TOPOLOŠKE MATRICE

$\underset{\sim}{B}_a = [b_{ik}]_{pxb}$ – matrica kontura gdje su
p – broj kontura
b – broj grana
c – broj čvorova
m – broj nezavisnih kontura

$m = b - n = b - c + 1$
$\underset{\sim}{B} = [b_{ik}]_{mxb}$ – matrica nezavisnih kontura
$\underset{\sim}{B} = [\underset{\sim}{B}_{T\,mxn} \mid \underset{\sim}{B}_{Lmxm}]_{mxb}$
$\underset{\sim}{B}_f = [\underset{\sim}{B}_{fT\,mxn} \mid \underset{\sim}{1}_{\,mxm}]_{mxb}$
$r\left(\underset{\sim}{B}_a\right) = r\left(\underset{\sim}{B}\right) = r\left(\underset{\sim}{B}_f\right) = m$

Matrica presjeka
$\underset{\sim}{Q}_a = \left[q_{ik}\right]_{sxb}$
$\underset{\sim}{Q} = \left[q_{ik}\right]_{nxb}$
$\underset{\sim}{Q} = \left[\underset{\sim}{Q}_T \mid \underset{\sim}{Q}_L\right]$ – matrica nezavisnih presjeka
$\underset{\sim}{Q}_f = \left[\underset{\sim}{1}_{\,nxn} \mid \underset{\sim}{Q}_{L\;nxm}\right]$ – matrica osnovnih presjeka
s – broj presjeka
n – broj nezavisnih presjeka
$r\left(\underset{\sim}{Q}_a\right) = r\left(\underset{\sim}{Q}\right) = r\left(\underset{\sim}{Q}_f\right) = n$

Matrica čvorova ili matrica incidencije
$\underset{\sim}{A}_a = \left[a_{ik}\right]_{cxb}$
$\underset{\sim}{A} = \left[a_{ik}\right]_{nxb}$
$\underset{\sim}{A} = \left[\underset{\sim}{A}_T \mid \underset{\sim}{A}_L\right]$ – matrica nezavisnih čvorova
$\underset{\sim}{A}_f = \left[\underset{\sim}{1}_{\,nxn} \mid \underset{\sim}{A}_{L\;nxm}\right]$ – matrica osnovnih čvorova
$r\left(\underset{\sim}{A}_a\right) = r\left(\underset{\sim}{A}\right) = r\left(\underset{\sim}{A}_f\right) = n$

Centralna topološka teorema ima oblik

$\underset{\sim}{Q}_a \underset{\sim}{B}_a^{\,T} = \underset{\sim}{0}$	$\underset{\sim}{Q}\underset{\sim}{B}^T = \underset{\sim}{0}$	$\underset{\sim}{Q}\underset{\sim}{B}_f^{\,T} = \underset{\sim}{0}$
$\underset{\sim}{A}_a \underset{\sim}{B}_a^{\,T} = \underset{\sim}{0}$	$\underset{\sim}{A}\underset{\sim}{B}^T = \underset{\sim}{0}$	$\underset{\sim}{Q}_f \underset{\sim}{B}^T = \underset{\sim}{0}$
$\underset{\sim}{B}_a \underset{\sim}{Q}_a^{\,T} = \underset{\sim}{0}$	$\underset{\sim}{B}\underset{\sim}{Q}^T = \underset{\sim}{0}$	$\underset{\sim}{A}\underset{\sim}{B}_f^{\,T} = \underset{\sim}{0}$
$\underset{\sim}{B}_a \underset{\sim}{A}_a^{\,T} = \underset{\sim}{0}$	$\underset{\sim}{B}\underset{\sim}{A}^T = \underset{\sim}{0}$	$\underset{\sim}{Q}_f \underset{\sim}{B}_f^{\,T} = \underset{\sim}{0}$

Kirhofovi zakoni
Naponi i struje grana mogu se zapisati

$$\underset{\sim}{u} = \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_b \end{bmatrix} \qquad \underset{\sim}{i} = \begin{bmatrix} i_1 \\ i_2 \\ \vdots \\ i_b \end{bmatrix}$$

Kirhofov zakon za struje je

$$\underset{\sim}{Q}_a \underset{\sim}{i} = \underset{\sim}{0}$$

$$\underset{\sim}{A}_a \underset{\sim}{i} = \underset{\sim}{0}$$

Kirhofov zakon za napone je

$$\underset{\sim}{B}_a \underset{\sim}{u} = \underset{\sim}{0}$$

Može se napisati

$$\underset{\sim}{i} = \underset{\sim}{B}^T \underset{\sim}{j}$$

gdje je

$\underset{\sim}{j}$ – proizvoljan sistem nezavisnih struja

takođe važi

$$\underset{\sim}{u} = \underset{\sim}{Q}^T \underset{\sim}{v} = \underset{\sim}{A}^T \underset{\sim}{v}$$

$\underset{\sim}{v}$ – proizvoljan sistem nezavisnih napona

Važne relacije su

$$\underset{\sim}{B} = \underset{\sim}{B}_L \underset{\sim}{B}_f \rightarrow B_f = \underset{\sim}{B}_L^{-1} \underset{\sim}{B}$$

$$\underset{\sim}{Q} = \underset{\sim}{Q}_T \underset{\sim}{Q}_f \rightarrow \underset{\sim}{Q}_f = \underset{\sim}{Q}_T^{-1} \underset{\sim}{Q}$$

1. Data je matrica osnovnih presjeka $\underset{\sim}{Q}_f$. Odrediti matricu osnovnih kontura $\underset{\sim}{B}_f$ za isto stablo.

Rješenje

Prema centralnoj topološkoj teoremi važi

$$\underset{\sim}{B}_f \underset{\sim}{Q}_f^T = 0$$

Ako se zna da važi

$$\underset{\sim}{B}_f = \left[\underset{\sim}{B}_{fT} \;\middle|\; \underset{\sim}{1}_{mxm} \right] \qquad \underset{\sim}{Q}_f = \left[\underset{\sim}{1}_{nxn} \;\middle|\; \underset{\sim}{Q}_{fL} \right]$$

onda je

$$\underset{\sim}{Q}_f^T = \begin{bmatrix} \underset{\sim}{1}_{nxn} \\ \hline \underset{\sim}{Q}_{fL}^T \end{bmatrix}$$

zamjenom u početnu relaciju

$$\left[\underset{\sim}{B}_{fT} \mid \underset{\sim}{1}_{mxm}\right]\left[\frac{\underset{\sim}{1}_{nxn}}{\underset{\sim}{Q}_{fL}{}^{T}}\right]=\underset{\sim}{0}$$

$$\underset{\sim}{B}_{fT}+\underset{\sim}{Q}_{fL}{}^{T}=\underset{\sim}{0}$$

$$\underset{\sim}{B}_{fT}=-\underset{\sim}{Q}_{fL}{}^{T}$$

sada je tražena matrica

$$\underset{\sim}{B}_{f}=\left[\underset{\sim}{B}_{fT} \mid \underset{\sim}{1}_{mxm}\right]=\left[-\underset{\sim}{Q}_{fL}{}^{T} \mid \underset{\sim}{1}_{mxm}\right]$$

2. Data je matrica nezavisnih kontura za stablo T, $\underset{\sim}{B}=\left[\underset{\sim}{B}_{T} \mid \underset{\sim}{B}_{L}\right]$. Odrediti matricu osnovnih presjeka $\underset{\sim}{Q}_{f}$ za isto stablo.

Rješenje

$$\underset{\sim}{Q}_{f}=\left[\underset{\sim}{1}_{nxn} \mid \underset{\sim}{Q}_{fL}\right]$$

Prema centralnoj topološkoj teoremi važi

$$\underset{\sim}{Q}_{f}\underset{\sim}{B}^{T}=\underset{\sim}{0}$$

slijedi

$$\left[\underset{\sim}{1}_{nxn} \mid \underset{\sim}{Q}_{fL}\right]\left[\frac{\underset{\sim}{B}_{T}{}^{T}}{\underset{\sim}{B}_{L}{}^{T}}\right]=\underset{\sim}{0}$$

$$\underset{\sim}{B}_{T}{}^{T}+\underset{\sim}{Q}_{fL}\underset{\sim}{B}_{L}{}^{T}=\underset{\sim}{0}$$

$$\underset{\sim}{Q}_{fL}\underset{\sim}{B}_{L}{}^{T}=-\underset{\sim}{B}_{T}{}^{T} \quad /\left(\underset{\sim}{B}_{L}{}^{T}\right)^{-1}$$

$$\underset{\sim}{Q}_{fL}=-\underset{\sim}{B}_{T}{}^{T}\left(\underset{\sim}{B}_{L}{}^{T}\right)^{-1}=-\underset{\sim}{B}_{T}{}^{T}\left(\underset{\sim}{B}_{L}{}^{-1}\right)^{T}=-\left(\underset{\sim}{B}_{L}{}^{-1}\underset{\sim}{B}_{T}\right)^{T}$$

sada je tražena matrica

$$\underset{\sim}{Q}_{f}=\left[\underset{\sim}{1}_{nxn} \mid -\left(\underset{\sim}{B}_{L}{}^{-1}\underset{\sim}{B}_{T}\right)^{T}\right]$$

$\underset{\sim}{B}_{L}$ je kvadratna nesingularna matrica pa postoji inverzna matrica.

3. Data je matrica nezavisnih presjeka $\underset{\sim}{Q}$ za izabrano stablo. Potrebno je odrediti matricu nezavisnih kontura $\underset{\sim}{B}$ za isto stablo.

Rješenje

$$\underset{\sim}{Q}=\left[\underset{\sim}{Q}_{T} \mid \underset{\sim}{Q}_{L}\right] \qquad \underset{\sim}{B}=\left[\underset{\sim}{B}_{T} \mid \underset{\sim}{B}_{L}\right]$$

Prema centralnoj topološkoj teoremi

$$\underset{\sim}{B}\underset{\sim}{Q}^T = \underset{\sim}{0}$$

$$\left[\underset{\sim}{B}_T \mid \underset{\sim}{B}_L\right]\left[\begin{array}{c}\underset{\sim}{Q}_T{}^T \\ \hline \underset{\sim}{Q}_L{}^T\end{array}\right] = \underset{\sim}{0}$$

$$\underset{\sim}{B}_T\underset{\sim}{Q}_T{}^T + \underset{\sim}{B}_L\underset{\sim}{Q}_L{}^T = \underset{\sim}{0}$$

$$\underset{\sim}{B}_T\underset{\sim}{Q}_T{}^T = -\underset{\sim}{B}_L\underset{\sim}{Q}_L{}^T \quad / \left(\underset{\sim}{Q}_T{}^T\right)^{-1}$$

$$\underset{\sim}{B}_T = -\underset{\sim}{B}_L\underset{\sim}{Q}_L{}^T\left(\underset{\sim}{Q}_T{}^T\right)^{-1} = -\underset{\sim}{B}_L\left(\underset{\sim}{Q}_T{}^{-1}\underset{\sim}{Q}_L\right)^T$$

sada je

$$\underset{\sim}{B} = \left[\underset{\sim}{B}_T \mid \underset{\sim}{B}_L\right] = \underset{\sim}{B}_L\underbrace{\left[-\left(\underset{\sim}{Q}_T{}^{-1}\underset{\sim}{Q}_L\right)^T \;\middle|\; \underset{\sim}{1}_{mxm}\right]}_{\underset{\sim}{B}_f} = \underset{\sim}{B}_L\underset{\sim}{B}_f$$

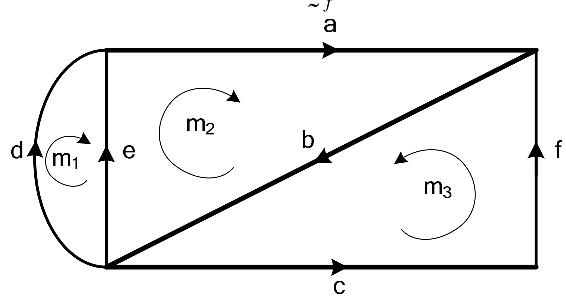
Slijedi jedna značajna relacija

$$\underset{\sim}{B} = \underset{\sim}{B}_L\underset{\sim}{B}_f \rightarrow B_f = \underset{\sim}{B}_L{}^{-1}\underset{\sim}{B}$$

a analogno važi i

$$\underset{\sim}{Q} = \underset{\sim}{Q}_T\underset{\sim}{Q}_f \rightarrow \underset{\sim}{Q}_f = \underset{\sim}{Q}_T{}^{-1}\underset{\sim}{Q}$$

4. Dat je graf kola kao na slici, i matrica nezavisnih kontura $\underset{\sim}{B}$. Za izabrano stablo T={a,b,c} odrediti matricu osnovnih kontura $\underset{\sim}{B}_f$.



$$\underset{\sim}{B} = \begin{array}{c} \\ \mu_1 \\ \mu_2 \\ \mu_3 \end{array}\!\!\begin{array}{c} \begin{array}{cccccc} a & b & c & d & e & f \end{array} \\ \left[\begin{array}{ccc:ccc} 0 & 0 & 0 & 1 & -1 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{array}\right] \end{array} = \left[\underset{\sim}{B}_T \mid \underset{\sim}{B}_L\right]$$

Rješenje

Može se napisati

$$\underset{\sim}{B} = \left[\underset{\sim}{B}_T \mid \underset{\sim}{B}_L\right] = \underset{\sim}{B}_L\underbrace{\left[\underset{\sim}{B}_L{}^{-1}\underset{\sim}{B}_T \mid \underset{\sim}{1}_{mxm}\right]}_{\underset{\sim}{B}_f} = \underset{\sim}{B}_L\underset{\sim}{B}_f$$

Iz zadatka važi

$$\underset{\sim}{B}_L = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \rightarrow \quad \underset{\sim}{B}_L^{-1} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

pa je

$$\underset{\sim}{B}_L^{-1}\underset{\sim}{B}_T = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

dobija se

$$\underset{\sim}{B}_f = \begin{array}{rl} & \begin{array}{ccc:ccc} a & b & c & d & e & f \end{array} \\ \begin{array}{r} \mu_1 \\ \mu_2 \\ \mu_3 \end{array} & \left[\begin{array}{ccc:ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{array}\right] \end{array}$$

5. Data je matrica osnovnih kontura iz prethodnog zadatka za stablo T_1={a,b,c}. Odrediti matricu $\underset{\sim}{B}_f$ za stablo T_2={a,b,f}.

$$\underset{\sim}{B}_f = \begin{array}{rl} & \begin{array}{ccc:ccc} a & b & c & d & e & f \end{array} \\ \begin{array}{r} \mu_1 \\ \mu_2 \\ \mu_3 \end{array} & \left[\begin{array}{ccc:ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{array}\right] \end{array}$$

<u>Rješenje</u>

Za novo stablo, potrebno je promjeniti red kolona matrice i dobija se

$$\underset{\sim}{B} = \begin{array}{rl} & \begin{array}{ccc:ccc} a & b & f & c & d & e \end{array} \\ \begin{array}{r} \mu_1 \\ \mu_2 \\ \mu_3 \end{array} & \left[\begin{array}{ccc:ccc} 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 & 0 \end{array}\right] \end{array} = [\underset{\sim}{B}_{T2} \;\vdots\; \underset{\sim}{B}_{L2}]$$

slijedi

$$\underset{\sim}{B}_2 = [\underset{\sim}{B}_{T2} \;\vdots\; \underset{\sim}{B}_{L2}] = \underset{\sim}{B}_{L2}\underbrace{\left[\underset{\sim}{B}_{L2}^{-1}\underset{\sim}{B}_{T2} \;\vdots\; \underset{\sim}{1}_{mxm}\right]}_{\underset{\sim}{B}_{f2}} = \underset{\sim}{B}_{L2}\underset{\sim}{B}_{f2} \rightarrow \underset{\sim}{B}_{f2} = \left[\underset{\sim}{B}_{L2}^{-1}\underset{\sim}{B}_{T2} \;\vdots\; \underset{\sim}{1}_{mxm}\right]$$

ostatak je analogan prethodnom zadatku.

6. Zadata je matrica nezavisnih čvorova A jednog orjentisanog grafa električnog kola:

$$\underset{\sim}{A} = \begin{array}{rl} & \begin{array}{cccccc} a & b & c & d & e & f \end{array} \\ \begin{array}{r} 1 \\ 2 \\ 3 \end{array} & \left[\begin{array}{cccccc} 1 & -1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & -1 & 0 & 0 \end{array}\right] \end{array}$$

Za stablo T={b,c,e} formirati matricu osnovnih kontura $\underset{\sim}{B}_f$ i matricu osnovnih presjeka $\underset{\sim}{Q}_f$ i nacrtati graf.

Rješenje

Ako se preuredi matrica A tako da se prvo napišu kolone koje odgovaraju granama stable pa onda kolone koje odgovaraju granama kostabla

$$\underset{\sim}{A} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{cccccc} b & c & e & a & d & f \end{array} \\ \begin{bmatrix} -1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & -1 & 1 & 1 \\ 0 & 1 & 0 & 0 & -1 & 0 \end{bmatrix} \end{array} = \left[\underset{\sim}{A}_T \;\vdots\; \underset{\sim}{A}_L \right]$$

Prema centralnoj topološkoj teoremi

$$\underset{\sim}{B}_f \underset{\sim}{A}^T = \underset{\sim}{0} \qquad\qquad \underset{\sim}{B}_f = \left[\underset{\sim}{B}_{fT} \;\vdots\; \underset{\sim}{1}_{mxm} \right]$$

slijedi

$$\left[\underset{\sim}{B}_{fT} \;\vdots\; \underset{\sim}{1}_{mxm} \right] \left[\begin{array}{c} \underset{\sim}{A}_T{}^T \\ \hline \underset{\sim}{A}_L{}^T \end{array} \right] = \underset{\sim}{0}$$

$$\underset{\sim}{B}_{fT} \underset{\sim}{A}_T{}^T + \underset{\sim}{A}_L{}^T = \underset{\sim}{0}$$

$$\underset{\sim}{B}_{fT} \underset{\sim}{A}_T{}^T = -\underset{\sim}{A}_L{}^T \;\; / \left(\underset{\sim}{A}_T{}^T \right)^{-1}$$

$$\underset{\sim}{B}_{fT} = -\underset{\sim}{A}_L{}^T \left(\underset{\sim}{A}_T{}^T \right)^{-1} = -\left(\underset{\sim}{A}_T{}^{-1} \underset{\sim}{A}_L \right)^T \rightarrow \underset{\sim}{B}_f = \left[-\left(\underset{\sim}{A}_T{}^{-1} \underset{\sim}{A}_L \right)^T \;\vdots\; \underset{\sim}{1}_{mxm} \right]$$

Prema zadatim podacima

$$\underset{\sim}{A}_T{}^{-1} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix}$$

$$\underset{\sim}{B}_f = \begin{array}{c} \\ \mu_1 \\ \mu_2 \\ \mu_3 \end{array} \begin{array}{c} \begin{array}{ccc|ccc} b & c & e & a & d & f \end{array} \\ \left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array} \right] \end{array}$$

$$\underset{\sim}{Q}_f = \left[\underset{\sim}{1}_{nxn} \;\vdots\; \underset{\sim}{Q}_{fL} \right]$$

iz prvog zadatka važi

$$\underset{\sim}{Q}_{fL} = -\underset{\sim}{B}_{fT}{}^T = \underset{\sim}{A}_T{}^{-1} \underset{\sim}{A}_L = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 1 & -1 & -1 \end{bmatrix}$$

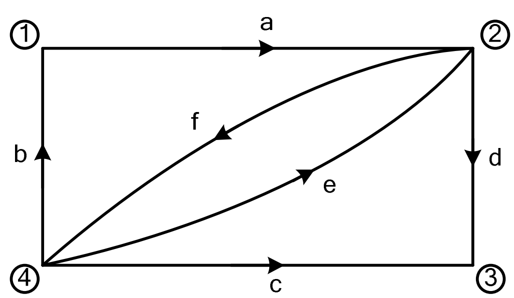
pa je tražena matrica osnovnih presjeka

$$
\begin{array}{c c}
 & \begin{array}{cccccc} b & c & e & a & d & f \end{array} \\
\underset{\sim}{Q}_f = \begin{array}{c} v_1 \\ v_2 \\ v_3 \end{array} & \left[\begin{array}{ccc:ccc} 1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 1 & -1 & -1 \end{array}\right]
\end{array}
$$

Potrebno je sada formirati graf kola. Poznato je da matrica A_a jednoznačno definiše graf. Na osnovu poznate matrice A formira se matrica A_a. Takođe je poznato da se matrica nezavisnih čvorova dobija iz matrice čvorova A_a tako što se ukloni vrsta koja odgovara odabranom referentnom čvoru. Dakle, potrebno je dodati tu nedostajuću vrstu već poznatoj matrici A kako bi se dobila tražena matrica A_a. Kako bi se odredili elementi nedostajuće vrste koristiće se osobina matrice A_a da u svakoj koloni ima dva elementa različita od 0 (1 i -1 jer svaka grana ima dva kraja koja pripadaju posebnim čvorovima), tj. da je suma svih elemenata matrice po koloni jednaka 0. Tada je

$$
\begin{array}{c c}
 & \begin{array}{cccccc} a & b & c & d & e & f \end{array} \\
\underset{\sim}{A}_a = \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 & 1 & -1 \end{bmatrix}
\end{array}
$$

pa je traženi graf



7. Zadata je matrica osnovnih presjeka grafa električnog kola.

$$
\begin{array}{c c}
 & \begin{array}{ccccccc} a & b & c & d & e & f & g \end{array} \\
\underline{Q_f} = \begin{array}{c} v_1 \\ v_2 \\ v_3 \\ v_4 \end{array} & \begin{bmatrix} -1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 \end{bmatrix}
\end{array}
$$

a) Odrediti koje grane grafa obrazuju stablo grafa T.
b) Koristeći centralnu topološku teoremu odrediti matricu osnovnih kontura $\underline{B_f}$.
c) Nacrtati odgovarajući graf.
d) Provjeriti KZ za struje za osnovni presjek.

Rješenje

a) Zadata matrica se preuredi tako da je prva submatrica jedinična matrica reda 4x4:

$$
\underline{Q}_f =
\begin{array}{c}
 \\ \nu_1 \\ \nu_2 \\ \nu_3 \\ \nu_4 \\ \\
\end{array}
\begin{array}{c}
\begin{array}{ccccccc} g & b & c & e & a & d & f \end{array} \\
\left[\begin{array}{cccc:ccc}
1 & 0 & 0 & 0 & -1 & 0 & 0 \\
0 & 1 & 0 & 0 & 1 & 0 & 1 \\
0 & 0 & 1 & 0 & 0 & 1 & 1 \\
0 & 0 & 0 & 1 & 0 & 1 & 0
\end{array}\right] \\
\begin{array}{cc} \underline{1_{nn}} & \underline{Q_{fl}} \end{array}
\end{array}
$$

Stablo obrazuju grane $T=\{g, b, c, e\}$, a kostablo $L=\{a, d, f\}$, $m=3$.

b)

$$\underline{B}_f \underline{Q}_f^{\;T} = \underline{0}$$

$$\underline{B}_f = \left[\underline{B}_{fT} \vdots 1_{mm}\right]$$

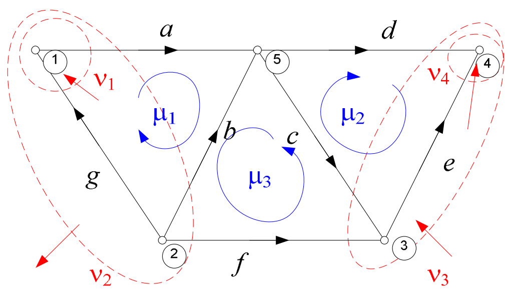
$$\underline{B}_{fT} = -\underline{Q}_{fL}^{\;T} = -\begin{bmatrix} -1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix}^T$$

$$
\underline{B}_f =
\begin{array}{c}
 \\ \mu_1 \\ \mu_2 \\ \mu_3 \\ \\
\end{array}
\begin{array}{c}
\begin{array}{ccccccc} g & b & c & e & a & d & f \end{array} \\
\left[\begin{array}{cccc|ccc}
1 & -1 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & -1 & -1 & 0 & 1 & 0 \\
0 & -1 & -1 & 0 & 0 & 0 & 1
\end{array}\right] \\
\begin{array}{cc} \underline{\mathrm{B}_{fT}} & \underline{1_{mm}} \end{array}
\end{array}
$$

c) $\mathrm{T}=\{g, b, c, e\}$, $\mathrm{L}=\{a, d, f\}$

μ_1, μ_2, μ_3 – fundamentalne konture

$\nu_1, \nu_2, \nu_3, \nu_4$ – fundamentalni presjeci.



d) Nezavisne jednačine na osnovu IKZ:

$$\underline{Q}_f \underline{i} = \underline{0}$$

$$\underline{i} = \begin{bmatrix} i_g \\ i_b \\ i_c \\ i_e \\ \dots \\ i_a \\ i_d \\ i_f \end{bmatrix} = \begin{bmatrix} i_T \\ \dots \\ i_L \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} i_g \\ i_b \\ i_c \\ i_e \\ i_a \\ i_d \\ i_f \end{bmatrix} = 0$$

Odakle se dobija set jednačina:

$i_g - i_a = 0$

$i_b + i_a + i_f = 0$

$i_c + i_d + i_f = 0$

$i_c + i_e = 0$

8. Ako je $\underline{i}$ matrica kolone struja grana grafa, $\underline{i_L}=[i_d\ i_e\ i_f]^T$ matrica kololone struja spojnica, a matrica kolone struja grafa stabla $\underline{i_T} = \begin{bmatrix} -i_d + i_f \\ i_d - i_f \\ i_d - i_e \end{bmatrix}$, odrediti matricu osnovnih kontura $\underline{B_f}$ i nacrtati odgovarajući graf.

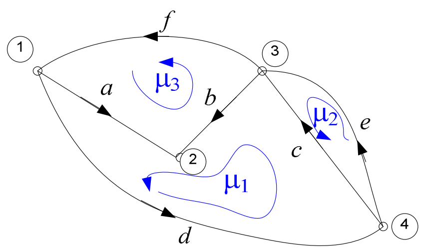
Rješenje

$$\underline{i} = \begin{bmatrix} \underline{i_T} \\ \underline{i_L} \end{bmatrix} = \begin{bmatrix} i_a \\ i_b \\ i_c \\ i_d \\ i_e \\ i_f \end{bmatrix} = \begin{bmatrix} -i_d + i_f \\ i_d - i_f \\ i_d - i_e \\ i_d \\ i_e \\ i_f \end{bmatrix} = \begin{bmatrix} -1 & 0 & 1 \\ 1 & 0 & -1 \\ 1 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} i_d \\ i_e \\ i_f \end{bmatrix} = \underline{B_f}^T \underline{i_L}$$

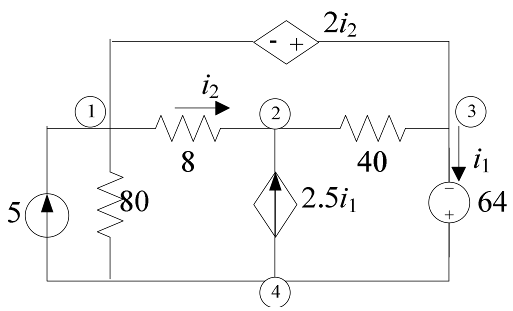
$\underline{B_f} = \left[\underline{B_{fT}} \vdots 1_{mm} \right]$

$$\underline{B_f} = \begin{array}{c} \\ \mu_1 \\ \mu_2 \\ \mu_3 \end{array} \begin{array}{c} \begin{array}{cccccccc} a & b & c & & d & e & f \end{array} \\ \begin{bmatrix} -1 & 1 & 1 & \vdots & 1 & 0 & 0 \\ 0 & 0 & -1 & \vdots & 0 & 1 & 0 \\ 1 & -1 & 0 & \vdots & 0 & 0 & 1 \end{bmatrix} \end{array}$$

Graf ima 6 grana sa m=3 osnovne konture i n=3 osnovna presjeka, c=4 čvora.



9. Koristeći topološke matrice, metodom nezavisnih napona (nezavisni naponi su određeni osnovnim presjecima) odrediti struje i_1 i i_2. U kom režimu rade pojedini izvori u kolu?



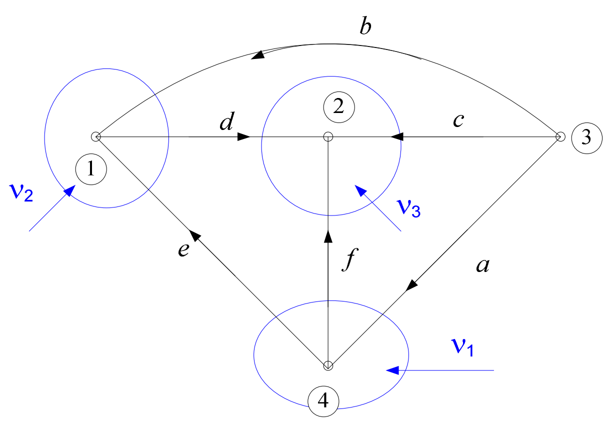
Rješenje

$c = 4$

$b = 6$

$n = c - 1 = 3$ - 3 grane stabla

$m = b - n = 3$ - 3 grane kostabla



T={a, b, c}, L={d, e, f}.

Naponi grana stabla čine nezavisan sistem napona, struje spojnica čine nezavisan sistem struja, pa zato za grane stabla biramo grane sa nezavisnim i zavisnim naponskim izvorima, a do punog broja grana dopunjavamo pasivnim granama.

$\underline{U} = \underline{Q_f}^T \underline{U_T}$

$$\underline{Q_f} = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ \\ \end{array} \begin{array}{ccccccc} a & b & c & & d & e & f \\ \end{array}$$

$$\underline{Q_f} = \begin{array}{c} v_1 \\ v_2 \\ v_3 \end{array}\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & -1 & -1 \\ 0 & 1 & 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{array}\right] \qquad \underline{Q_{fT}} \quad \underline{Q_{fL}}$$

$$U = \begin{bmatrix} u_a \\ u_b \\ u_c \\ \cdots \\ u_d \\ u_e \\ u_f \end{bmatrix} = \begin{bmatrix} U_T \\ \cdots \\ U_L \end{bmatrix}$$

$$\begin{bmatrix} u_a \\ u_b \\ u_c \\ u_d \\ u_e \\ u_f \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 1 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_a \\ u_b \\ u_c \end{bmatrix}, \qquad \begin{aligned} u_d &= -u_b + u_c \\ u_e &= -u_a + u_b \\ u_f &= -u_a + u_c \end{aligned}$$

(naponi spojnica izraženi preko nezavisnih napona, tj. napona stabla)

$u_a = -64V$

$u_b = 2i_2$

$i_2 = \frac{u_d}{8} \Rightarrow u_b = 2\frac{u_d}{8} = \frac{u_d}{4}$

$u_d = -\frac{u_d}{4} + u_c \Rightarrow u_c = \frac{5}{4}u_d$

$u_e = 64 + \frac{u_d}{4}$

$u_f = 64 + \frac{5}{4}u_d$

Da bi se odredio napon u_d, potrebo je još napisati IKZ za neki od čvorova (ν_3):

$i_d + 2.5i_1 + i_c = 0 \Rightarrow \frac{u_d}{8} + 2.5i_1 + \frac{u_c}{40} = 0$

$\frac{u_d}{8} + 2.5i_1 + \frac{1}{40}\frac{5}{4}u_d = 0 \Rightarrow u_d = -16i_1$

IKZ za čvor 4 (ν_1):

$$i_1 = 2.5i_1 + \frac{u_e}{80} + 5 \Rightarrow -1.5i_1 = \frac{1}{80}\left(64 + \frac{u_d}{4}\right) + 5$$

$$-\frac{3}{2}i_1 = \frac{4}{5} - \frac{i_1}{20} + 5$$

$$\frac{29}{20}i_1 + \frac{29}{5} = 0 \Rightarrow i_1 = -4A$$

Odakle se dobijaju:

$u_d = 64V$

$i_2 = 8A$

$u_b = 16V$

$u_c = 80V$

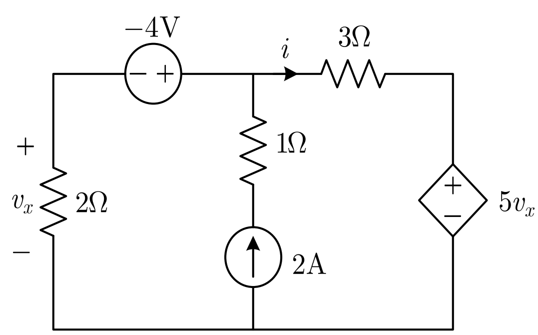
$u_e = 80V$

$u_f = 144V$

Da bi se odredio režim rada, potrebno je odrediti snage:

- strujni izvor 5A: $-U_eI$=(-80)5=-400W<0 – potrošač
- naponski izvor 64V: Ui_1=64(-4)=--256W<0 – potrošač
- SKNI: $2i_2(-i_b)$; $i_b + \frac{u_c}{40} + i_1 = 0 \Rightarrow i_b = -2 + 4 = 2A$, pa je $2i_2(-i_b)$=-32W<0 – potrošač
- SKSI: $2.5i_1(-u_f)$=(-10)(-144)=1440W>0 – izvor

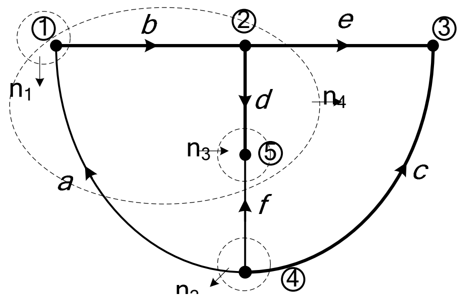
10. U kolu datom na slici odrediti napon v_x i struju i
    a) Primjenom metoda nezavisnih napona
    b) Primjenom metoda nezavisnih konturnih struja



Rješenje

Na početku potrebno je formirati graf kola i označiti stablo grafa.

a) Kako se zadatkom traži upotreba metoda nezavisnih napona, potrebno je označiti nezavisne presjeke prema prethodno definisanom stablu



Sa slike se vidi da je stablo grafa T={b,c,d,e} a kostablo L={a,f}, pa je matrica osnovnih presjeka:

$$\underset{\sim}{Q}_f = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \end{array} \begin{array}{c} \begin{array}{cccc:cc} b & c & d & e & a & f \end{array} \\ \left[\begin{array}{cccc:cc} 1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & -1 & -1 \end{array}\right] \end{array} \tag{1}$$

Naponi grana su prema definiciji

$$\underset{\sim}{U} = \begin{bmatrix} U_b \\ U_c \\ U_d \\ U_e \\ \hdashline U_a \\ U_f \end{bmatrix} \qquad \underset{\sim}{U} = \underset{\sim}{Q}_f{}^T \underset{\sim}{U}_T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 1 & 0 & -1 \\ 0 & 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} U_b \\ U_c \\ U_d \\ U_e \end{bmatrix} = \begin{bmatrix} U_b \\ U_c \\ U_d \\ U_e \\ \hdashline -U_b + U_c - U_e \\ U_c + U_d - U_e \end{bmatrix} \tag{2}$$

dakle, naponi grana kostabla su

$$U_a = -U_b + U_c - U_e \tag{3}$$

$$U_f = U_c + U_d - U_e \tag{4}$$

posmatrajući graf i zadate parametre kola može se pisati

$$U_b = -(-4\,\mathrm{V}) = 4\,\mathrm{V} \tag{5}$$

$$\mathrm{U_c} = -5v_x \tag{6}$$

$$U_a = -v_x = -2i_a \tag{7}$$

zamjenom relacija (5) i (6) u relaciju (3)

$$U_a = -4 + U_c - U_e \tag{8}$$

a zamjenom relacija (6) i (7) u relaciju 1.4

$$U_a = -4 + 5U_a - U_e \rightarrow U_e = 4U_a - 4 \tag{9}$$

ukoliko posmatramo KZS za čvor 2

$$i_b = i_d + i_e \tag{10}$$

ako se zna da važi

$$i_b = i_a = \frac{1}{2}U_a \qquad i_d = -2\,\mathrm{A} \qquad i_e = \frac{1}{3}U_e \tag{11}$$

zamjenom relacija (11) u relaciju (10) dobija se

$$\frac{1}{2}U_a = -2 + \frac{1}{3}U_e \tag{12}$$

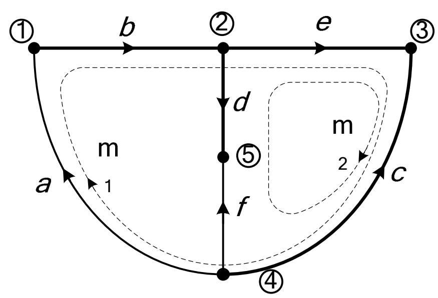
Sada relacije (9) i (12) čine sistem dvije jednačine sa dvije nepoznate, pa su rješenja

$$U_a = 4\,\mathrm{V} \qquad U_e = 12\,\mathrm{V}$$

Sada su traženi napon i struja

$$v_x = -U_a = -4\,\mathrm{V} \qquad i = i_e = \frac{1}{3}U_e = 4\,\mathrm{A}$$

b) Ukoliko se primjeni metod nezavisnih kontura, potrebno je označiti nezavisne konture na grafu



Sa slike se vidi da je stablo grafa T={b,c,d,e} a kostablo L={a,f}, pa je matrica osnovnih kontura:

$$\underset{\sim}{B}_f = \begin{array}{c} \\ \mu_1 \\ \mu_2 \end{array} \begin{array}{c} \begin{array}{cccc:cc} b & c & d & e & a & f \end{array} \\ \left[\begin{array}{cccc:cc} 1 & -1 & 0 & 1 & 1 & 0 \\ 0 & -1 & -1 & 1 & 0 & 1 \end{array}\right] \end{array}$$

a prema definiciji metoda, struje grana grafa se dobijaju prema relaciji

$$\underset{\sim}{i} = \underset{\sim}{B}_f{}^T \underset{\sim}{i}_L$$

sada važi

$$\begin{bmatrix} i_b \\ i_c \\ i_d \\ i_e \\ i_a \\ i_f \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -1 & -1 \\ 0 & -1 \\ 1 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} i_a \\ i_f \end{bmatrix} = \begin{bmatrix} i_a \\ -i_a - i_f \\ -i_f \\ i_a + i_f \\ i_a \\ i_f \end{bmatrix}$$

dakle, dobija se

$$\begin{aligned} i_b &= i_a \\ i_c &= -i_a - i_f \\ i_d &= -i_f \\ i_e &= i_a + i_f \end{aligned} \tag{13}$$

posmatrajući šemu vidi se da važi:

$$i_f = 2\,\text{A} \tag{14}$$

pa je

$$\begin{aligned} i_b &= i_a \\ i_c &= -i_a - 2 \\ i_d &= -2 \\ i_e &= i_a + 2 \end{aligned} \tag{15}$$

takođe, sa šeme se vidi

$$\begin{aligned} U_a &= -v_x = 2i_a \\ U_c &= -5v_x = 5U_a \\ U_b &= -(-4) = 4\,\text{V} \end{aligned} \tag{16}$$

ako se napišu jednačine za KZN za konturu μ_1

$$U_a + U_b + U_e - U_c = 0 \tag{17}$$

ako se relacije (16) zamjene u relacije (17) dobija se

$$2i_a + 4 + U_e - 5U_a = 0 \tag{18}$$

a za napon grane e važi

$$U_e = 3i_e$$

tada je

$$2i_a + 4 + 3i_e - 10i_a = 0$$

$$3i_e - 8i_a = -4 \tag{19}$$

Sada četvrta jednačina sistema (15) i jednačina (19) čine sistem sa dvije nepoznate čija su rješenja

$$3(i_a + 2) - 8i_a = -4 \to -5i_a = -10 \to i_a = 2\text{A} \to i_e = 4\text{A}$$

gdje je i_e tražena struja, a nepoznati napon v_x je prema prvoj relaciji sistema (16)

$$v_x = -U_a = -2i_a = -4\,\text{V}$$