160k\Omega}V_2 = 0$$

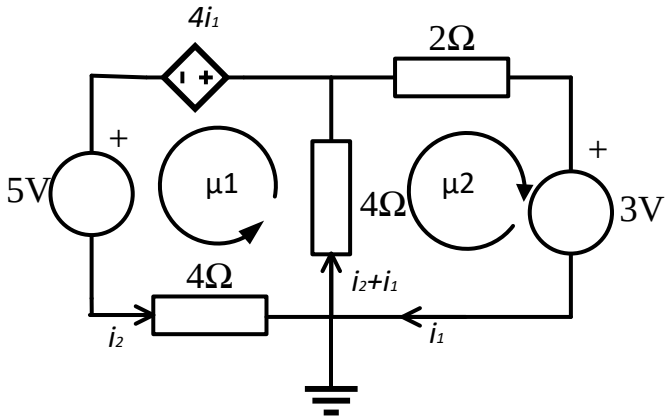
$$(\frac{1}{16k\Omega}+\frac{1}{40k\Omega}+\frac{1}{160k\Omega})V_2 - \frac{1}{160k\Omega}V_1 = \frac{2.5V}{16k\Omega}$$

Rešavanjem sistema jednačina dobijamo: $V_3 = -5V$

3. Za kolo sa slike izračunati struju $i_1(t)$.



Rešenje:



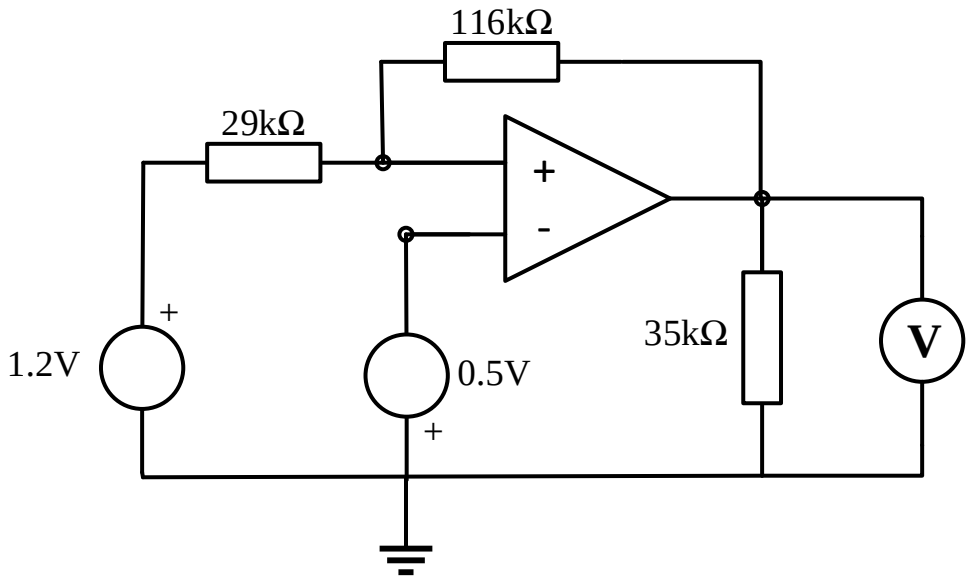
Primjenom drugog Kirhohovog zakona nad konturama μ_1 i μ_2 dobijamo sistem jednačina:

$$5 + 4i_2 + 4i_1 + 4i_2 + 4i_1 = 0$$

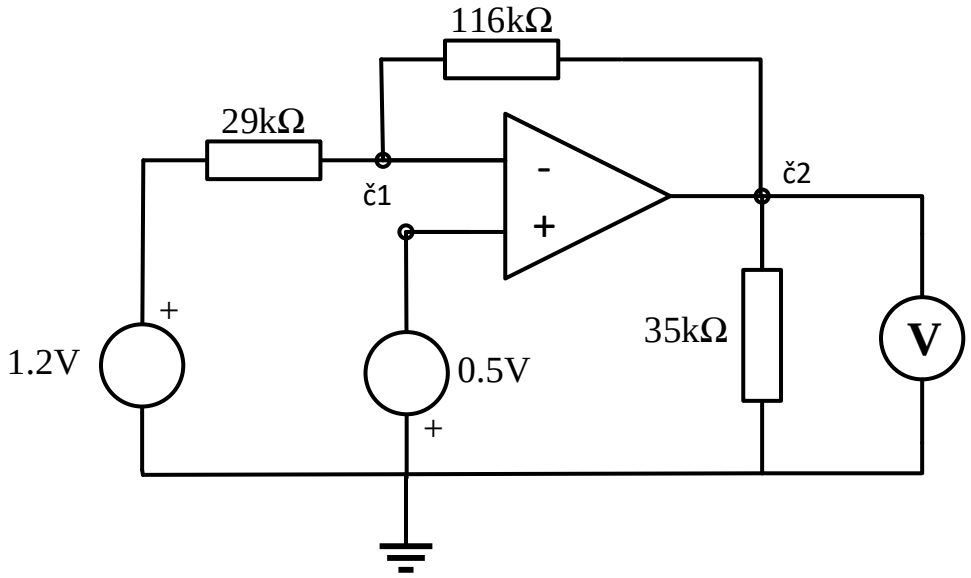
$$3 + 6i_1 + 4i_2 = 0$$

Rešavanjem sistema dolazimo do rešenja: $i_1 = -0.25A$

4. Odrediti pokazivanje voltmetra u kolu sa slike.



Rešenje:



Napon voltmetra jedan je potencijalu čvora 2.

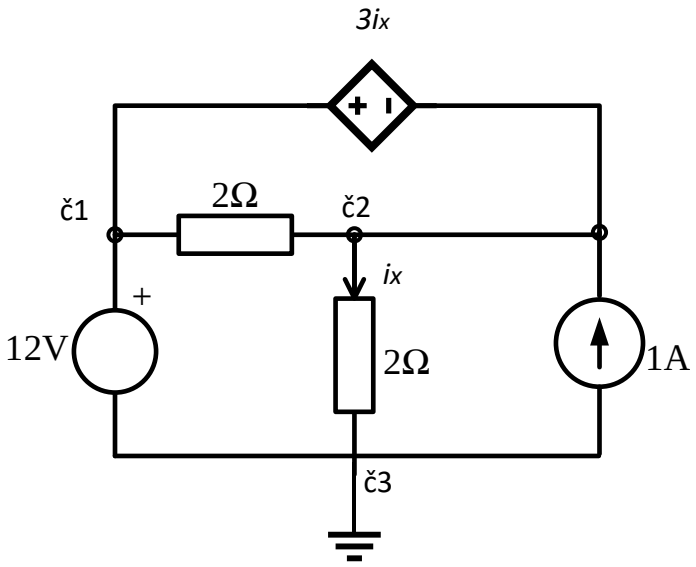
$V_+ = V_- = V_1 = -0.5V$

Primjenom metoda potencijala čvorova dobijamo:

$$(\frac{1}{29k\Omega} + \frac{1}{116k\Omega})V_1 - \frac{1}{116k\Omega}V_2 = \frac{1.2V}{29k\Omega}$$

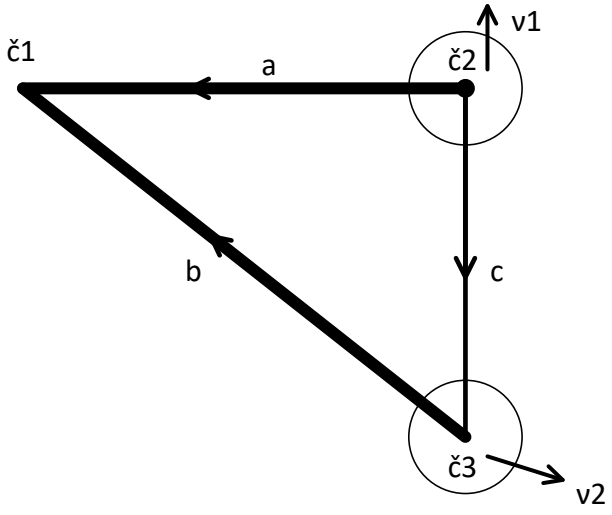
$V_2 = -7.3V$

5. Primjenom metoda osnovih presjeka odrediti struju i_x u kolu sa slike.



Rešenje:

Graf kola:



Matrica osnovnih presjeka:

$$\underset{\sim}{Q_f} = \begin{array}{c} \\ \end{array} \begin{array}{c} \text{a}\ \ \text{b}\ \ \text{c} \\ \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \end{bmatrix} \end{array}$$

$$\underset{\sim}{U} = \underset{\sim}{Q}_f^T \cdot \underset{\sim}{u}_T$$

$$\begin{bmatrix} u_a \\ u_b \\ u_c \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} u_a \\ u_b \end{bmatrix}$$

$$u_c = u_a - u_b$$

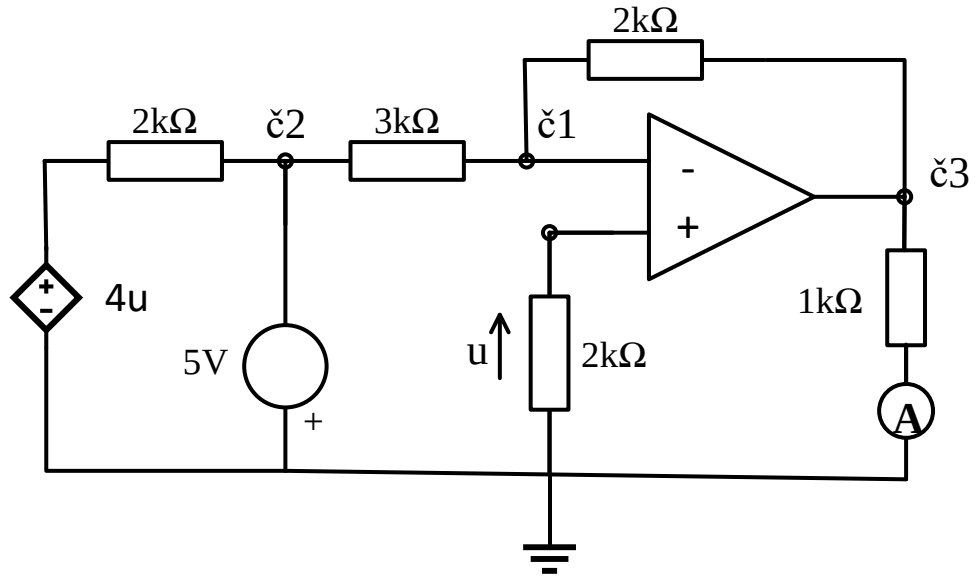
Sa slike se vidi da je:

$$u_b = -12V$$
$$u_a = -3i_x = -3\frac{u_c}{2}$$
$$u_c = -3i_x + 12$$

Takođe, vidi se da je $u_c = 2i_x$, pa je rešenje:

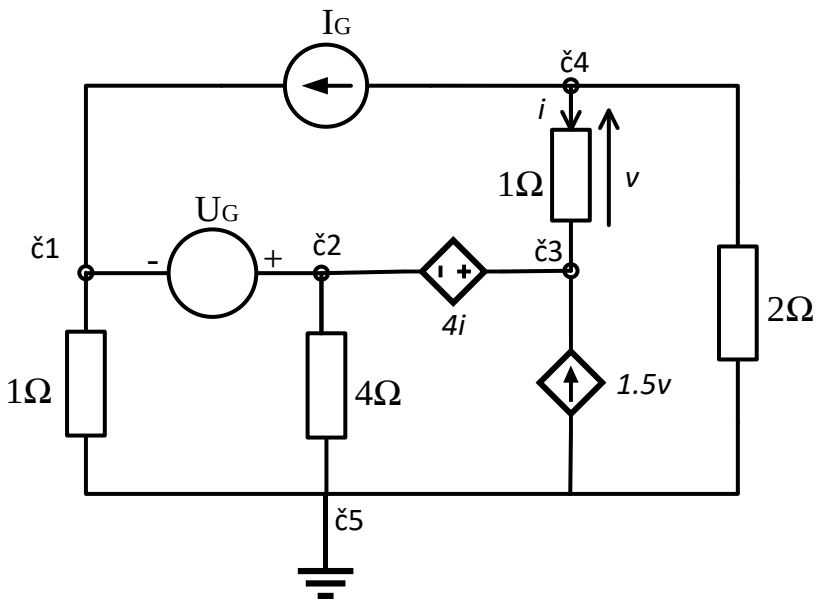
$$i_x = \frac{12}{5}A$$

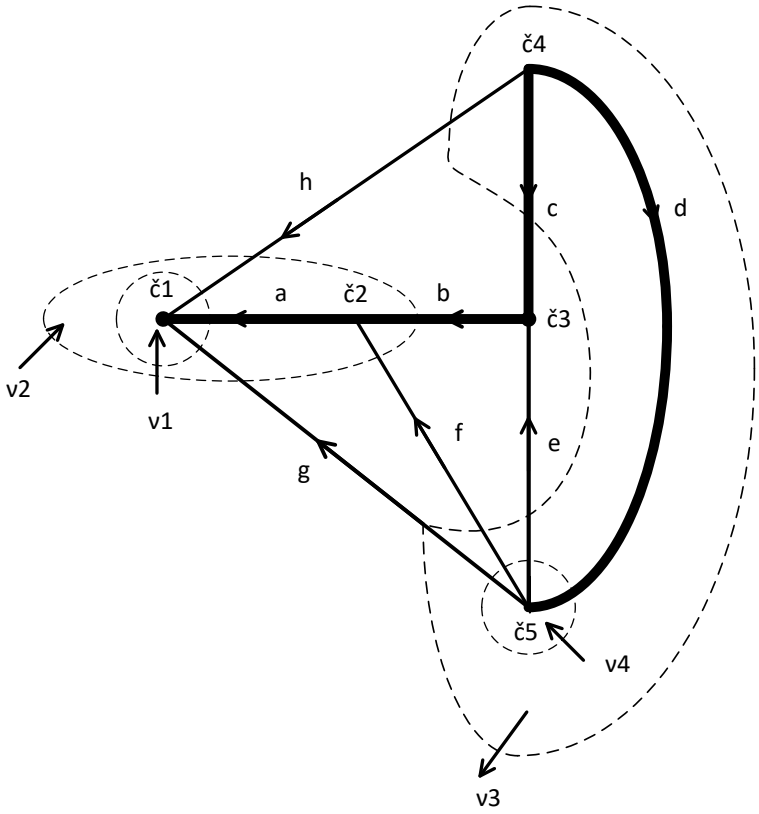
6. Odrediti struju ampermetra u kolu sa slike.



$$V_1 = 0V$$
$$V_2 = -5V$$
$$u = 0V$$
$$\frac{V_2}{3} = \frac{-V_3}{2}$$
$$i_A = \frac{V_3}{1K\Omega} = \frac{10}{3}mA$$

7. Primjenom metoda osnovnih presjeka odrediti napone svih grana u kolu sa slike, ako je $U_G = 10V$ a I_G=4A.





$$Q_f = \begin{matrix} a & b & c & d & e & f & g & h \end{matrix}$$
$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & -1 & -1 & -1 & 0 \end{bmatrix}$$

$$\underset{\sim}{U} = \underset{\sim}{Q}_f^T \cdot \underset{\sim}{u}_T$$

$$\begin{bmatrix} u_a \\ u_b \\ u_c \\ u_d \\ u_e \\ u_f \\ u_g \\ u_h \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & -1 \\ 0 & 1 & 1 & -1 \\ 1 & 1 & 1 & -1 \\ 1 & 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} u_a \\ u_b \\ u_c \\ u_d \end{bmatrix}$$

$$u_e = u_c - u_d$$
$$u_f = u_b + u_c - u_d$$
$$u_g = u_a + u_b + u_c - u_d$$
$$u_h = u_a + u_b + u_c$$

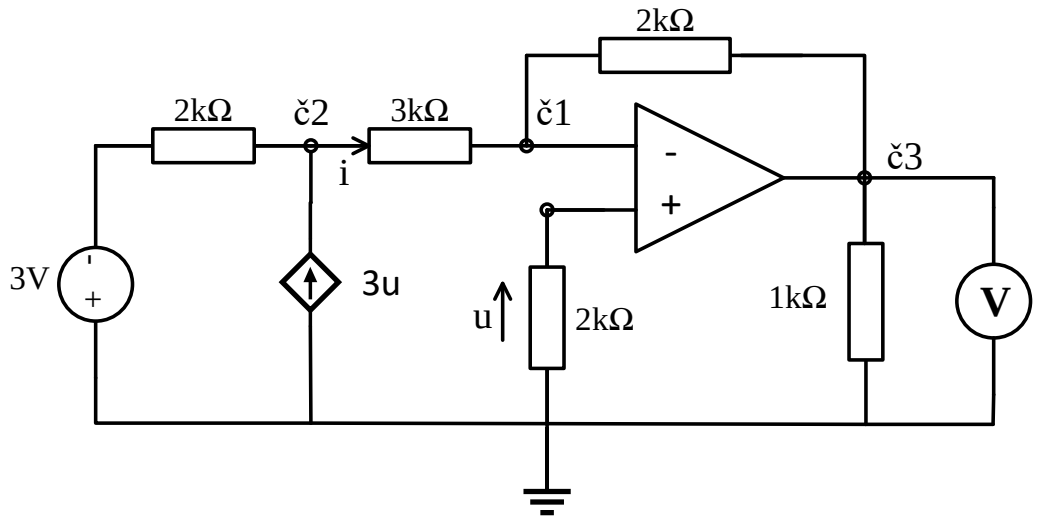
Dodatne jednačine:

$$u_a = U_G$$
$$u_b = 4u_c$$
$$0.5u_d = 1.5u_c + 0.25u_f + u_g$$
$$I_G + u_c + 0.5u_d = 0$$

Rešavanjem sistema jednačina dobijaju se svi naponi u kolu:

$$\begin{bmatrix} u_a \\ u_b \\ u_c \\ u_d \\ u_e \\ u_f \\ u_g \\ u_h \end{bmatrix} = \begin{bmatrix} 10V \\ -8.53V \\ -2.13V \\ -3.73V \\ 1.6V \\ -6.93V \\ 3.06V \\ -0.66V \end{bmatrix}$$

8. Izračunati napon voltmetra u kolu sa slike.



$V_1 = 0$

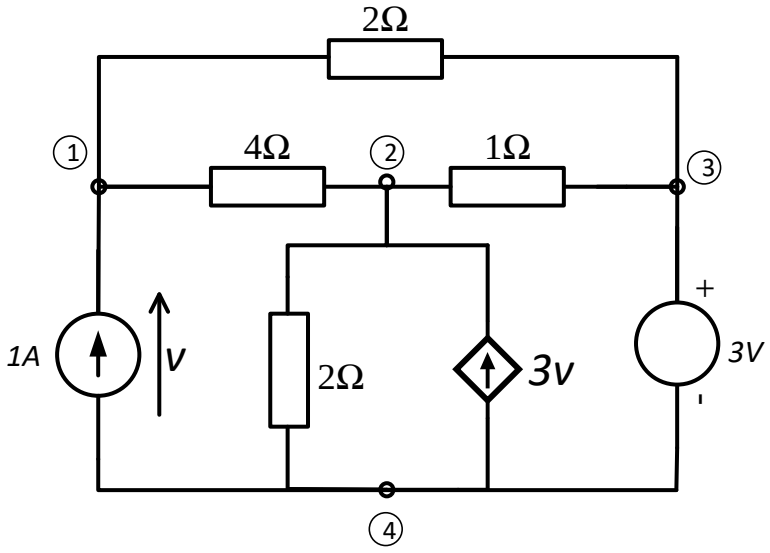
$u = 0$

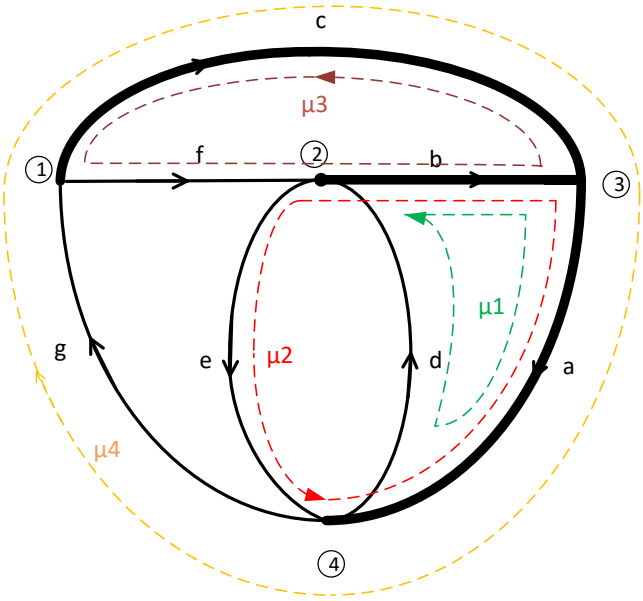
$i = \frac{-3V - 0}{5k\Omega} = -\frac{3}{5}mA$

$\Rightarrow -\frac{3}{5}mA = -\frac{V_3}{2k\Omega}$

$V_3 = \frac{6}{5}V$

9. Primjenom metoda nezavisih struja odrediti struje svih grana u kolu sa slike.





$$\underset{\sim}{i} = \underset{\sim}{B_f^T} \cdot \underset{\sim}{i_L}$$

$$B_f = \begin{array}{c} \begin{matrix} a & b & c & d & e & f & g \end{matrix} \\ \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ -1 & -1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

$$\begin{bmatrix} i_a \\ i_b \\ i_c \\ i_d \\ i_e \\ i_f \\ i_g \end{bmatrix} = \underset{\sim}{B_f^T} \cdot \begin{bmatrix} i_d \\ i_e \\ i_f \\ i_g \end{bmatrix}$$

$$i_a = i_d - i_e + i_g$$
$$i_b = i_d - i_e + i_f$$
$$i_c = -i_f + i_g$$

Sa slike vidimo da je:

$$i_g = 1A$$
$$i_d = 3v$$

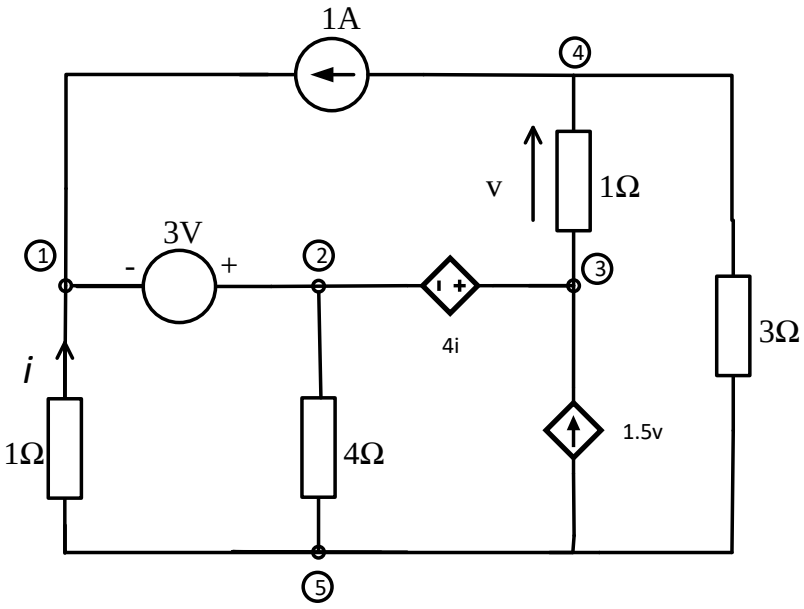
Dodatne jednačine dobijamo primjenom II Kirhohovog zakona:

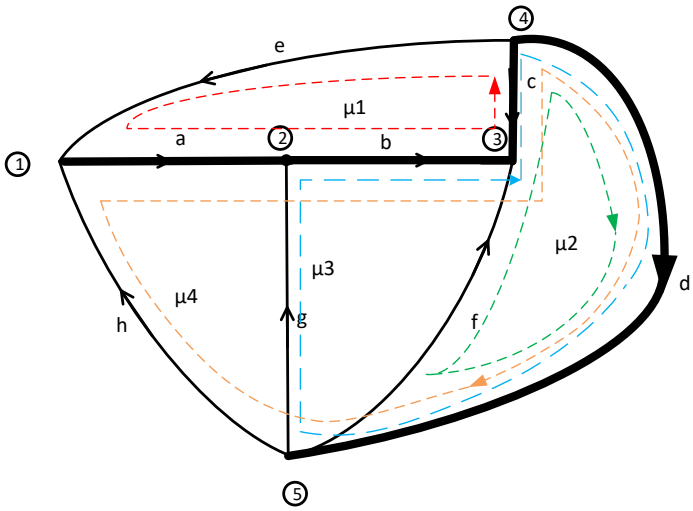
$$v - 2i_e - 4i_f = 0 \Rightarrow i_d = 6i_e + 12i_f$$
$$2i_e - i_b = U_G = 3$$
$$2i_c - i_b - 4i_f = 0$$

Ovim je kompletiran sistema jednačina na osnovu kojeg se mogu dobiti sve struje u kolu.

$$\begin{bmatrix} i_a \\ i_b \\ i_c \\ i_d \\ i_e \\ i_f \\ i_g \end{bmatrix} = \begin{bmatrix} 171/8 \\ 71/4 \\ 29/8 \\ 123/4 \\ 83/8 \\ -21/8 \\ 1 \end{bmatrix}$$

10. Primjenom metoda nezavisnih struja odrediti sve struje u kolu sa slike.





$$\underset{\sim}{i} = \underset{\sim}{B_f^T} \cdot \underset{\sim}{i_L}$$

$$B_f = \begin{array}{c} \begin{matrix} a & b & c & d & e & f & g & h \end{matrix} \\ \begin{bmatrix} 1 & 1 & -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & -1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & -1 & 1 & 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

$$\begin{bmatrix} i_a \\ i_b \\ i_c \\ i_d \\ i_e \\ i_f \\ i_g \\ i_h \end{bmatrix} = \underset{\sim}{B_f^T} \cdot \begin{bmatrix} i_e \\ i_f \\ i_g \\ i_h \end{bmatrix}$$

$$i_a = i_e + i_h$$
$$i_b = i_e + i_g + i_h$$
$$i_c = -i_e - i_f - i_g - i_h$$
$$i_d = i_f + i_g + i_h$$

Sa slike vidimo da važi:

$$i_e = 1A$$
$$i_f = 1.5 i_c$$

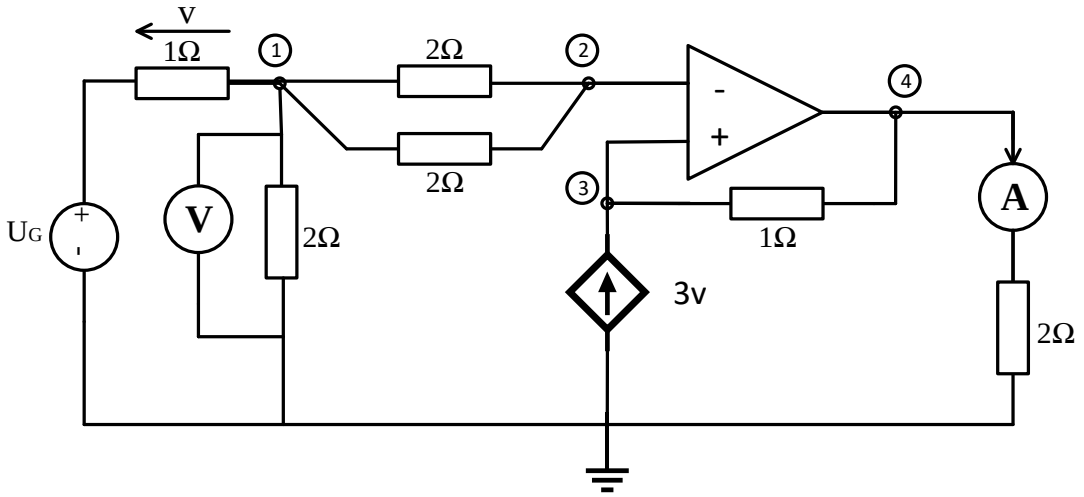
Primjenom II Kirhohovog zakona dolazimo do dodatnih jednačina:

$$i_h - 4 i_g = 3$$
$$4 i_g - 4 i_h - i_c + 3 i_d = 0$$

Sistem je sada kompletan.

$$\begin{bmatrix} i_a \\ i_b \\ i_c \\ i_d \\ i_e \\ i_f \\ i_g \\ i_h \end{bmatrix} = \begin{bmatrix} -23/5 \\ -27/4 \\ 27/10 \\ -37/10 \\ 1 \\ 81/20 \\ -43/20 \\ -28/5 \end{bmatrix}$$

11. Odrediti pokazivanje ampermetra u kolu sa slike. Pokazivanje voltmetra je 4V.



U_v - napon voltmetra

$V_1 = U_V = 4V$

$(1+0.5+0.5+0.5)V_1 - V_2 = U_G \Rightarrow V_2 = 10 - U_G$

$(0.5+0.5)V_2 - (0.5+0.5)V_1 = 0 \Rightarrow V_2 = V_1 = 4V$

$4 = 10 - U_G \Rightarrow U_G = 6V$

$V_3 - V_4 = 3v$

$V_3 = V_2 = 4V$

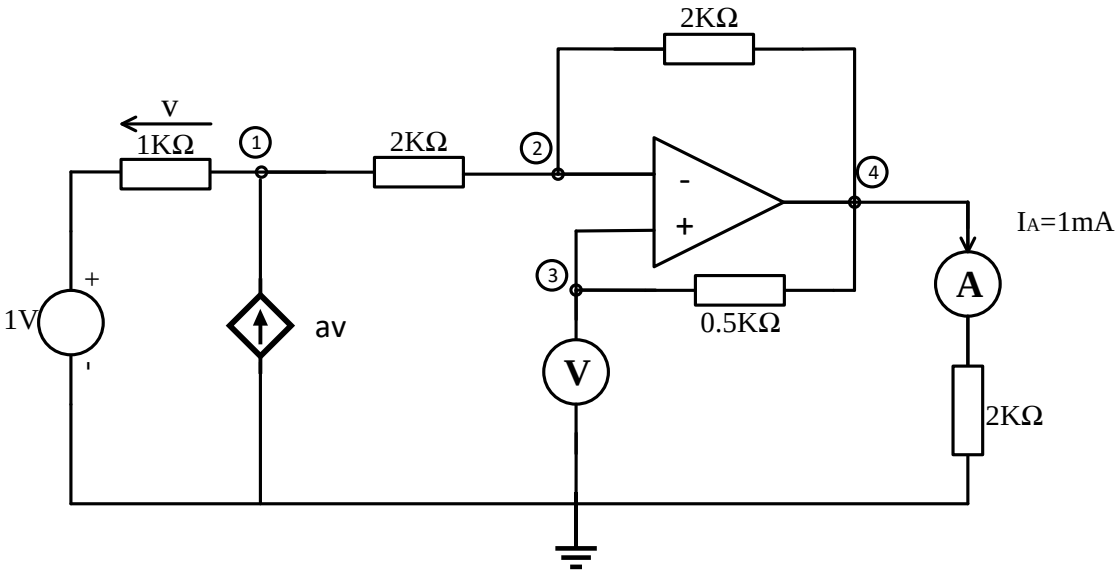
II Kirhohov zakon:

$-U_G + v + 4V = 0 \Rightarrow v = 2V$

$V_4 = 4V - 6V = -2V$

$I_A = \frac{V_4}{2\Omega} = -1A$

12. Odrediti pokazivanje voltmetra u kolu sa slike.



Primjenom metoda potencijala čvorova dobijamo sledeće jednačine:

$$(0.5+0.5)V_2 - 0.5V_1 - 0.5V_4 = 0$$
$$(1+0.5)V_1 - 0.5V_2 = 1 + av \cdot 10^3$$

Takođe važi:

$$V_4 = 2K\Omega \cdot I_A = 2V$$
$$2V_3 - 2V_4 = 0 \Rightarrow V_2 = V_3 = V_4 = V_1 = 2V$$

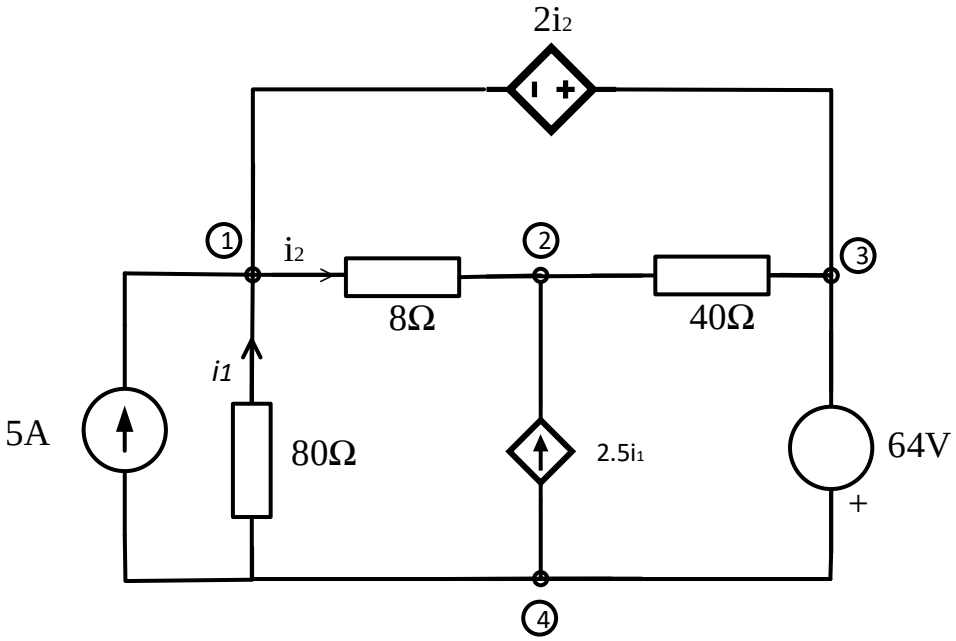
Primjenom II Kirhohovog zakona dolazimo do jednačine:

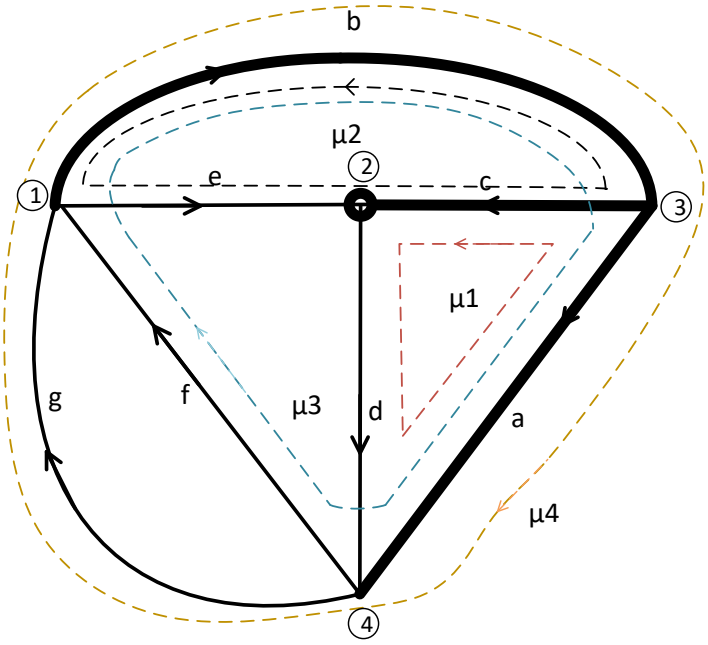
$$-1 + v + V_1 = 0$$

Odavde slijedi:

$$v = -1$$
$$a = -0.001$$
$$U_v = V_3 = 2V$$

13. Primjenom metoda nezavisnih struja, odrediti struju i_2.





$$\underset{\sim}{i} = B_f^T \cdot \underset{\sim}{i}_L$$

$$B_f = \begin{array}{c} \begin{matrix} a & b & c & d & e & f & g \end{matrix} \\ \begin{bmatrix} -1 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & -1 & -1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

$$\begin{bmatrix} i_a \\ i_b \\ i_c \\ i_d \\ i_e \\ i_f \\ i_g \end{bmatrix} = \underset{\sim}{B_f^T} \cdot \begin{bmatrix} i_d \\ i_e \\ i_f \\ i_g \end{bmatrix}$$

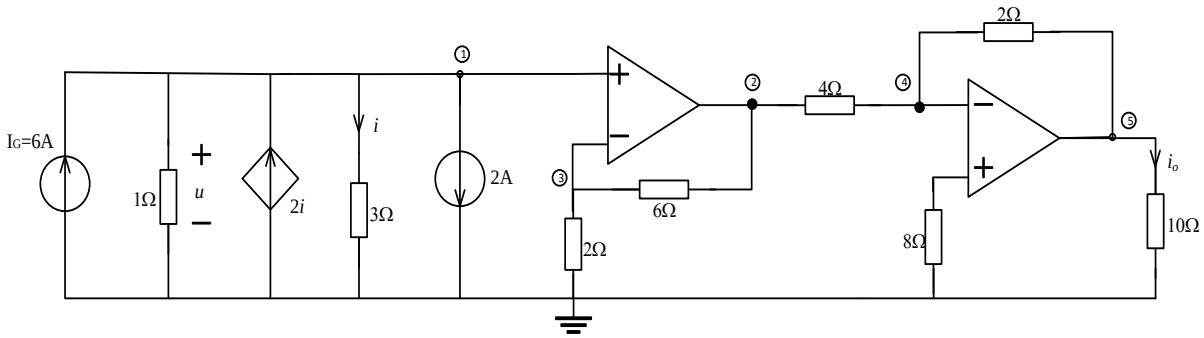
$$i_a = -i_d + i_f + i_g$$
$$i_b = -i_e + i_f + i_g$$
$$i_c = i_d - i_e$$
$$i_g = 5A$$
$$i_d = 2.5 i_f$$

Primjenom II Kirhohovog zakona dolazimo do seta dodatnih jednačina:

$$8i_e - 40i_c + 2i_e = 0$$
$$64 + 40i_c - 8i_e - 80i_f = 0$$

Rešavanjem Sistema dolazimo do rešenja: $i_2 = i_e = 1.68A$

14. U kolu sa slike odrediti struju i_o I snagu strujnog generator I_G.



Rešenje:

Primjena metoda potencijala čvorova nad čvorovima 1, 3 I 4:

$$(\frac{1}{3}+1)V_1 = 4+2i = 4+2\frac{V_1}{3} \quad \Rightarrow V_1 = u = V_3 = 4V$$

$$(\frac{1}{2}+\frac{1}{6})V_3 - \frac{1}{6}V_2 = 0 \quad \Rightarrow V_2 = 4V_3 = 16V$$

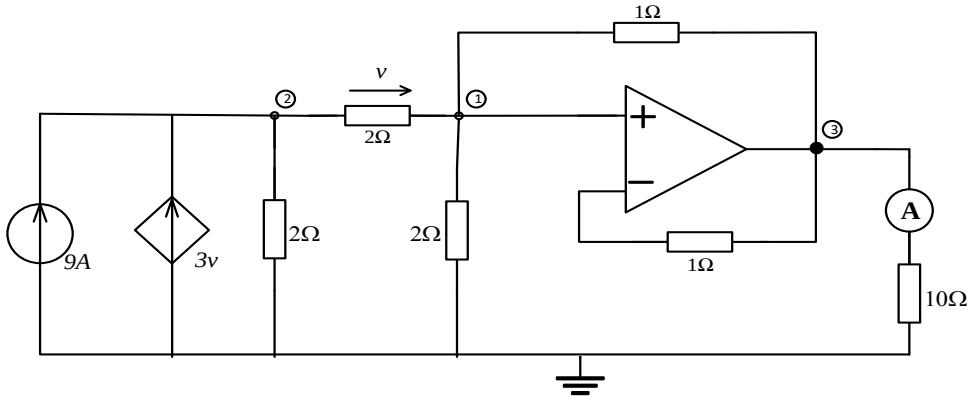
$$\frac{-1}{4}V_2 - \frac{1}{2}V_5 = 0 \quad \Rightarrow V_5 = -8V$$

$$V_4 = 0$$

$$i_o = \frac{-8V}{10\Omega} = -\frac{4}{5}A$$

$$P_{I_G} = I_G \cdot V_1 = 24W$$

15. U kolu sa slike odrediti struju ampermetra:



Primjenom metoda potencijala čvorova dobijamo:

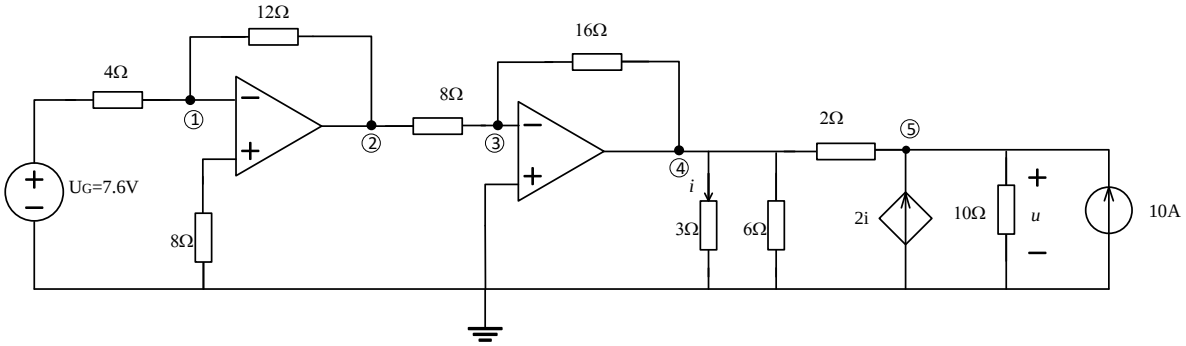
$$2V_1 - 0.5V_2 - V_3 = 0$$

$$V_2 - 0.5V_1 = 9 + 3v$$

Kako je $V_1 = V_3$, a $v = V_1 - V_2$, rešavanjem sistema jednačina dobija se:

$$I_A = \frac{1}{5}A$$

16. U kolu sa slike odrediti napon *u* i snagu generatora U_G.



Primjenom metoda potencijala čvorova dobijamo sledeće jednačine:

$$(\frac{1}{4}+\frac{1}{12})V_1-\frac{1}{12}V_2=\frac{7.6}{4}$$
$$(\frac{1}{8}+\frac{1}{16})V_3-\frac{1}{8}V_2-\frac{1}{16}V_4=0$$
$$(\frac{1}{2}+\frac{1}{10})V_5-\frac{1}{2}V_4=10+2i$$

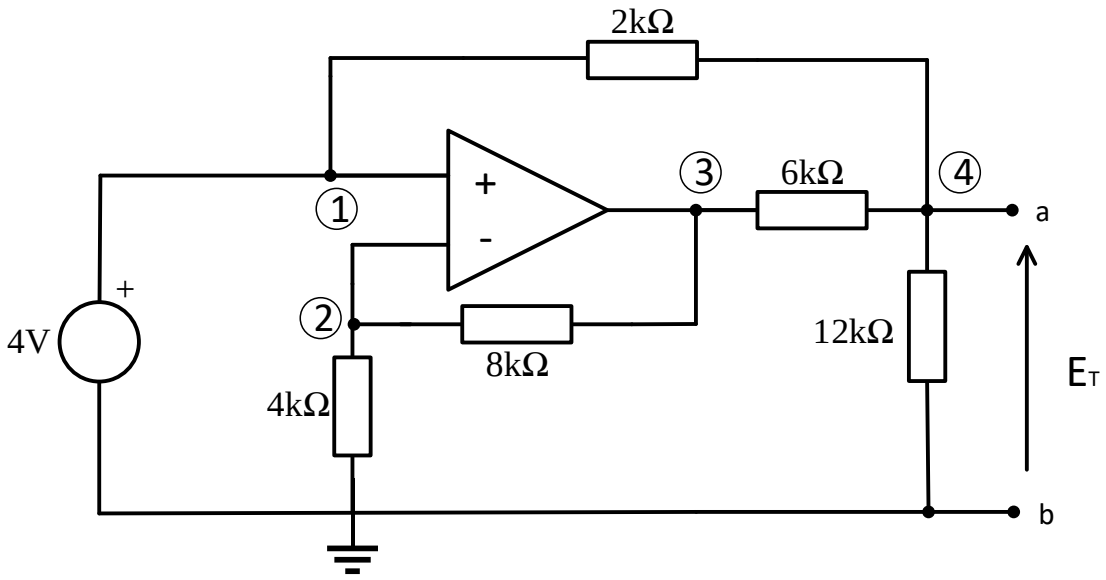
Sa slike vidimo da važi:

$$V_1=0$$
$$V_3=0$$
$$u=V_5$$
$$i=\frac{V_4}{3}$$

Rešavanjem sistema jednačina dolazimo do rešenja:

$$V_2=-22.8V$$
$$V_4=-2V_2=45.6V$$
$$u=105.33V$$
$$P_{U_G}=U_G\cdot\frac{V_1-V_2}{12}=14.44W$$

17. Ekvivalentirati kolo Teveninovim generatorom između tačaka a I b.



Prvo tražimo napon Teveninovog generatora:

$V_1 = V_2 = 4V$

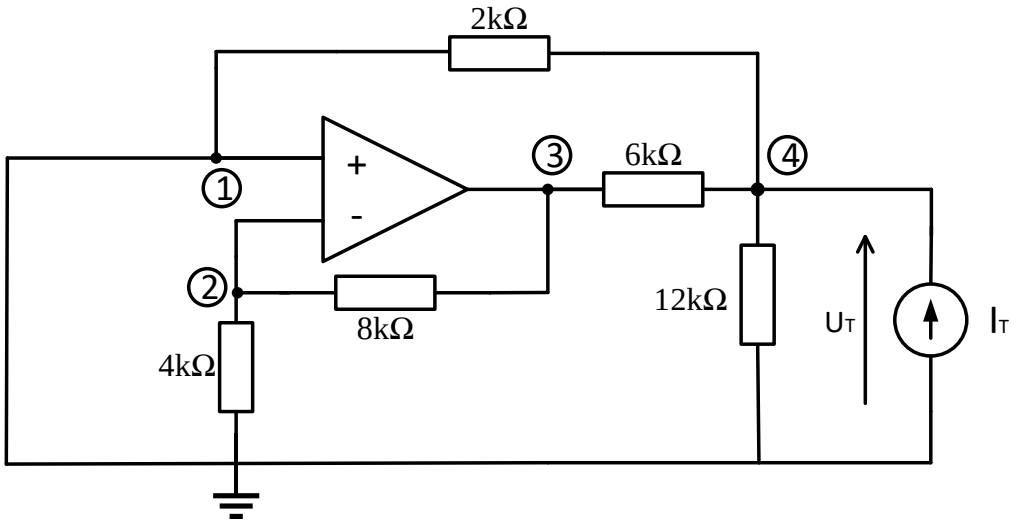
$(\frac{1}{4}+\frac{1}{8})V_2 - \frac{1}{8}V_3 = 0$

$(\frac{1}{6}+\frac{1}{2}+\frac{1}{12})V_4 - \frac{1}{6}V_3 - \frac{1}{2}V_1 = 0$

$V_3 = 12V$

$V_1 = V_4 = E_T = 5.33V$

Otpornost Teveninovog generator određujemo tako što iskjučimo nezavisne izvore u kolu, a zatim uvedemo testni generator između tačaka a I b.



Primjenom metoda potencijala čvorova dobijamo jednačine:

$$(\frac{1}{6}+\frac{1}{2}+\frac{1}{12})V_4-\frac{1}{6}V_3-\frac{1}{2}V_1=I_T\cdot 10^3$$
$$(\frac{1}{4}+\frac{1}{8})V_2-\frac{1}{8}V_3=0$$

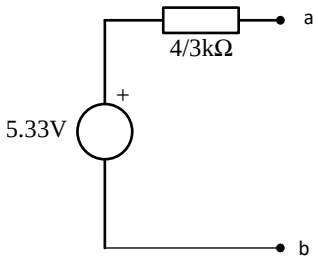
Kako je:

$$V_1=V_2=0$$
$$U_T=V_4=\frac{4}{3}I_T\cdot 10^3$$

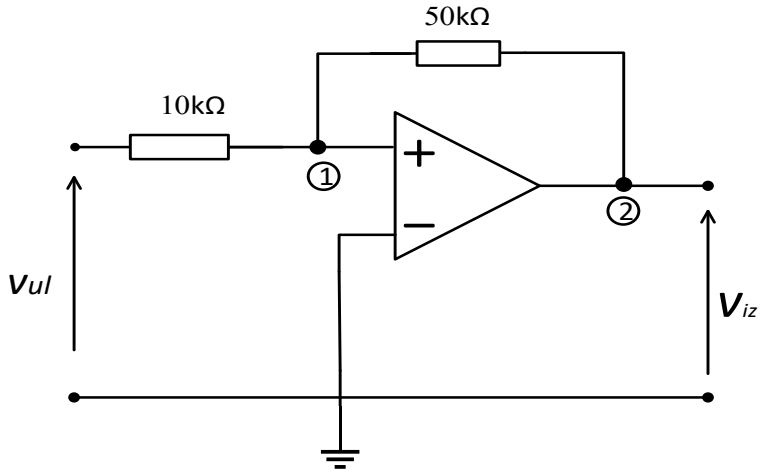
Otpornost Teveninovog generatora je:

$$R_T=\frac{U_T}{I_T}=\frac{4}{3}k\Omega$$

Ekvivalentna šema kola je:



18. Odrediti pojačanje realnog operacionog pojačavače sa slike.



$$V_1 = V_+$$
$$v_{iz} = A \cdot (V_+ - V_-)$$
$$A = \frac{v_{iz}}{V_+}$$

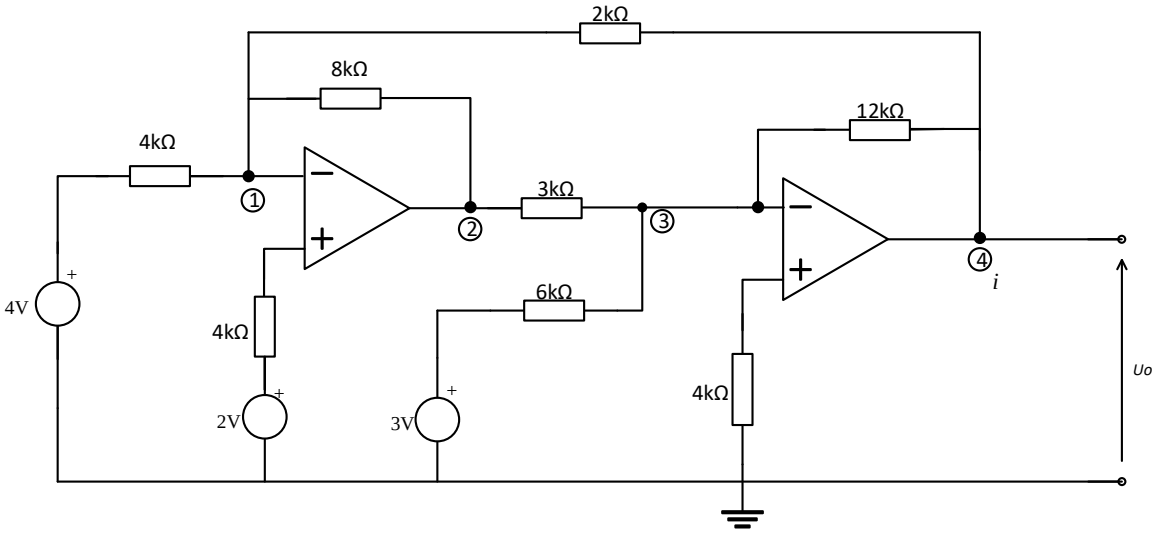
Primjenom metoda potencijala čvorova dobijamo:

$$(\frac{1}{10} + \frac{1}{50}) \cdot V_1 - \frac{1}{50} V_2 = \frac{v_{ul}}{10}$$

$$V_1 = V_+ = \frac{5v_{ul} + v_{iz}}{6}$$

$$A = \frac{6v_{iz}}{5v_{ul} + v_{iz}}$$

19. Naći napon U_o u kolu sa slike.



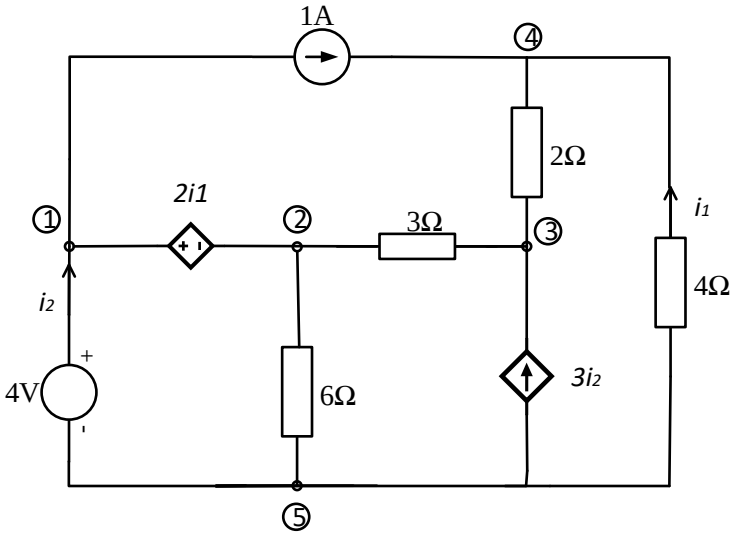
Primjenom metoda potencijala čvorova dolazimo do seta jednačina:

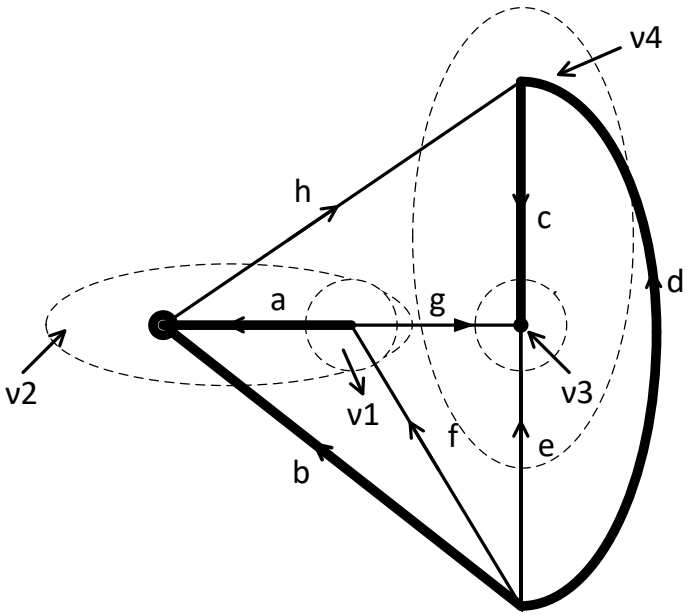
$$(\frac{1}{4} + \frac{1}{8} + \frac{1}{2}) V_1 - \frac{1}{8} V_2 - \frac{1}{2} V_4 = 1$$
$$(\frac{1}{3} + \frac{1}{6} + \frac{1}{12}) V_3 - \frac{1}{3} V_2 - \frac{1}{12} V_4 = \frac{1}{2}$$

Kako je V_1= 2V a V_3=0, dobijamo:

$$U_o = V_4 = 2V$$

20. Primjenom metoda nezavisnih napona odrediti struju i_2.





$$Q_f = \begin{array}{c} \begin{matrix} a & b & c & d & e & f & g & h \end{matrix} \\ \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & -1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & -1 & -1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 & 1 \end{bmatrix} \end{array}$$

$$\underset{\sim}{U} = \underset{\sim}{Q_f^T} \cdot \underset{\sim}{u_T}$$

$$\begin{bmatrix} u_a \\ u_b \\ u_c \\ u_d \\ u_e \\ u_f \\ u_g \\ u_h \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ -1 & 1 & 0 & 0 \\ 1 & -1 & 1 & 1 \\ 0 & -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_a \\ u_b \\ u_c \\ u_d \end{bmatrix}$$

$$u_e = u_c + u_d$$
$$u_f = -u_a + u_b$$
$$u_g = u_a - u_b + u_c + u_d$$
$$u_h = -u_b + u_d$$

Dodatne jednačine:

$$u_a = -\frac{u_d}{2}$$
$$u_b = -4V$$

Primjenom I Kirhohovog zakona nad čvorovima 4 I 5 kompletiramo sistem jednačina:

$$1 + \frac{1}{4}u_d = \frac{1}{2}u_c$$
$$\frac{u_g}{3} + 3i_2 + \frac{u_c}{2} = 0$$
$$\frac{u_d}{4} + 3i_2 + \frac{u_f}{6} + i_2 = 0$$

Rešavanjem sistema jednačina dobija se: $i_2 = 1.041A$

21. Na osnovu zadate matrice osnovnih kontura izvesti matricu osnovnih presjeka kola:

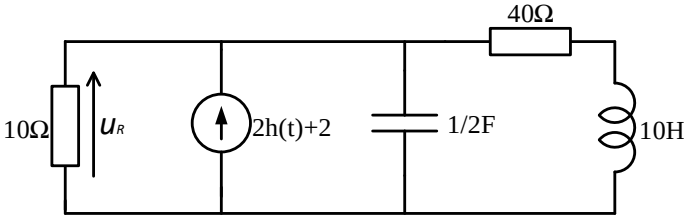
$$\underset{\sim}{B_f} = \begin{bmatrix} 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\underset{\sim}{Q_f} = \left| 1_{nxn}; \underset{\sim}{Q_{fL}} \right|$$

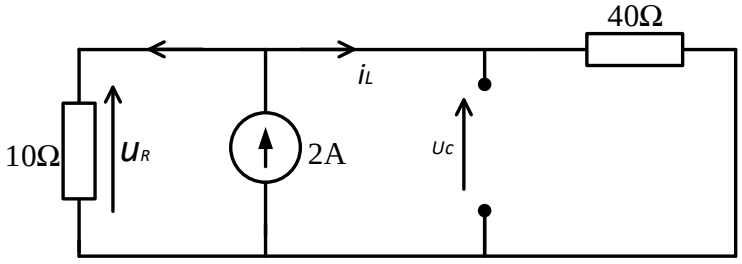
$$\underset{\sim}{B_f} = \left| \underset{\sim}{B_{fT}}; 1_{mxm} \right|$$

$$\underset{\sim}{Q_f} \cdot \underset{\sim}{B_f} = \underset{\sim}{0} \Rightarrow \underset{\sim}{Q_f} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & -1 & -1 \\ 0 & 1 & 0 & 0 & 0 & -1 & -1 & -1 \\ 0 & 0 & 1 & 0 & -1 & -1 & -1 & 0 \\ 0 & 0 & 0 & 1 & -1 & -1 & -1 & -1 \end{bmatrix}$$

22. Odrediti napon U_R za $t > 0$, ako je za $t < 0$ kolo bilo u stacionarnom stanju.



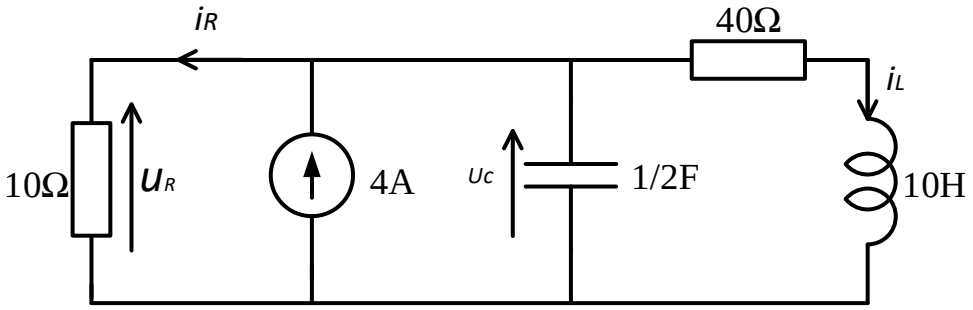
Analiza kola za t<0:



$$i_L(0^-) = \frac{10}{10+40} \cdot 2 = 0.4A$$

$$u_C(0^-) = 40 \cdot 0.4 = 16V$$

Analiza kola za t>0:



$$4 = i_R + i_C + i_L \Rightarrow i_L = 4 - i_R - i_C = 4 - \frac{u_R}{10} - C\frac{du_C}{dt}$$

$$u_R = u_C \Rightarrow i_L = 4 - \frac{u_R}{10} - C\frac{du_R}{dt}$$

$$u_R = 40i_L + u_L = 40(4 - \frac{u_R}{10} - C\frac{du_R}{dt}) + L\frac{d}{dt}(4 - \frac{u_R}{10} - C\frac{du_R}{dt})$$

$$u_R = 160 - 4u_R - 20\frac{du_R}{dt} - \frac{du_R}{dt} - 5\frac{d^2u_R}{dt^2}$$

$$(5D^2 + 21D + 5)u_R = 160$$

$$(D^2 + 4.2D + 1)u_R = 32$$

$$s_1 = -0.25$$

$$s_2 = -3.95$$

$$u_{R_h}(t) = Ae^{-0.25t} + Be^{-3.95t}$$

$$u_{R_p}(t) = 32$$

$$u_R(t) = Ae^{-0.25t} + Be^{-3.95t} + 32$$

$$u_C(0_-) = u_C(0_+) \Rightarrow A + B + 32 = 16 \Rightarrow A = -16 - B$$

$$i_L(0_-) = i_L(0_+) \Rightarrow 4 - \frac{u_R(0_+)}{10} - C\frac{du_R(0_+)}{dt} = 0.4A$$

$$4 - 1.6 - 0.5(-0.25A - 3.95B) = 0.4$$

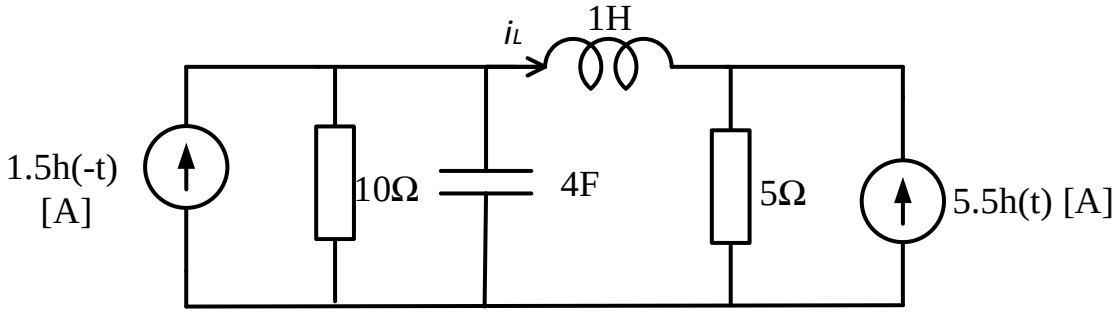
$$0.25(-16 - B) - 3.95B = -2$$

$$-4.2B = 2 \Rightarrow B = -0.476$$

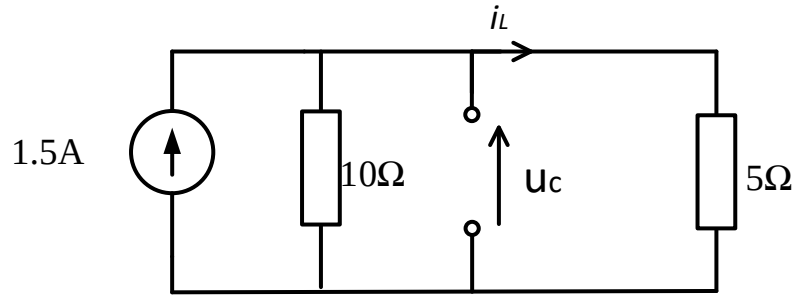
$$A = -15.524$$

$$u_R(t) = -15.524e^{-0.25t} + -0.476e^{-3.95t} + 32$$

23. Odrediti napon na kondezatoru za $t > 0$, ako je za $t < 0$ kolo bilo u stacionarnom stanju.



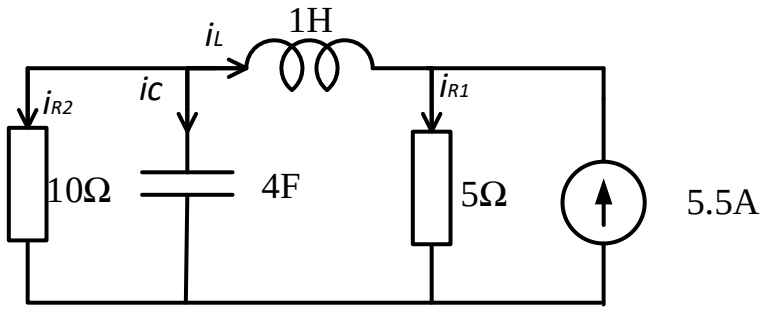
Za t<0:



$$i_L(0^-) = \frac{10}{15} \cdot 1.5 = 1A$$

$$u_C(0^-) = 5V$$

Za t>0:



$$5.5 + i_L = i_{R1}$$

$$i_L + i_C + i_{R2} = 0$$

$$u_C = u_{R2}$$

$$i_L = -i_C - i_{R2} = -4\frac{du_c}{dt} - \frac{u_c}{10}$$

$$i_{R1} = 5.5 - 4\frac{du_c}{dt} - \frac{u_c}{10}$$

$$u_C - u_L - u_{R1} = 0$$

$$u_C - L\frac{di_L}{dt} - 5i_{R1} = 0$$

$$u_C - 1\frac{d}{dt}(-4\frac{du_c}{dt} - \frac{u_c}{10}) - 5(5.5 - 4\frac{du_c}{dt} - \frac{u_c}{10}) = 0$$

$$u_C + 4\frac{d^2u_C}{dt^2} + \frac{1}{10}\frac{du_c}{dt} - 27.5 + 20\frac{du_c}{dt} + \frac{1}{2}u_C = 0$$

$$(4D^2 + 20.1D + 1.5)u_C = 27.5$$

$$(D^2 + 5.025D + 0.375)u_C = 6.875$$

Rešenje homogene diferencijalne jednačine je:

$$u_{C_h}(t) = Ae^{-0.07t} + Be^{-4.95t}$$

Partikularno rešenje je:

$$u_{C_p}(t) = 18.33$$

Potpuni odziv:

$u_C(t) = Ae^{-0.07t} + Be^{-4.95t} + 18.33$

Iz početnih uslova važi:

$u_C(0^-) = u_C(0^+) = A + B + 18.33 = 5 \Rightarrow A = -13.33 - B$

$i_L(0^-) = i_L(0^+) = 1A = -4\dfrac{du_c(0^+)}{dt} - \dfrac{u_c(0^+)}{10} = -4(-0.07A - 4.95B) - 0.5$

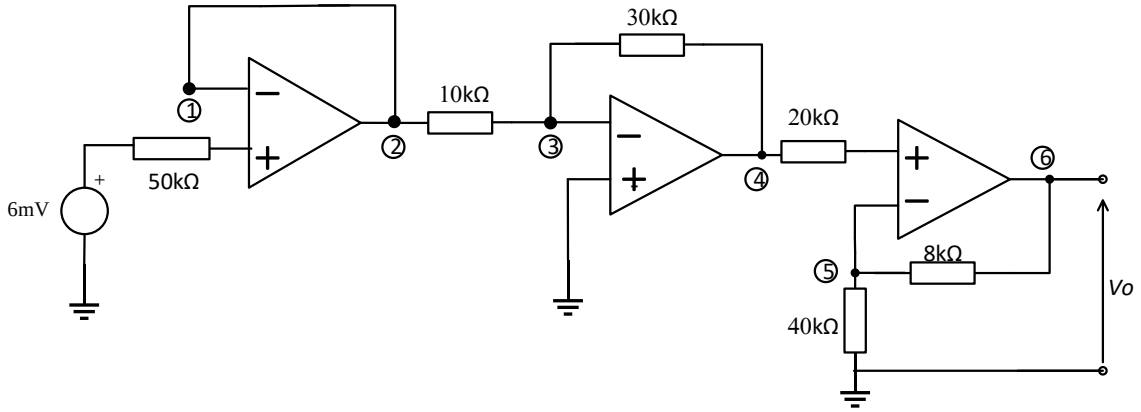
Rešavanjem sistema jednačina dobijamo:

$A = -13.6$

$B = 0.268$

$u_c(t) = -13.6e^{-0.07t} + 0.28e^{-4.95t} + 18.33$

24. Za kolo sa slike sa idealnim operacionim pojačavačima odrediti napon Vo.



$V_1 = V_2 = 6mV$

$V_3 = 0V$

$(\dfrac{1}{10} + \dfrac{1}{30})V_3 - \dfrac{1}{10}V_2 - \dfrac{1}{30}V_4 = 0$

$\Rightarrow -3V_2 - V_4 = 0$

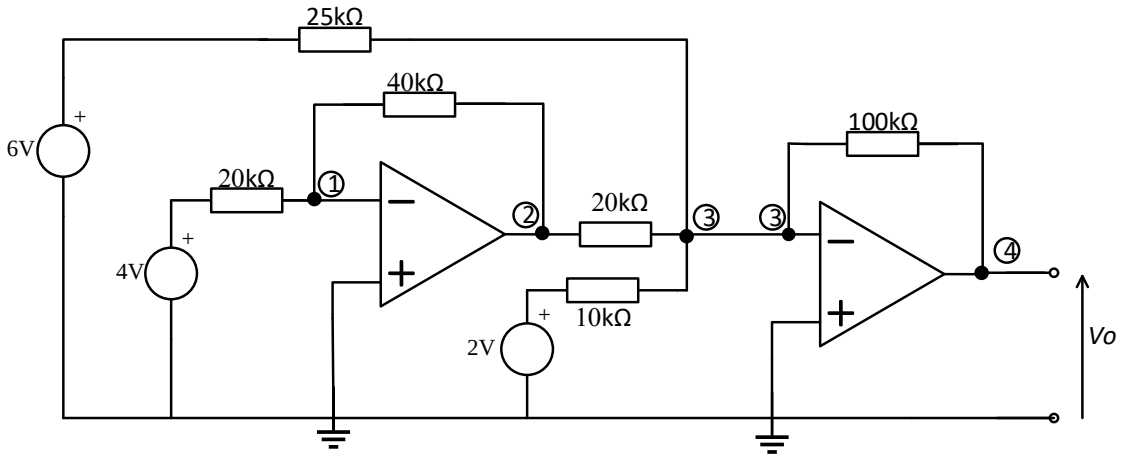
$V_4 = -3V_2 = -18mV$

$V_5 = V_4$

$(\dfrac{1}{40} + \dfrac{1}{8})V_5 - \dfrac{1}{8}V_6 = 0$

$V_o = V_6 = \dfrac{6}{5}V_5 = -21.6mV$

25. Za kolo sa slike sa idealnim operacionim pojačavačima odrediti napon Vo.



$V_1 = 0V$

$V_3 = 0V$

$\frac{4}{20K} = -\frac{V_2}{40K}$

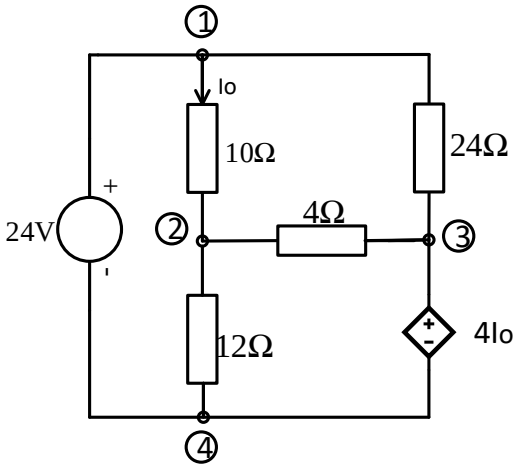
$V_2 = -8V$

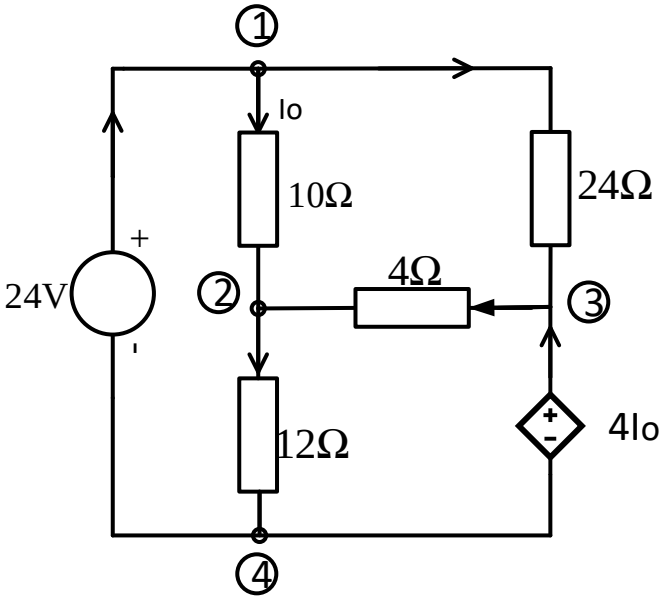
$(\frac{1}{20K} + \frac{1}{10K} + \frac{1}{25K} + \frac{1}{100K})V_3 - \frac{1}{20K}V_2 - \frac{1}{100K}V_4 = \frac{6}{25K} + \frac{2}{10K}$

$\frac{-8}{20} - \frac{1}{100}V_4 = \frac{6}{25} + \frac{2}{10}$

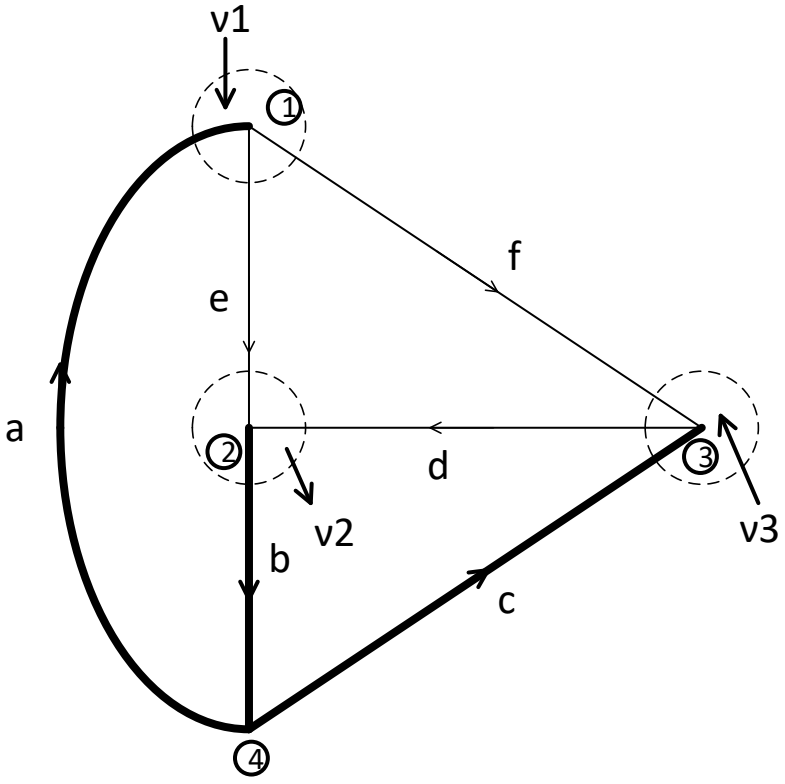
$\Rightarrow V_o = V_4 = -4V$

26. Koristeći topološke matrice, napisati jednačine prema metodu nezavisnih napona za kolo prema šemi sa slike i izračunati struju Io. Nezavisni naponi su određeni osnovnim presjecima.





Graf kola:



$$\underset{\sim}{Q_f} = \begin{array}{c} \begin{matrix} a & b & c & d & e & f \end{matrix} \\ \begin{bmatrix} 1 & 0 & 0 & 0 & -1 & -1 \\ 0 & 1 & 0 & -1 & -1 & 0 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{bmatrix} \end{array}$$

$$\underset{\sim}{U} = \underset{\sim}{Q_f^T} \cdot \underset{\sim}{u_T}$$

$$\begin{bmatrix} u_a \\ u_b \\ u_c \\ u_d \\ u_e \\ u_f \end{bmatrix} = \underset{\sim}{Q_f^T} \begin{bmatrix} u_a \\ u_b \\ u_c \end{bmatrix}$$

$$u_d = -u_b - u_c$$
$$u_e = -u_a - u_b$$
$$u_f = -u_a + u_c$$

Za kolo sa slike takođe važi:

$$u_a = -24V$$
$$u_c = -4I_o = -\frac{4}{10}u_e$$

Kada ovo uvrstimo u sistem jednačina dobijen preko topološke matrice, dobijamo:

$$u_d = -u_b + 0.4u_e$$
$$u_e = 24 - u_b$$
$$u_f = 24 - 0.4u_e$$

Kako je $u_b = 24 - u_e$, onda je:

$$u_d = -24 + u_e + 0.4u_e = -24 + 1.4u_e$$

Primjenom I Kirhohovog zakona dolazimo do jednačina:

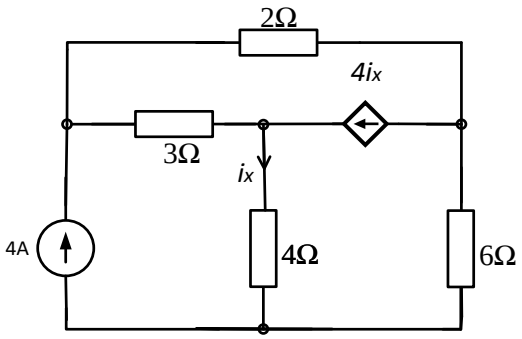
$$i_e + i_d = i_b$$
$$\Rightarrow \frac{u_e}{10} + \frac{u_d}{4} = \frac{u_b}{12}$$
$$1.2u_e + 3u_d = u_b$$

Iz prethodnih jednačina slijedi:

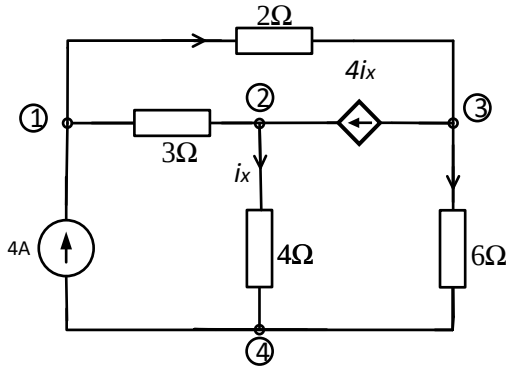
$$1.2u_e - 72 + 4.2u_e = 24 - u_e$$

Dobijamo da je $u_e = 15V$, a $I_o = \frac{u_e}{10} = 1.5A$

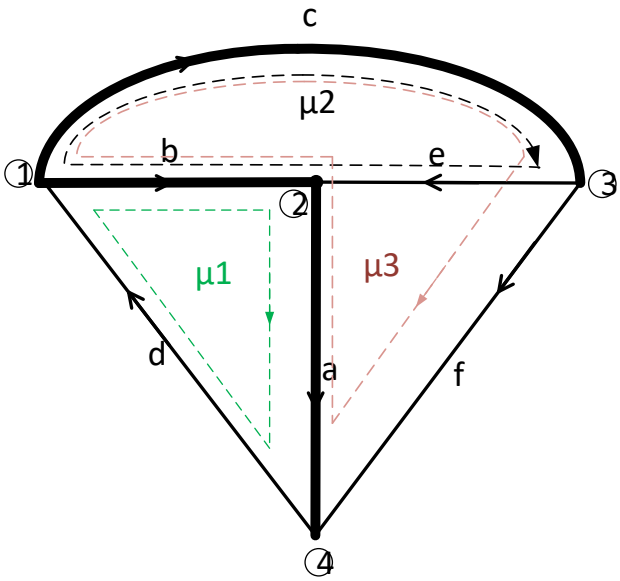
27. Koristeći topološke matrice, napisati jednačine prema metodu nezavisnih struja za kolo prema šemi sa slike i izračunati struju i_x. Nezavisne struje su određene osnovnim konturama.



Rešenje:



Odgovarajući graf:



$$\underset{\sim}{B_f} = \begin{array}{c} \begin{matrix} a & b & c & d & e & f \end{matrix} \\ \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 & 1 & 0 \\ -1 & -1 & 1 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

$$\underset{\sim}{i} = \underset{\sim}{B_f^T} \cdot \underset{\sim}{i_L}$$

$$\begin{bmatrix} i_a \\ i_b \\ i_c \\ i_d \\ i_e \\ i_f \end{bmatrix} = \underset{\sim}{B_f^T} \begin{bmatrix} i_d \\ i_e \\ i_f \end{bmatrix}$$

$$i_a = i_d - i_f$$
$$i_b = i_d - i_e - i_f$$
$$i_c = i_e + i_f$$

Za kolo sa slike takođe važi:

$$i_e = 4i_a$$
$$i_d = 4A$$

Kada ovo uvrstimo u sistem jednačina dobijen preko topološke matrice, dobijamo:

$$i_a = 4 - i_f$$
$$i_b = 4 - 4i_a - i_f$$
$$i_c = 4i_a + i_f$$

Kako je $i_f = 4 - i_a$, onda je:

$$i_b = 4 - 4i_a - 4 + i_a = -3i_a$$
$$i_c = 4i_a + 4 - i_a = 3i_a + 4$$

Primjenom II Kirhohovog zakona dolazimo do rešenja:

$$4i_a + 3i_b - 2i_c - 6i_f = 0 \Rightarrow$$

$$4i_a - 9i_a - 6i_a - 8 - 24 + 6i_a = 0$$
$$-5i_a = 32$$
$$i_a = i_x = -6.4A$$