University of Montenegro

Faculty of Electrical Engineering

SINGLE ITERATION ALGORITHM HARDWARE IMPLEMENTATION

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ALGORITHM STEPS

$$x(n)=\sum_{i=1}^{K}A_i e^{j\frac{2\pi k_i n}{N}}$$

- Signal definition

$$\mathrm{var}=M\,\frac{N-M}{N-1}\,(A_1^2+A_2^2+\ldots+A_K^2)$$

- Missing samples noise variance
- N-signal length
- M-number of available samples
- K-number of signal components

$$T=\frac{1}{N}\left(-\mathrm{var}^2\log(1-\sqrt[N]{P})\right)^{1/2}$$

- Threshold

$$V(f)=\sum_{m=1}^{M}v(m)e^{-j\frac{2\pi fm}{N}},\ f=1,\ldots,N$$

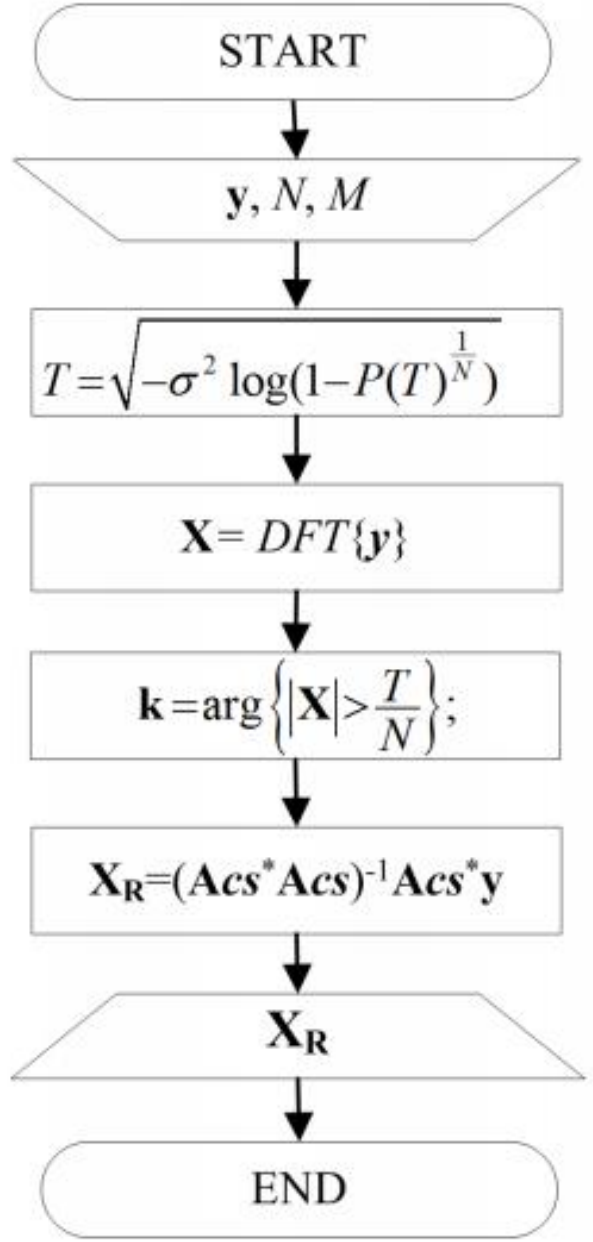
-Vector of initial DFT

$v(m)\rightarrow$ Vector of available signal samples

$$pos=\arg\left\{|V|>T\right\}$$

- Finding the positions above the threshold

ALGORITHM



START
y, N, M
$T = \sqrt{-\sigma^2 \log(1 - P(T)^{\frac{1}{N}})}$
$\mathbf{X} = DFT\{\mathbf{y}\}$
$\mathbf{k} = \arg\left\{|\mathbf{X}| > \frac{T}{N}\right\};$
$\mathbf{X_R} = (\mathbf{A}cs^*\mathbf{A}cs)^{-1}\mathbf{A}cs^*\mathbf{y}$
$\mathbf{X_R}$
END

ALGORITHM STEPS

Forming CS matrix :

$A_{N\times N}$ -DFT matrix

-**Rows**-positions of random measurements

$A_{CS_{M\times K}}$ -CS matrix

-**Columns**-positions of the V with amplitudes (absolute values) above the threshold

The optimization problem:

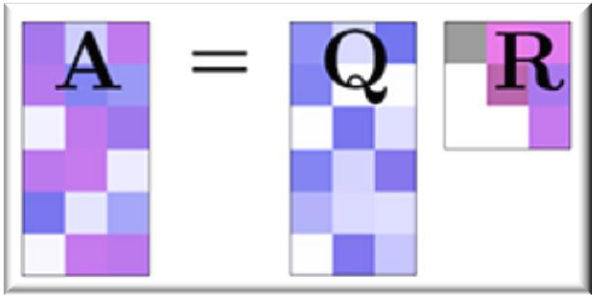
Hard to implement in hardware

$$X = (A_{CS}^{*} A_{CS})^{-1}(A_{CS}^{*} v)$$

QR decomposition → $A_{CS} = Q_{CS} R_{CS}$

QR Decomposition methods

- There are several methods for QR decomposition calculation:
  - Gram-Schmidt decomposition;
  - Householder transformation and
  - **Givens rotations**



- QR based on Givens rotations can be parallelized and has a low computational complexity

$$A = QR,$$

$$R = Q^T A,$$

$$Q^T Q = I.$$

- A-real or complex matrix; square or rectangular matrix
- Q-an orthogonal matrix; R - right triangular matrix

QR Decomposition

- Givens rotation matrix: $$G=\begin{bmatrix} c & s \\ -s & c \end{bmatrix}$$
  - $c^2 + s^2 = 1$, $c = \cos(\theta)$ i $s = \sin(\theta)$

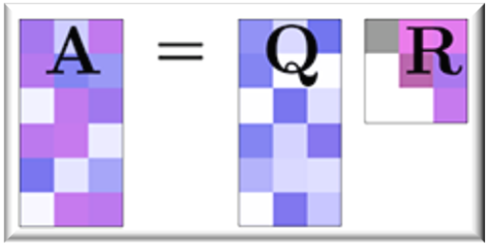
$$\begin{bmatrix} c & s \\ -s & c \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix}=\begin{bmatrix} \sqrt{a^2+b^2} \\ 0 \end{bmatrix}, \qquad c=\frac{a}{\sqrt{a^2+b^2}}, \qquad s=\frac{b}{\sqrt{a^2+b^2}}$$

  - General form of Givens rotation matrix:

$$G(i,j,\theta)=\begin{bmatrix} 1 & \cdots & 0 & \cdots & 0 & \cdots & 0 \\ \vdots & \ddots & \vdots & \cdots & \vdots & \cdots & \vdots \\ 0 & \cdots & C & \cdots & S & \cdots & 0 \\ \vdots & \cdots & \vdots & \ddots & \vdots & \cdots & \vdots \\ 0 & \cdots & -S & \cdots & C & \cdots & 0 \\ \vdots & \cdots & \vdots & \cdots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & \cdots & 0 & \cdots & 1 \end{bmatrix}\begin{matrix} \\ \\ i \\ \\ j \\ \\ \\ \end{matrix}$$

$$\qquad\qquad\qquad i \qquad\qquad j$$

Algorithm steps

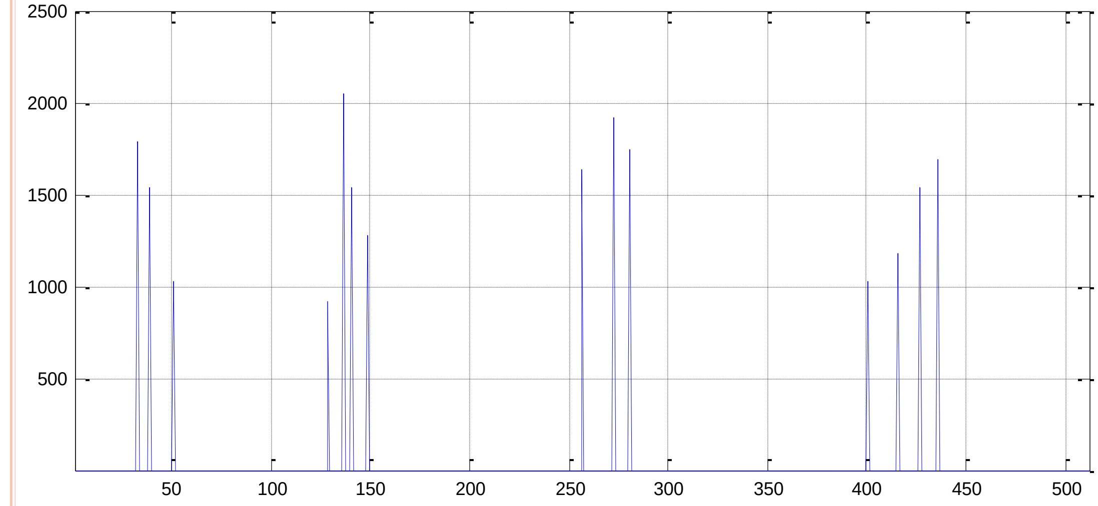


QR decomposition → $A_{CS} = Q_{CS} R_{CS}$

$$X = (A_{CS}^{*} A_{CS})^{-1} (A_{CS}^{*} v) \rightarrow X = \left((Q_{CS} R_{CS})^{*} (Q_{CS} R_{CS}) \right)^{-1} (A_{CS}^{*} v)$$

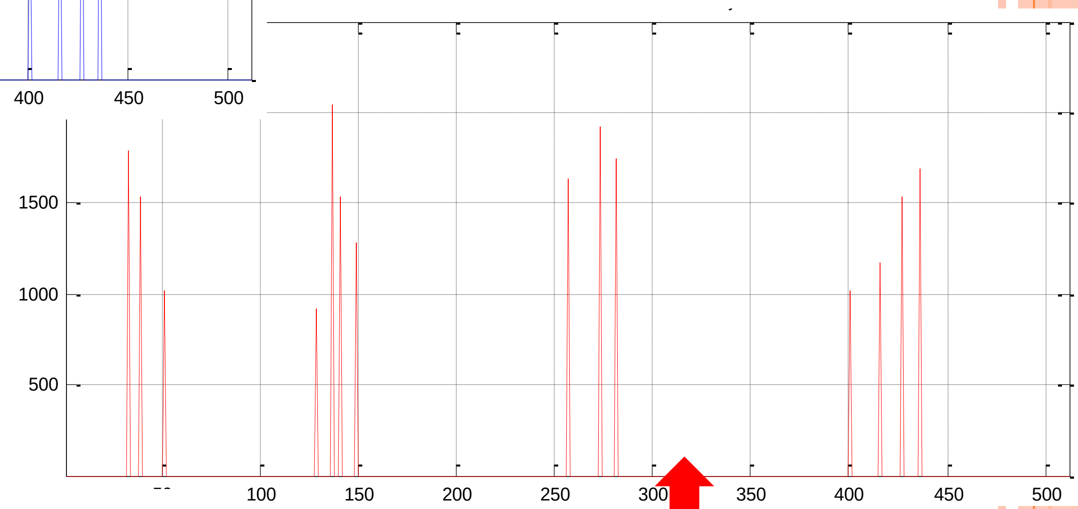
$$X = \left(R_{CS}{}^{*} Q_{CS}{}^{*} Q_{CS} R_{CS} \right)^{-1} (A_{CS}^{*} v) = (R_{CS}^{*} R_{CS})^{-1} (A_{CS}^{*} v)$$

$$X = R_{CS}^{-1} (R_{CS}{}^{-1})^{*} \cdot A_{CS}^{*} v$$

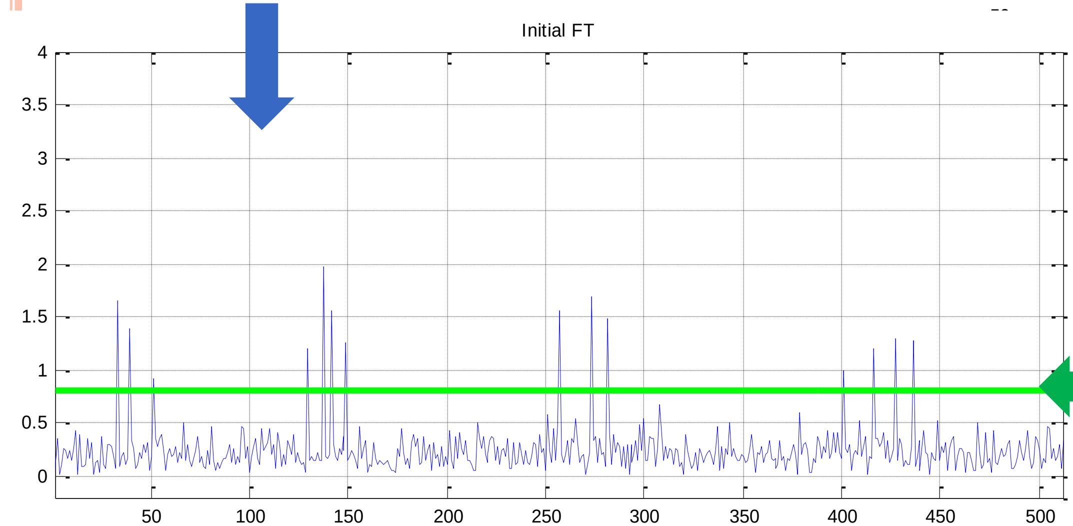


Original Fourier transform

220 out of 512 samples used



Initial Fourier transform



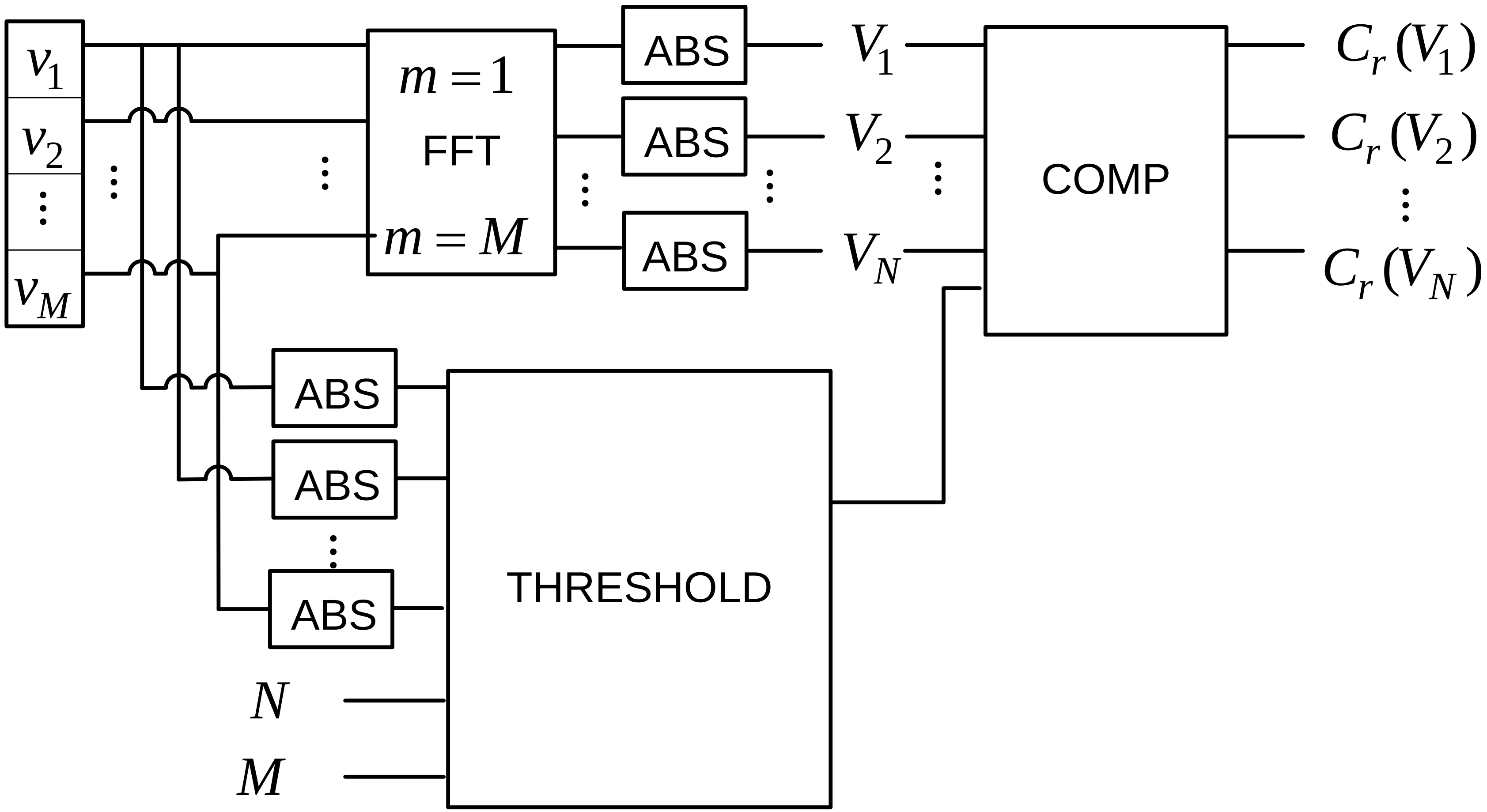
Reconstructed Fourier transform

Threshold

Block scheme

- *Part 1*: FFT calculation using the available signal samples and finding the positions of the FFT coefficients that are above the threshold;
- *Part 2*: Forming of the Compressive Sensing matrix;
- *Part 3*: QR decomposition and optimization problem solving;
- *Part 4*: Spectral positioning block

BLOCK SCHEME – PART 1

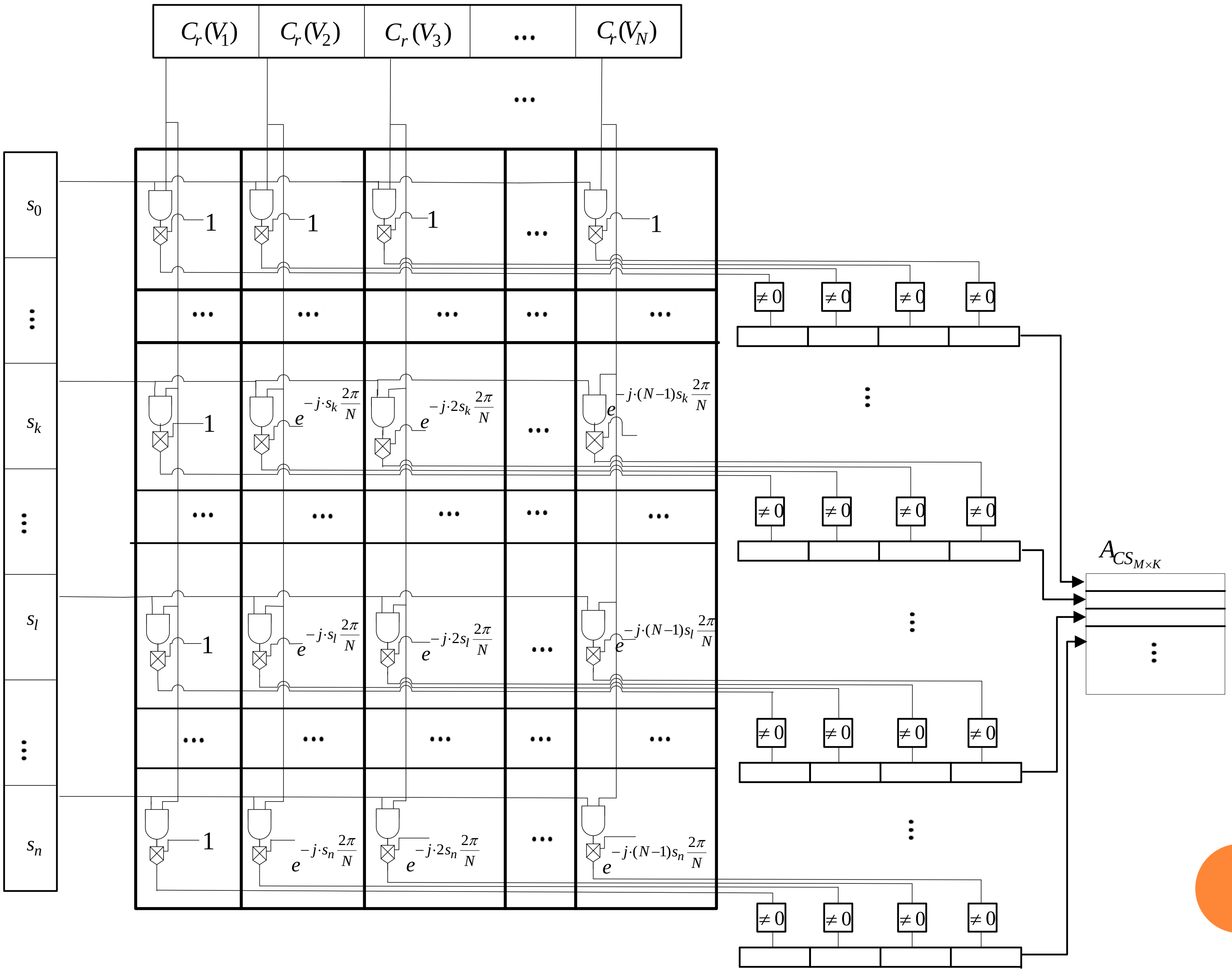


$v_1, \ldots, v_M$ ⇨ Available signal samples

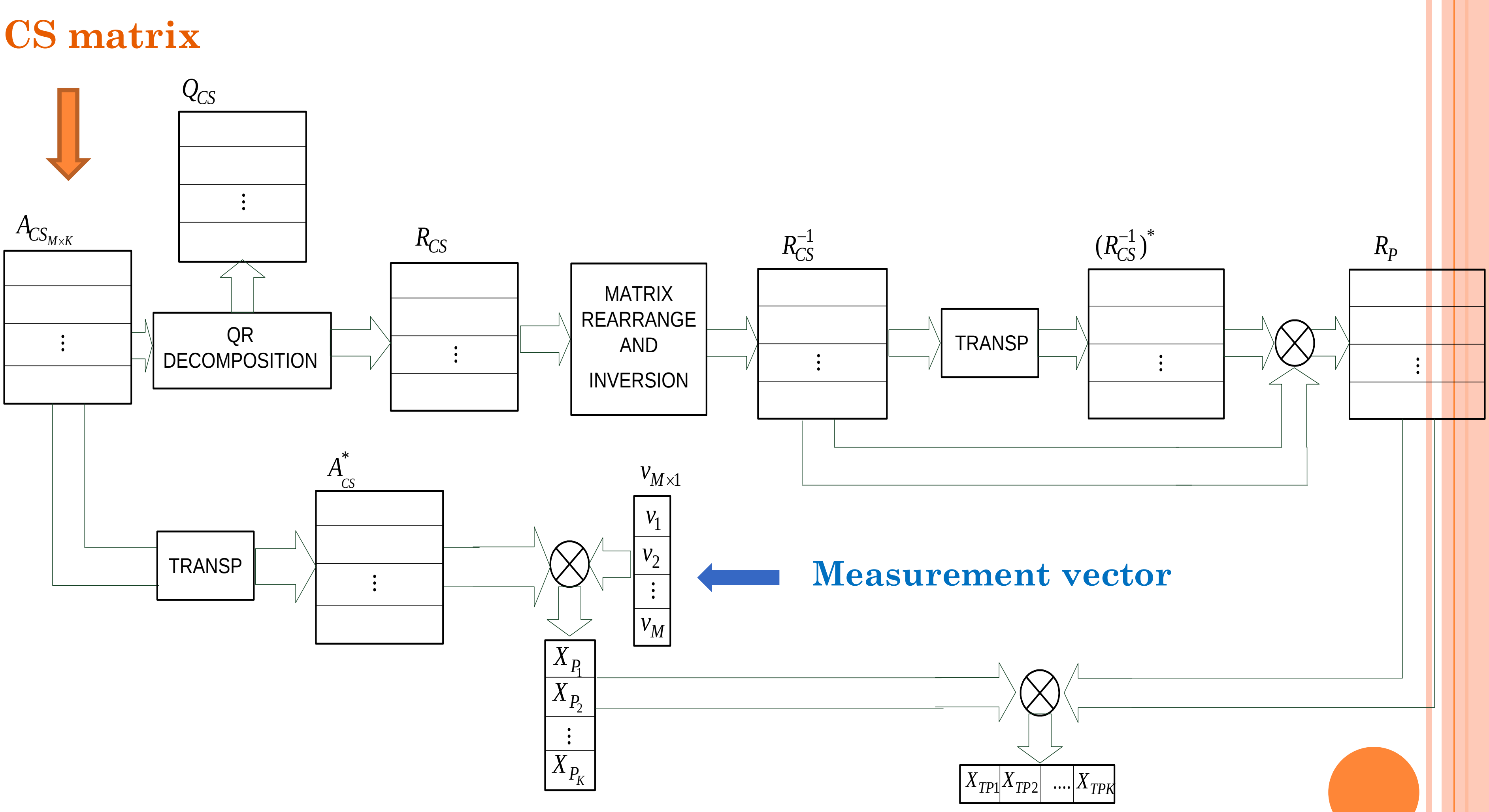
N-signal length

M-number of available samples

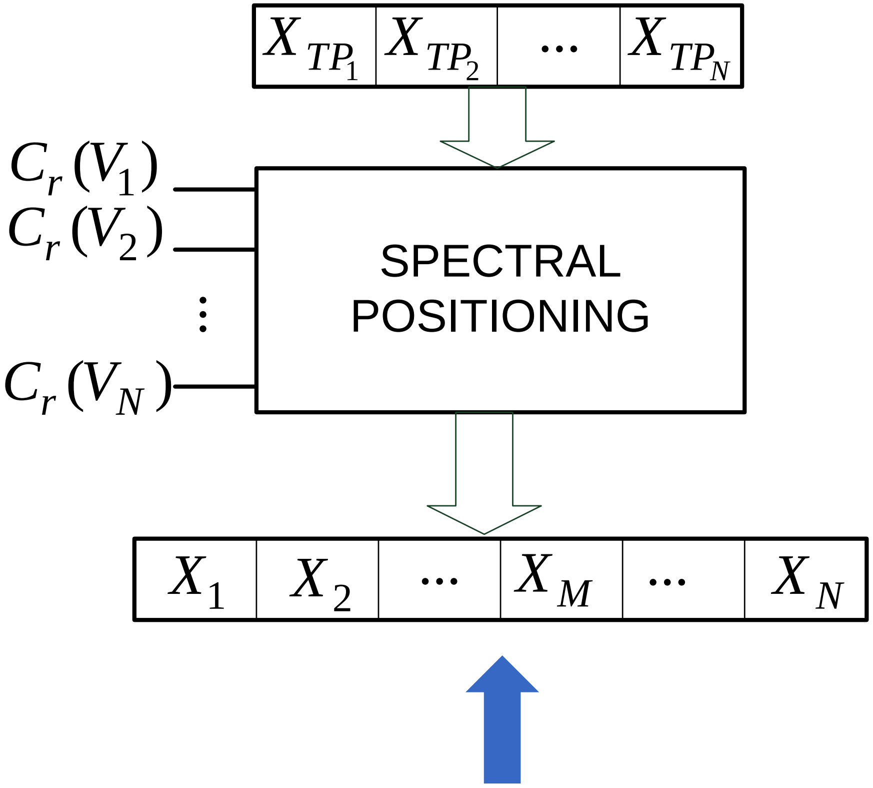
Block scheme – Part 2



Block scheme – Part 3

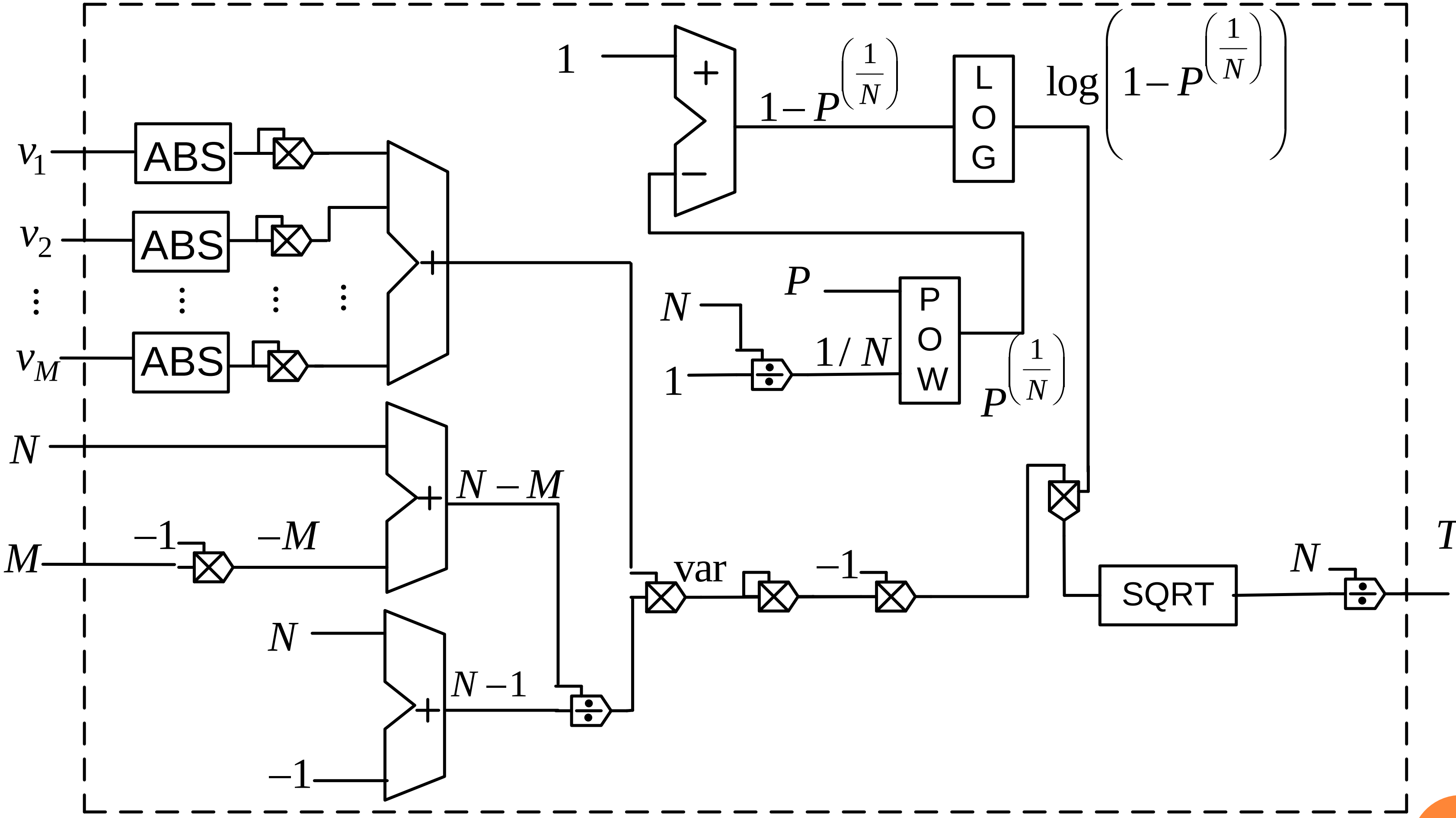


Block for spectral positioning



Reconstructed DFT

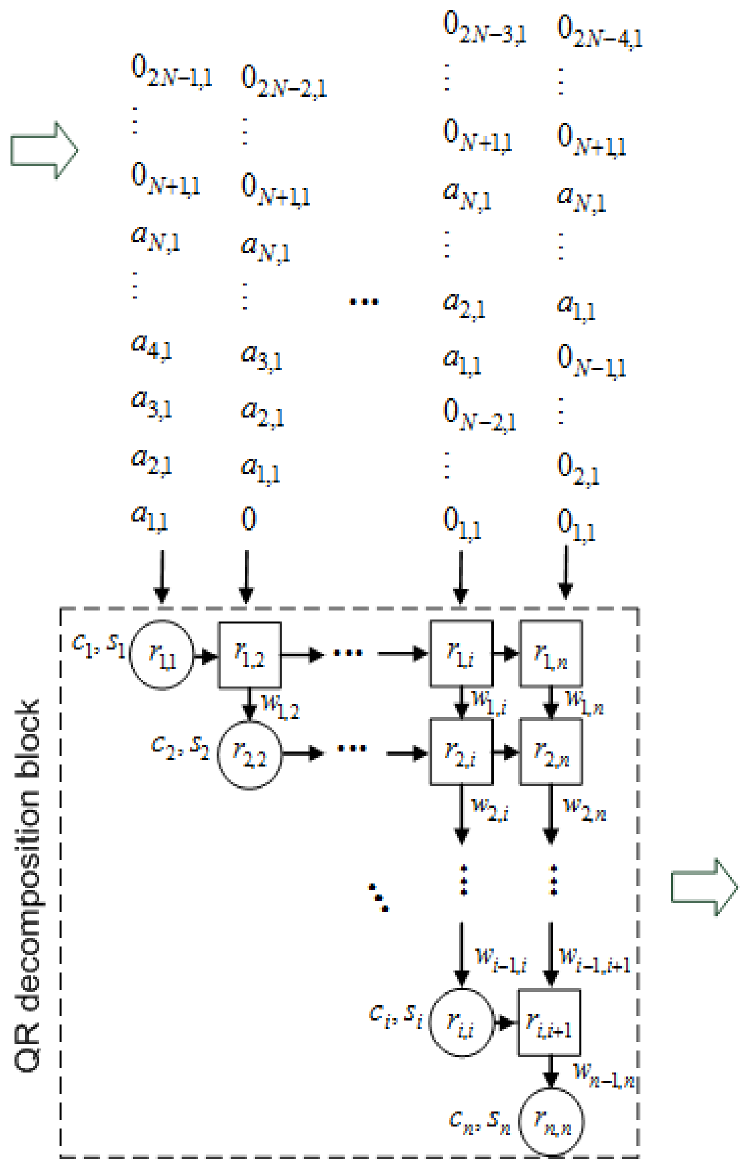
Block for THRESHOLD calculation



QR Decomposition

$$A = QR,$$
$$Q^T Q = I.$$
$$\Downarrow$$
$$A^{-1} = R^{-1} Q^T$$

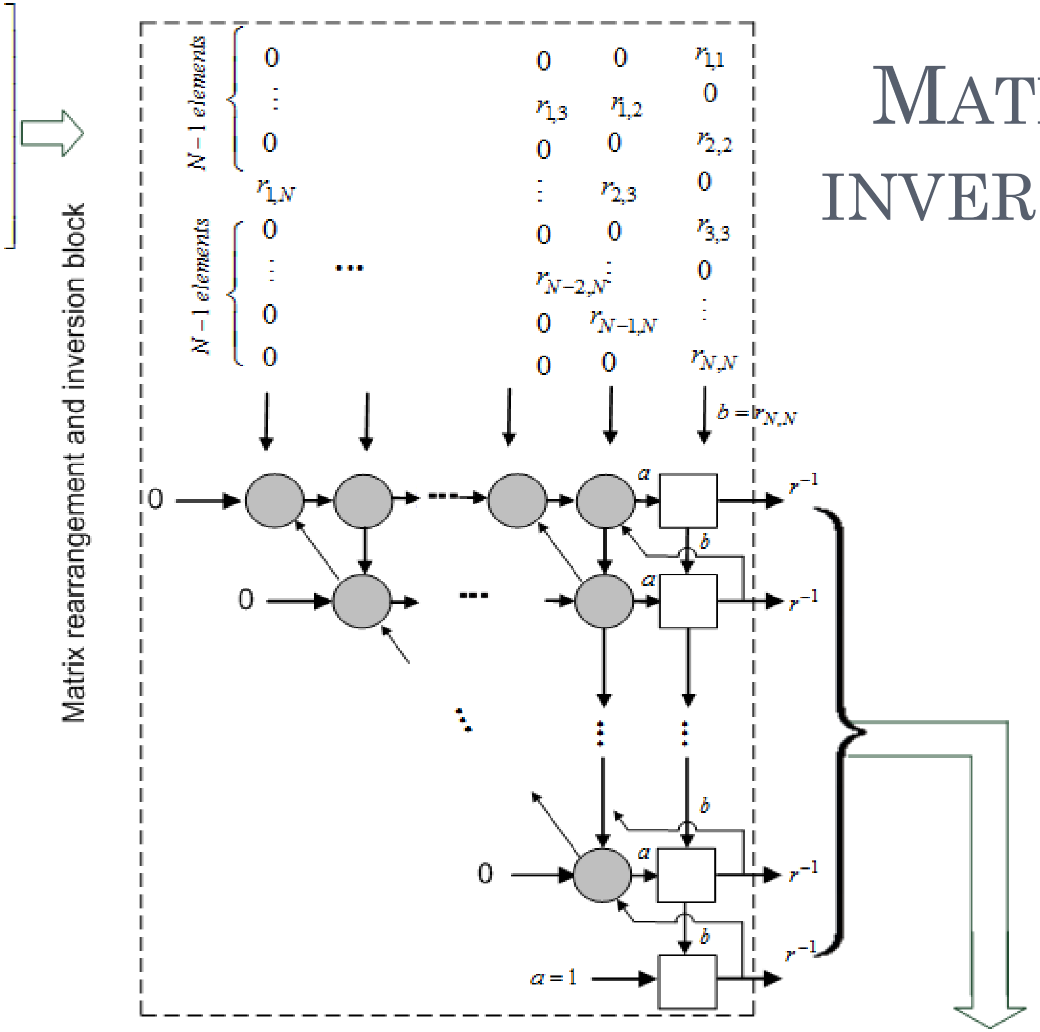
$$A_{CS} = \begin{bmatrix} a_{1,1} & a_{1,2} & a_{1,3} & \cdots & a_{1,N} \\ a_{2,1} & a_{2,2} & a_{2,3} & \cdots & a_{2,N} \\ a_{3,1} & a_{3,2} & a_{3,3} & \cdots & a_{3,N} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ a_{N,1} & a_{N,2} & a_{N,3} & \cdots & a_{N,N} \end{bmatrix}$$



$$R_{CS} = \begin{bmatrix} r_{1,1} & r_{1,2} & r_{1,3} & \cdots & r_{1,N-1} & r_{1,N} \\ 0 & r_{2,2} & r_{2,3} & \cdots & r_{2,N-1} & r_{2,N} \\ 0 & 0 & r_{3,3} & \cdots & r_{3,N-1} & r_{3,N} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & r_{N-1,N-1} & r_{N-1,N} \\ 0 & 0 & 0 & \cdots & 0 & r_{N,N} \end{bmatrix}$$

MATRIX INVERSION

$$R_{CS}=\begin{bmatrix} r_{1,1} & r_{1,2} & r_{1,3} & \cdots & r_{1,N-1} & r_{1,N} \\ 0 & r_{2,2} & r_{2,3} & \cdots & r_{2,N-1} & r_{2,N} \\ 0 & 0 & r_{3,3} & \cdots & r_{3,N-1} & r_{3,N} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & r_{N-1,N-1} & r_{N-1,N} \\ 0 & 0 & 0 & \cdots & 0 & r_{N,N} \end{bmatrix}$$



$$A = QR,$$

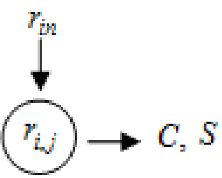
$$Q^T Q = I.$$

$$A^{-1} = R^{-1} Q^T$$

$$R_{CS}^{-1}=\begin{bmatrix} r_{1,1}^{-1} & r_{1,2}^{-1} & r_{1,3}^{-1} & \cdots & r_{1,N-1}^{-1} & r_{1,N}^{-1} \\ 0 & r_{2,2}^{-1} & r_{2,3}^{-1} & \cdots & r_{2,N-1}^{-1} & r_{2,N}^{-1} \\ 0 & 0 & r_{3,3}^{-1} & \cdots & r_{3,N-1}^{-1} & r_{3,N}^{-1} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & r_{N-1,N-1}^{-1} & r_{N-1,N}^{-1} \\ 0 & 0 & 0 & \cdots & 0 & r_{N,N}^{-1} \end{bmatrix}$$

$r_{N,N}^{-1}$	0	$r_{N-1,N-1}^{-1}$	···	$r_{3,3}^{-1}$	0	$r_{2,2}^{-1}$	0	$r_{1,1}^{-1}$
0	0	$r_{N-1,N}^{-1}$	···	$r_{3,N}^{-1}$	0	$r_{2,3}^{-1}$	0	$r_{1,2}^{-1}$
0	0	0	···	0	0	$r_{2,N}^{-1}$	0	$r_{1,3}^{-1}$
⋮		⋮	···				⋮	⋮
0	0	0		0	0	0	0	$r_{1,N}^{-1}$

Cell architectures for QR decomposition and matrix inversion:

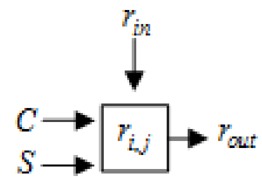


$$r_{in} = 0 \rightarrow \quad C \leftarrow 1, \quad S \leftarrow 0$$

$$r_{in} \neq 0 \rightarrow \quad C \leftarrow \frac{r_{i,j}}{\sqrt{r_{i,j}^2 + r_{in}^2}}$$

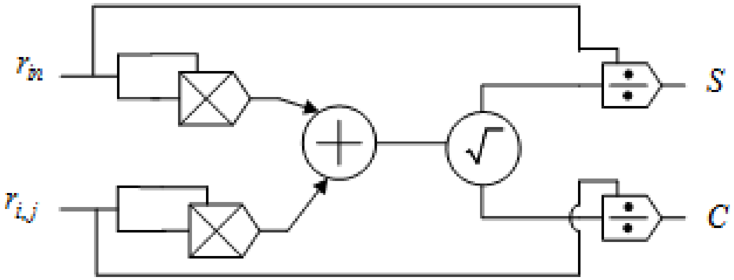
$$S \leftarrow \frac{r_{in}}{\sqrt{r_{i,j}^2 + r_{in}^2}}$$

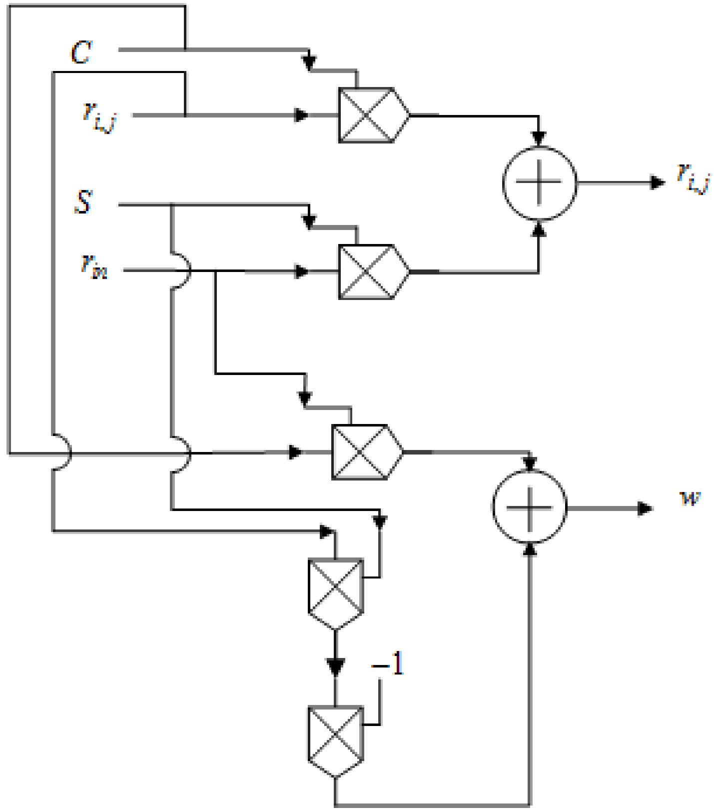
$$r_{i,j} \leftarrow \sqrt{r_{i,j}^2 + r_{in}^2}$$



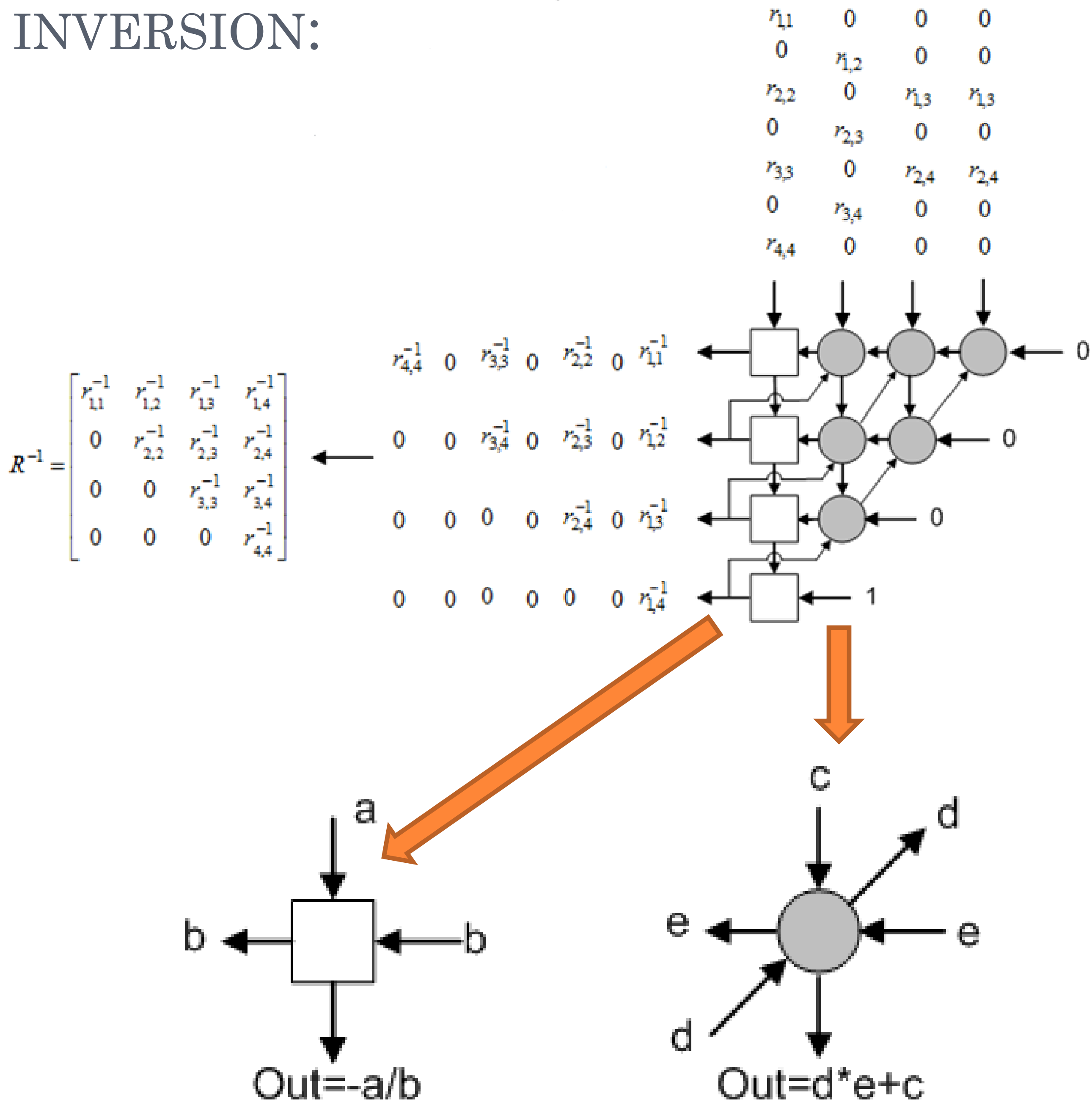
$$w \leftarrow -S \cdot r_{i,j} + C \cdot r_{in}$$

$$r_{i,j} \leftarrow C \cdot r_{i,j} + S \cdot r_{in}$$





Cell architectures for Triangular matrix inversion:



Thank you