Adaptive Gradient Based Algorithm for Complex Sparse Signal Reconstruction

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Introduction

- Reconstruction of missing/omitted samples of complex-valued signal is considered
- It is assumed that signals are sparse in the DFT domain with sparsity K, meaning that only K values of the signal’s DFT are non-zero.
- The proposed algorithm is based on gradient steepest descent method.

Problem formulation

Reconstruction of the missing samples can be formulated as constrained minimization problem:

$$\text{Minimize } \mathcal{M}(\mathrm{DFT}(y(n)))$$
$$\text{under constraints } y(n) = x(n) \text{ for } n \in \mathbb{N}_x$$

Where:

M is concentration (sparsity) measure (L1 norm)

$x(n)$ are available signal samples

$y(n)$ is reconstructed signal

N_x is set of available samples positions

Algorithm

Algorithm start with missing samples set to 0. For each missing sample n_i four signals are formed by varying missing sample real and imaginary part:

$$y_1^{(k)}(n) = \begin{cases} y^{(k)}(n) + \Delta & \text{for } n = n_i \\ y^{(k)}(n) & \text{for } n \neq n_i \end{cases}$$

$$y_2^{(k)}(n) = \begin{cases} y^{(k)}(n) - \Delta & \text{for } n = n_i \\ y^{(k)}(n) & \text{for } n \neq n_i \end{cases}$$

$$y_3^{(k)}(n) = \begin{cases} y^{(k)}(n) + j\Delta & \text{for } n = n_i \\ y^{(k)}(n) & \text{for } n \neq n_i \end{cases}$$

$$y_4^{(k)}(n) = \begin{cases} y^{(k)}(n) - j\Delta & \text{for } n = n_i \\ y^{(k)}(n) & \text{for } n \neq n_i \end{cases}$$

Algorithm - continued

Real and imaginary parts of the measure gradient vector are calculated

$$g_r(n_i) = \mathcal{M}\left[\mathrm{DFT}[y_1^{(k)}(n)]\right] - \mathcal{M}\left[\mathrm{DFT}[y_2^{(k)}(n)]\right]$$

$$g_i(n_i) = \mathcal{M}\left[\mathrm{DFT}[y_3^{(k)}(n)]\right] - \mathcal{M}\left[\mathrm{DFT}[y_4^{(k)}(n)]\right]$$

$$G^{(k)}(n_i) = g_r(n_i) + j\, g_i(n_i)$$

Values of the reconstructed signal are updated

$$y^{(k+1)}(n) = y^{(k)}(n) - \frac{1}{N} G^{(k)}(n)$$

Algorithm stopping criterion

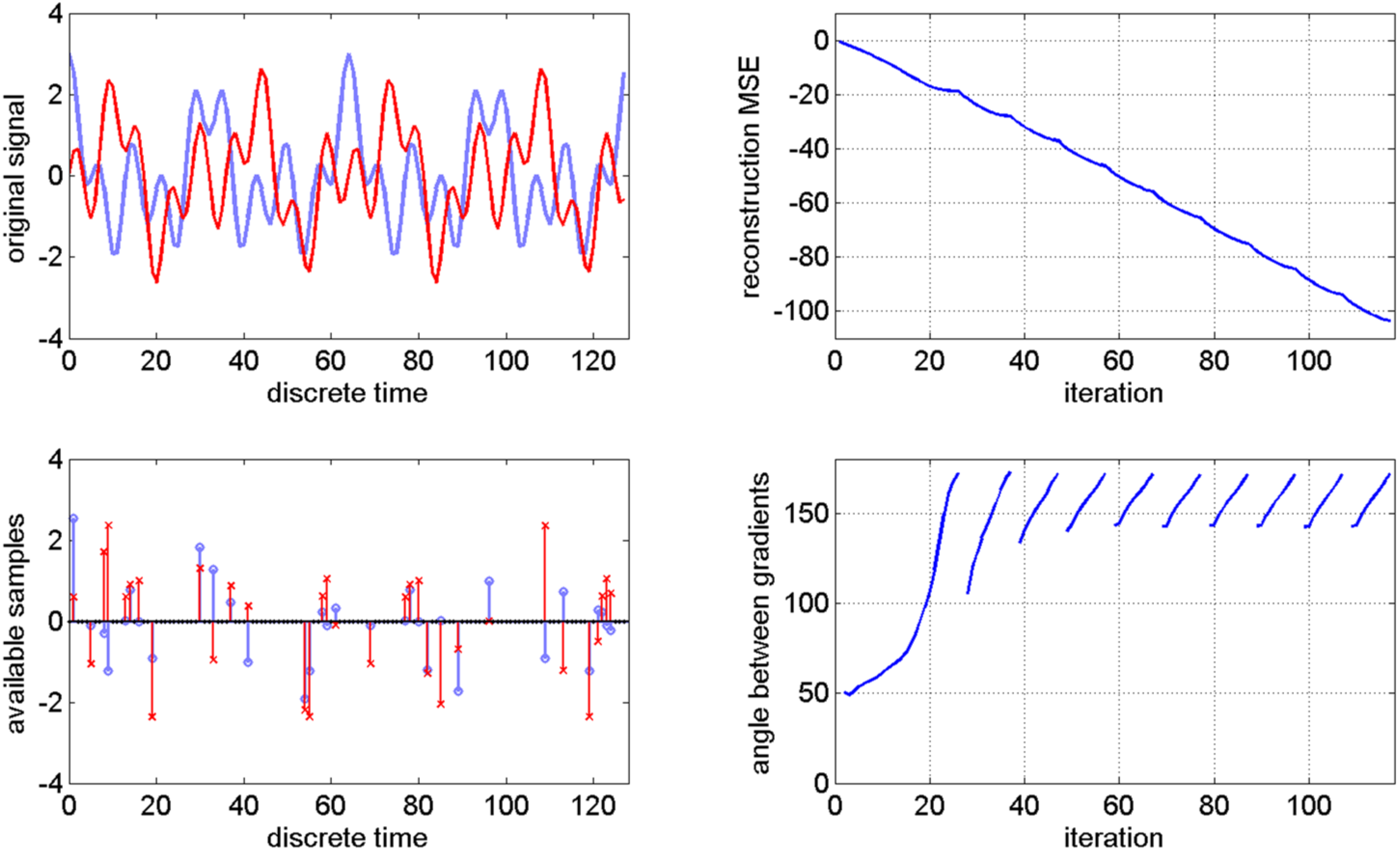
Angle between successive gradient vectors (complex valued) is calculated.

$$\beta = \arccos \frac{\Re[\langle \mathbf{G}^{(k-1)}, \mathbf{G}^{(k)} \rangle]}{||\mathbf{G}^{(k-1)}|| \cdot ||\mathbf{G}^{(k)}||}$$

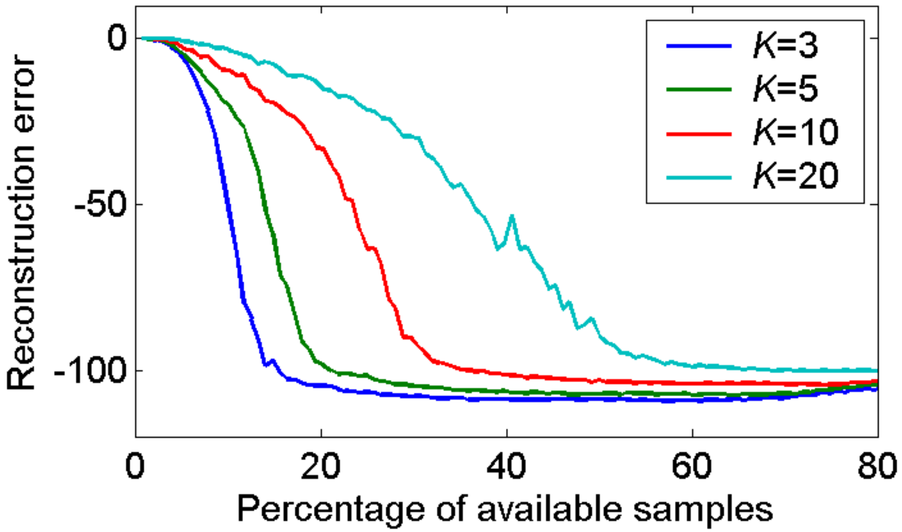
Angle close to 180° means that iterative procedure reached oscillatory state around true sample values.

Since reconstruction error is of Δ order iterative procedure should stop or reduction of Δ should be performed.

Reconstruction example



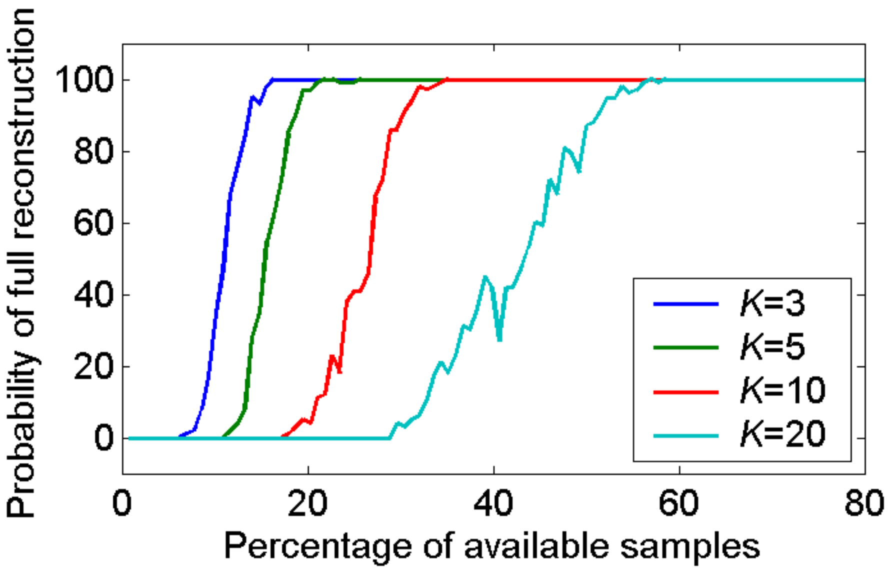
Statistical analysis

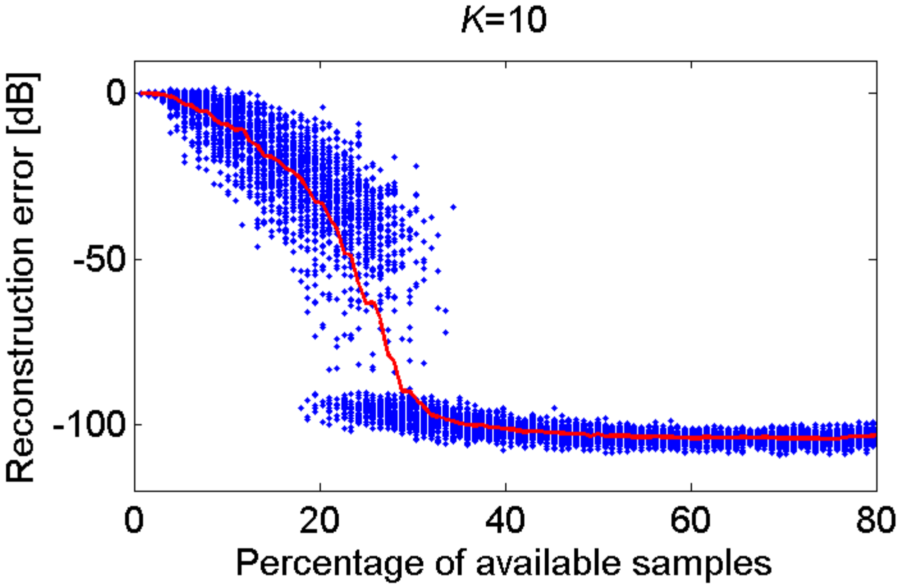


K – signal sparsity

Average over 100 realizations

Percentage of available samples is varied from 0% to 80%





Conclusion

- An algorithm for signal reconstruction is presented. It is assumed that the analyzed signal is sparse in the DFT domain, and that signal is complex valued.
- L1 norm is used as sparisty measure.
- Reconstruction performances are presented on example signal and statistical analysis of the proposed algorithm is performed.
- Reconstruction error and percentage of the full reconstruction events (reconstruction with error close to the desired precision) are considered.

Thanks for your attention.

Questions?