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Concentration Measures with an Adaptive Algorithm for Processing Sparse Signals

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Basic idea

- Sparse signals are of interest in many applications like radars, sonars, biomedicine, etc.
- Situation when some signal samples are missing or are omitted is common due to physical or measurement unavailability.
- Missed samples are decreasing concentration measure in sparse domain.
- This fact can be used for creating reconstruction algorithm which finds samples that are producing best concentration in sparse domain.

Concentration measures

- Concentration measure is used to find samples that are missing. Lowest concentration measure (best concentrated) indicates that samples have true values.
- Many concentration measures are used in signal processing (Rényi measure, ratio of l_4 and l_2 norms, etc.)
- Concentration measure used in this paper is defined as:

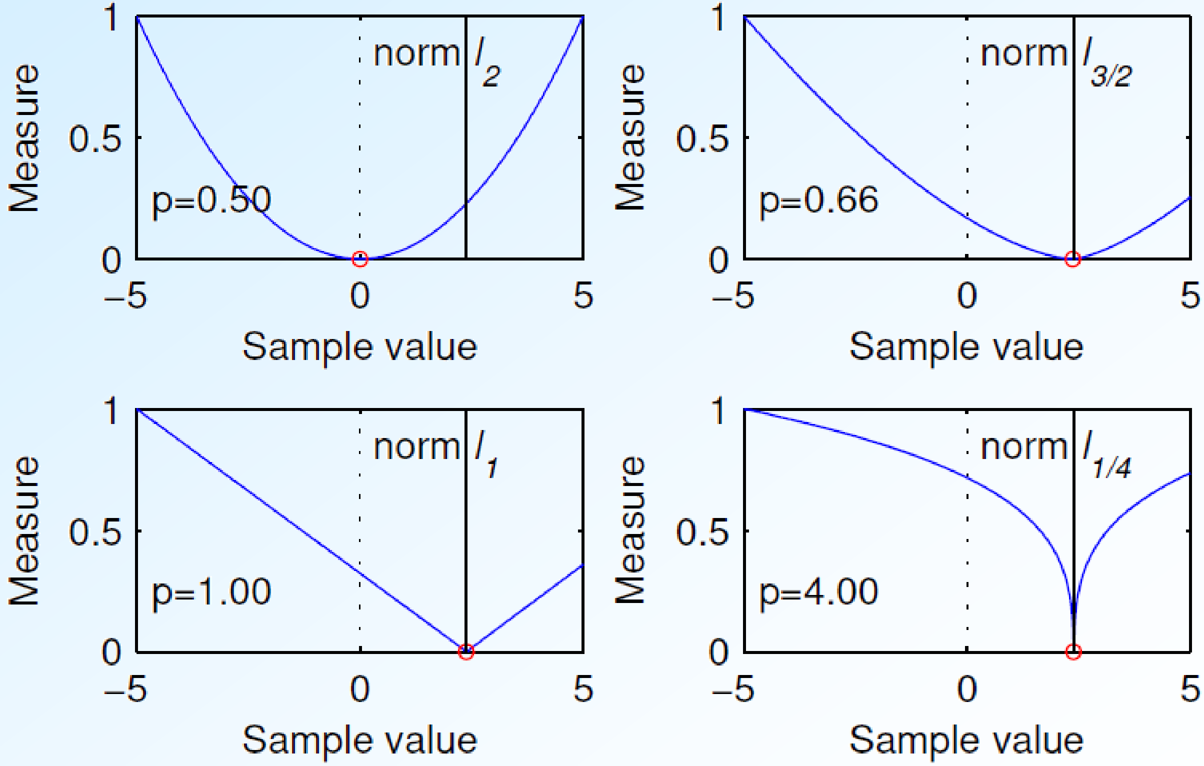
$$\mu[x] = \frac{1}{N} \sum_{n} \left| T[x(n)] \right|^{1/p}$$

Direct reconstruction

One sample is missing

Missing sample position is known.

We can vary value of missing sample, calculate concentration measure and, in some cases, find missing sample value (black vertical line)



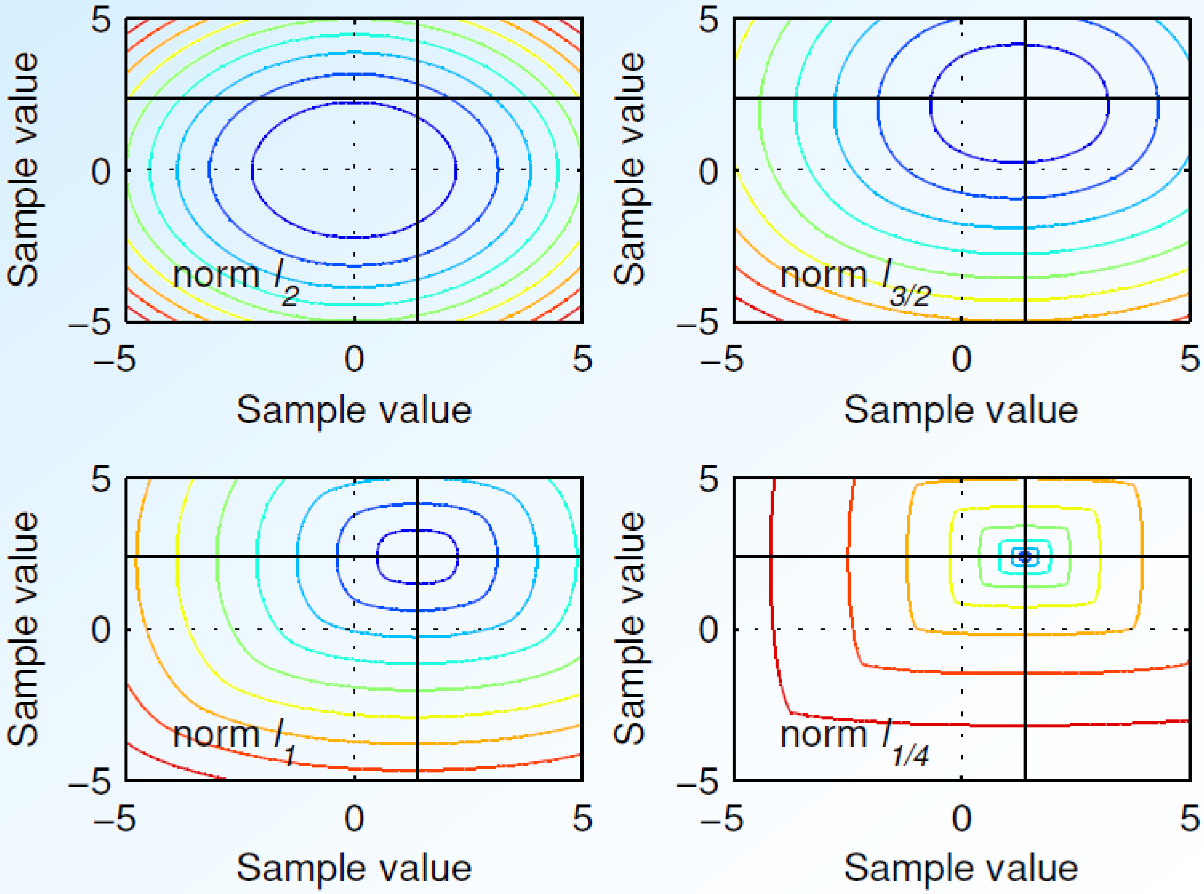
Direct reconstruction

Two samples are missing

2D search is performed

Measure minimum coincide with missing sample values in norm 1 and norm ¼ case

Norm 3/2 minimum is close to true values



Gradient reconstruction algorithm

Start with signal where all missing samples are set to 0.

For each missing sample we form two signals:

$$y_1^{(k)}(n) = \begin{cases} y^{(k)}(n) + \Delta & \text{for } n = n_i \\ y^{(k)}(n) & \text{for } n \neq n_i \end{cases}$$

$$y_2^{(k)}(n) = \begin{cases} y^{(k)}(n) - \Delta & \text{for } n = n_i \\ y^{(k)}(n) & \text{for } n \neq n_i \end{cases}$$

Δ is used here in order to decide should current sample value be increased or decreased

and calculate approximation of measure gradient as:

$$g(n_i) = \frac{\mathcal{M}_p\left[T[y_1^{(k)}(n)]\right] - \mathcal{M}_p\left[T[y_2^{(k)}(n)]\right]}{2\Delta}$$

T is signal transformation to sparse domain

Form gradient vector G and adjust missing sample values:

$$y^{(k+1)}(n) = y^{(k)}(n) - \mu G(n)$$

Reconstruction example

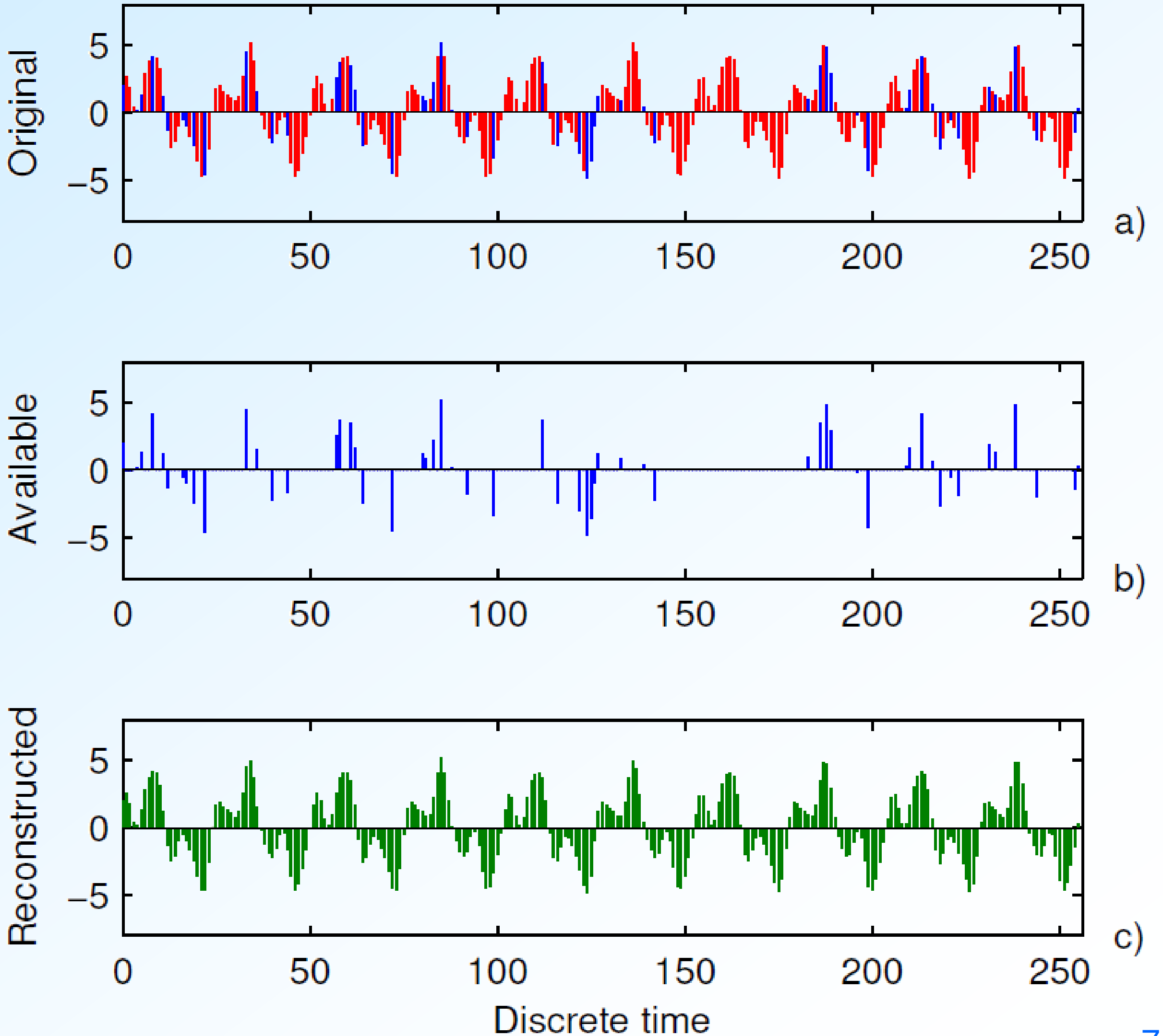
Signal with 200 missing samples is considered.

Missing samples are at arbitrary (known) positions.

Only 56 samples are available.

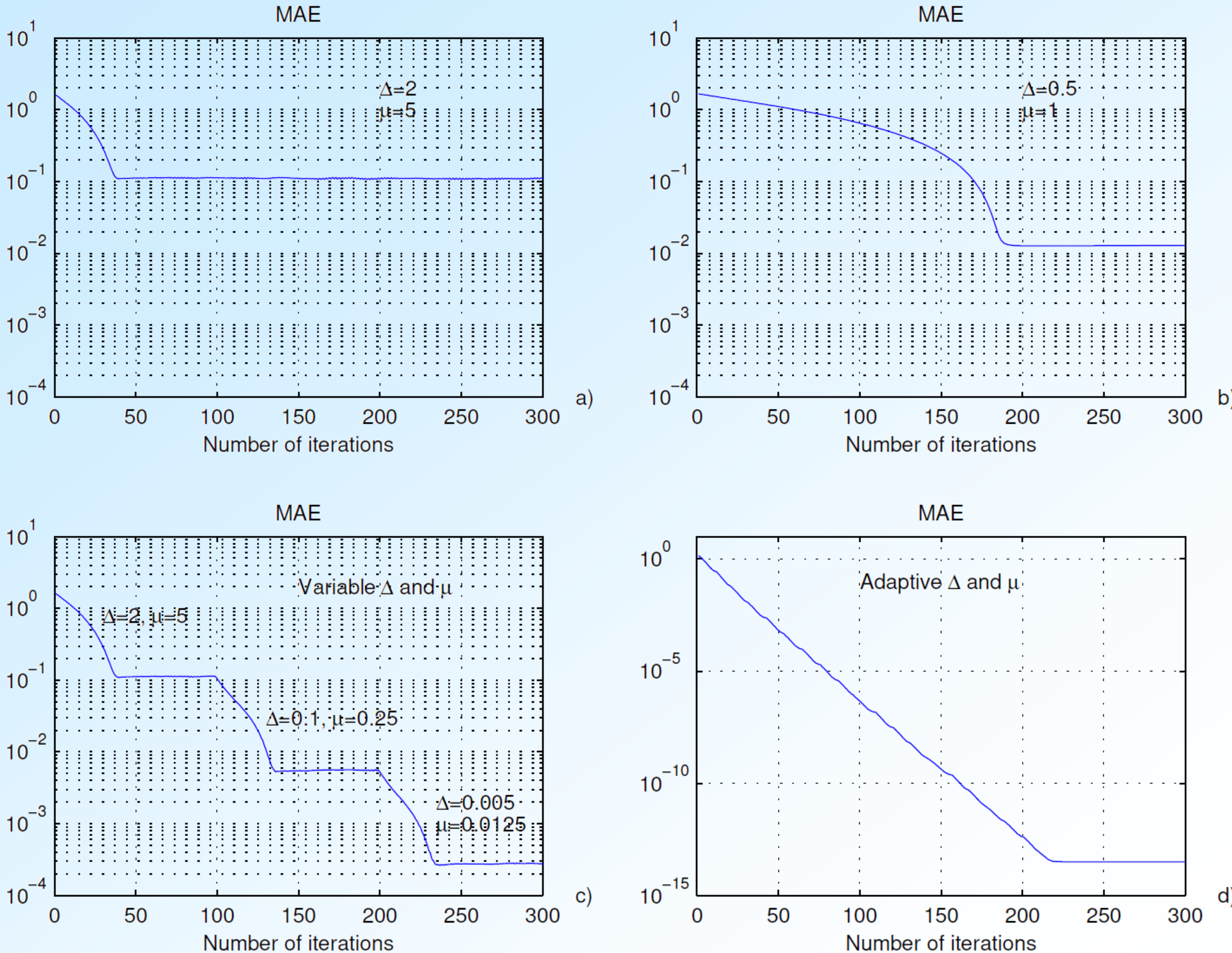
Signal is sparse in DFT domain.

Reconstruction is performed with proposed algorithm.



Algorithm parameters

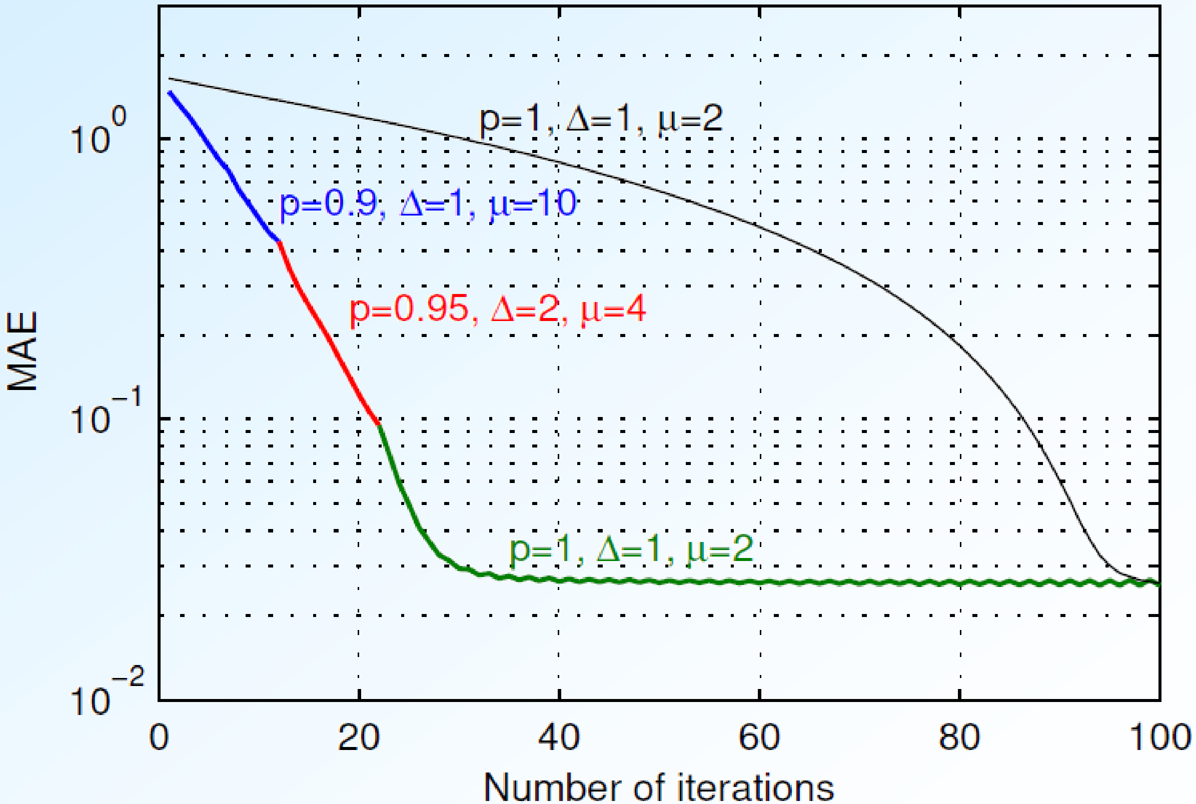
- Algorithm performances depend on parameters Δ and μ .
- Mean absolute error (MAE) is used for algorithm evaluation.
- Large Δ and μ – faster convergence and high MAE
- Small Δ and μ – slower convergence and lower MAE
- Varying or adaptive Δ and μ lead to low MAE with fast convergence.
- We can switch to smaller Δ and μ when we detect that correction of missing samples is small enough (details are given in the paper).


MAE
Δ=2
μ=5
Number of iterations
a)
MAE
Δ=0.5
μ=1
Number of iterations
b)
MAE
Variable Δ and μ
Δ=2, μ=5
Δ=0.1, μ=0.25
Δ=0.005
μ=0.0125
Number of iterations
c)
MAE
Adaptive Δ and μ
Number of iterations
d)

Varying measure parameter

Convergence speed can be further increased by using measures other than norm 1 at the beginning of iterative procedure.

Same MAE is obtained within 30 iterations compared with 100 iterations if constant algorithm parameters are used.



Conclusion

- In this paper we have analyzed the signal concentration measures application to missing samples reconstruction problem.
- In addition to the analysis of direct search, an algorithm for the signal reconstruction is presented and analyzed.
- Various setups of the algorithm parameters are considered including: constant, varying and self-adaptive parameter selection.
- It has been shown that the algorithm convergence can be significantly improved by using varying measure order.

Thank you

Questions?