Compressive sensing in Hermite transform domain

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Motivation

- Hermite transform domain is interesting for numerous applications:
  - Biomedical engineering
  - Protein structure analysis
  - Physics
  - Image proccessing
  - Tomography

Mathematical form

- Hermite functions can be defined with the following recursion:

$$\psi_0(t,\sigma) = \frac{1}{\sqrt[4]{\pi}} e^{-\frac{t^2}{2\sigma^2}}, \qquad \psi_1(t,\sigma) = \frac{\sqrt{2}t}{\sqrt[4]{\pi}} e^{-\frac{t^2}{2\sigma^2}},$$

$$\psi_p(t,\sigma) = t\sqrt{2/p}\ \psi_{p-1}(t,\sigma) - \sqrt{(p-1)/p}\ \psi_{p-2}(t,\sigma).$$

- Hermite transfrom (expansion) is given with:

$$f(t) = \sum_{p=0}^{\infty} c_p \psi_p(t,\sigma)$$

where: $c_p \approx \frac{1}{N} \sum_{n=1}^{N} \frac{\psi_p(x_n)}{[\psi_{N-1}(x_n)]^2} f(x_n)$

CS problem formulation

- Let us introduce:

$$\mathbf{W_H} = \frac{1}{N}\begin{bmatrix} \frac{\psi_0(1)}{(\psi_{N-1}(1))^2} & \frac{\psi_0(2)}{(\psi_{N-1}(2))^2} & \cdots & \frac{\psi_0(N)}{(\psi_{N-1}(N))^2} \\ \frac{\psi_1(1)}{(\psi_{N-1}(1))^2} & \frac{\psi_1(2)}{(\psi_{N-1}(2))^2} & \cdots & \frac{\psi_1(N)}{(\psi_{N-1}(N))^2} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\psi_{N-1}(1)}{(\psi_{N-1}(1))^2} & \frac{\psi_{N-1}(2)}{(\psi_{N-1}(2))^2} & \cdots & \frac{\psi_{N-1}(N)}{(\psi_{N-1}(N))^2} \end{bmatrix}$$

then we can write:

$$\mathbf{c} = \mathbf{W}_H \mathbf{f}$$

CS problem formulation

- If we define

$$\boldsymbol{\Psi} = \begin{bmatrix} \psi_0(1) & \psi_0(2) & \dots & \psi_0(N) \\ \psi_1(1) & \psi_1(2) & \cdots & \psi_1(N) \\ \vdots & \vdots & \ddots & \vdots \\ \psi_{N-1}(1) & \psi_{N-1}(2) & \cdots & \psi_{N-1}(N) \end{bmatrix} = \mathbf{W}_H^{-1}$$

then the matrix form of the inverse Hermite transform can be formulated:

$$\mathbf{f} = \mathbf{W}_H^{-1}\mathbf{c} = \boldsymbol{\Psi}\mathbf{c}$$

- Let assume that the compressive sensing is done using a random selection of M_A signal values modelled by a random measurement matrix :

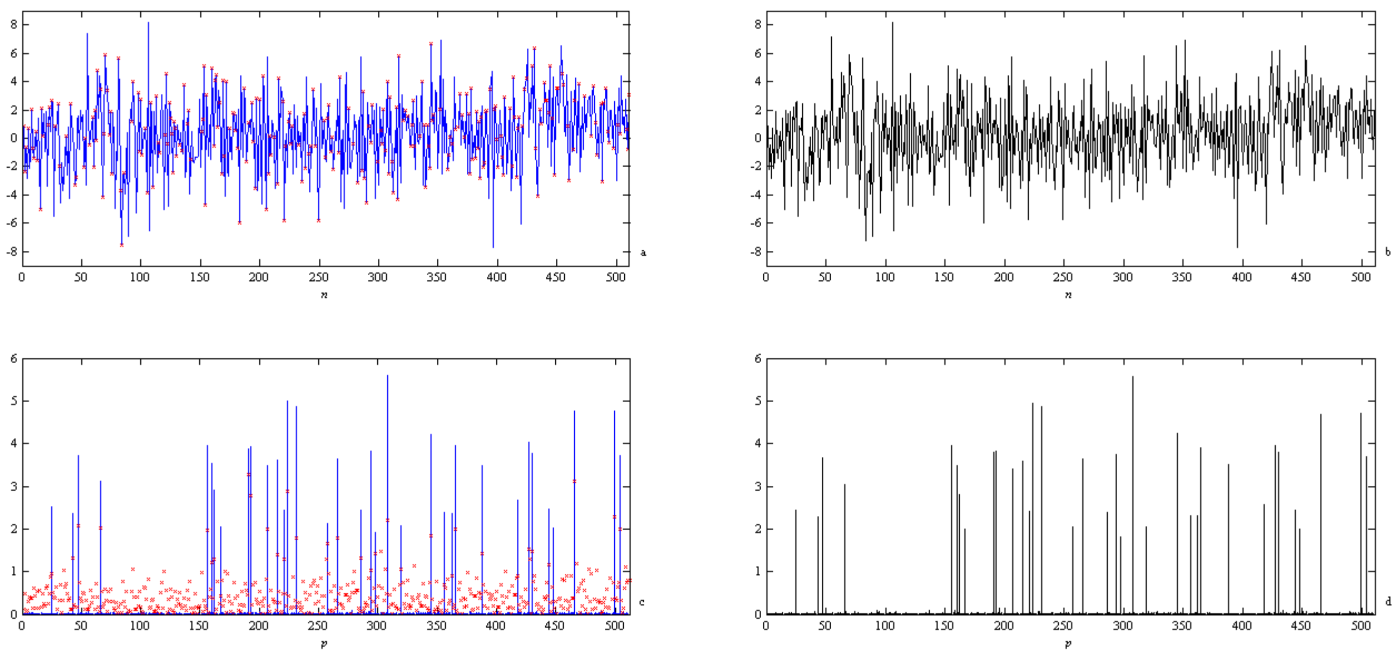
$$\mathbf{y_{cs}} = \boldsymbol{\Phi}\mathbf{f} = \boldsymbol{\Phi\Psi}\mathbf{c} = \mathbf{A_{cs}}\mathbf{c}$$

Finally, the reconstruction problem can be formulated as:

$$\min\|\mathbf{c}\|_{l_1} \; \mathit{subject\ to} \; \mathbf{y}_{cs} = \mathbf{A}_{cs}\mathbf{c}$$

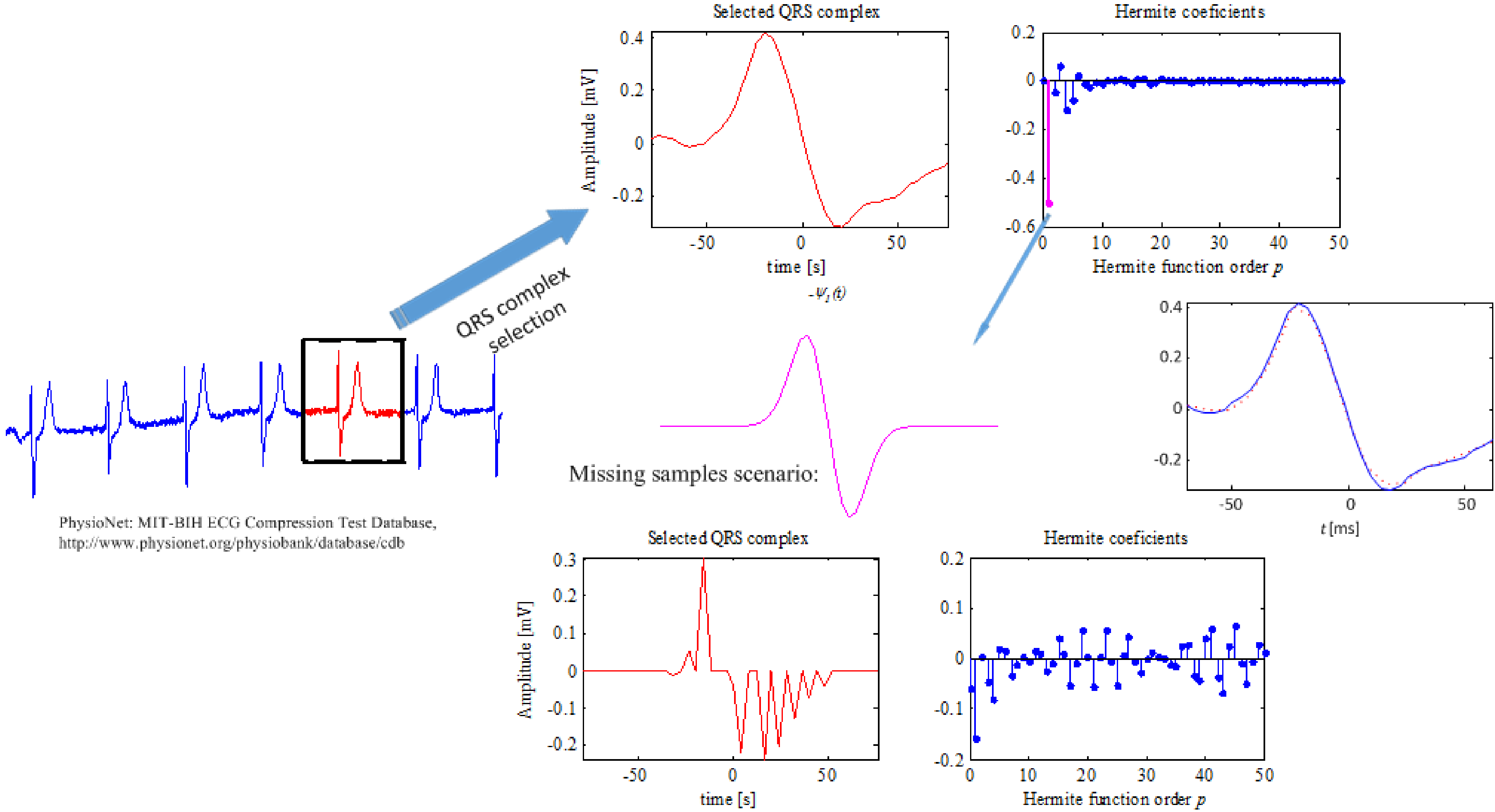
Our solutions

- We have proposed a gradient-based approach to solve the problem
- Example of reconstruction: a noisy signal of length 512 with 35 components, and SNR=30 dB is considered



Application in ECG signal analysis

- Real ECG signal was observed. Hermite domain is suitable for analysis, and, as we showed, for Compressive sensing



Theoretical analysis and development of new algorithms

- Missing samples produce noise in tranform domain
- Our aim was to determine statistical properties of that noise, and to use it in the development of new effective and fast algorithms
- Based on mean values and variances of this noise on the signal positions and non-signal positions the probability of wrong signal component detection was determined:

$$P_E = \int_0^\infty (1 - P_N(\xi)) f(\xi) d\xi =$$

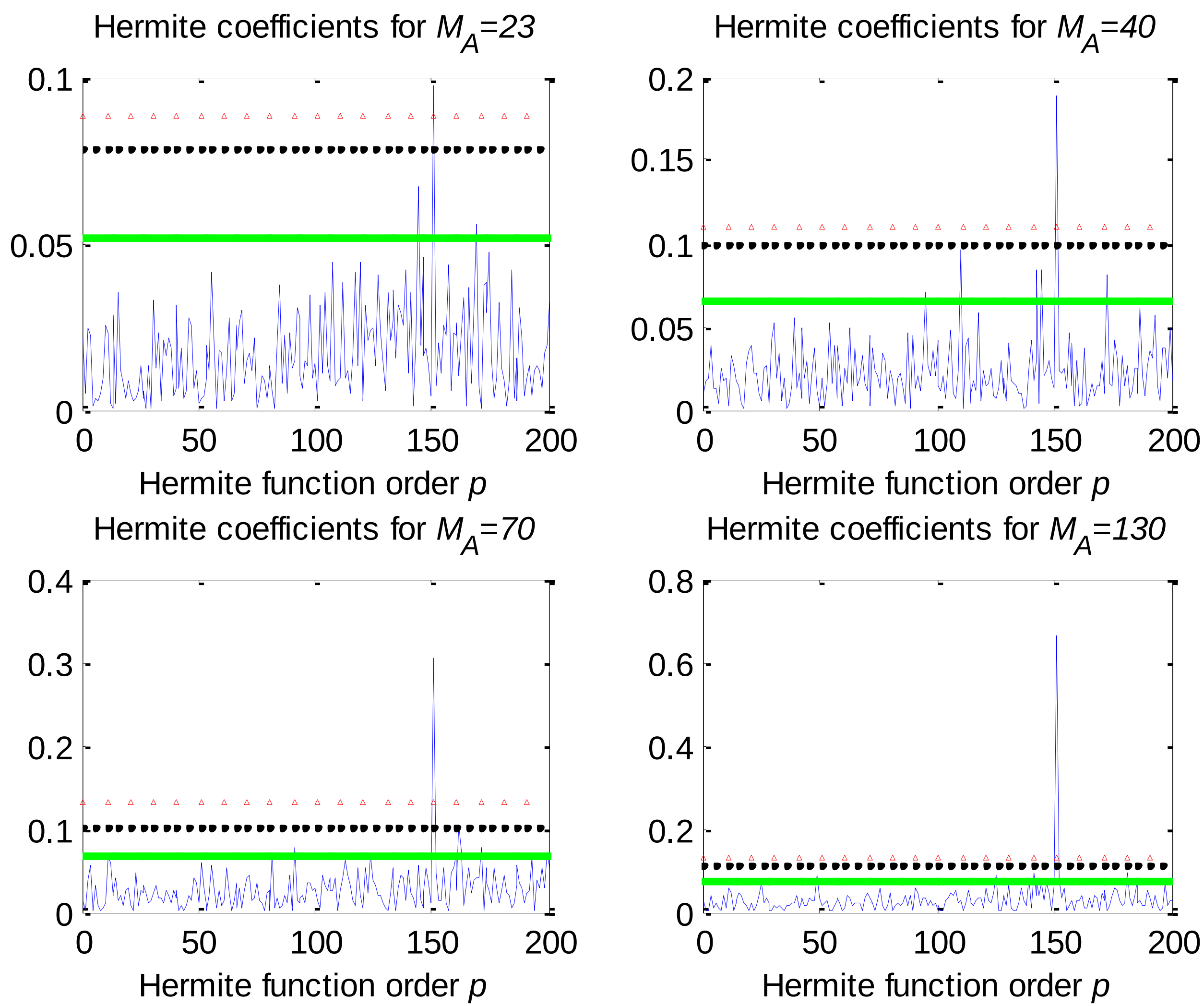
$$= \frac{1}{\sigma_N\sqrt{2\pi}} \int_0^\infty \left(1 - \operatorname{erf}\left(\frac{\xi}{\sqrt{2}\sigma_N}\right)^{N-1}\right)\left(\exp\left(-\frac{(\xi - \mu_S)^2}{2{\sigma_S}^2}\right) + \exp\left(-\frac{(-\xi - \mu_S)^2}{2{\sigma_S}^2}\right)\right) d\xi$$

and a treshold which separates signal and noisy components with a high probability was derived:

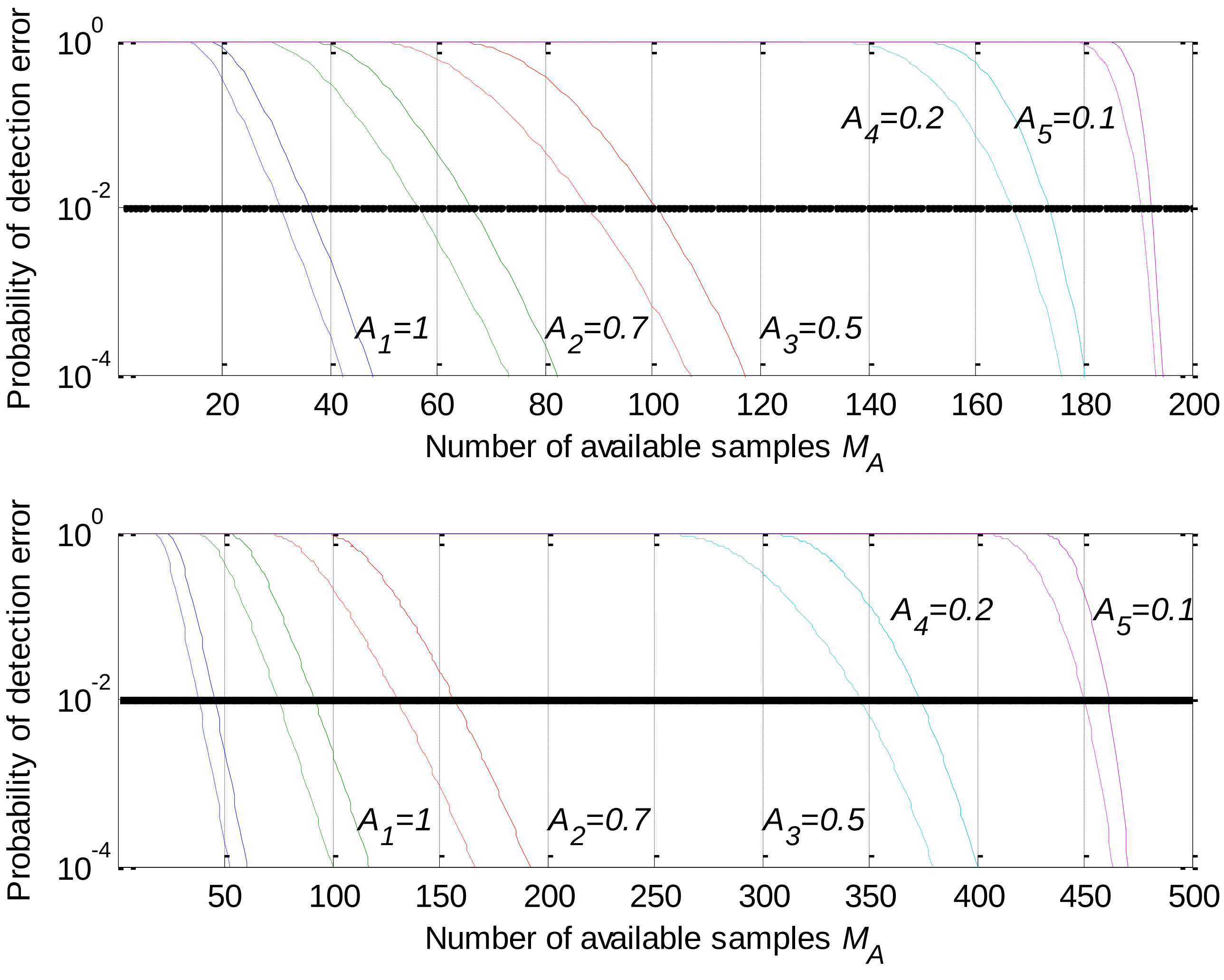
$$T = \sigma_N \sqrt{\left(-4/\pi - aL + \sqrt{(4/\pi + aL)^2 - 4aL}\right)/a}$$

with $L = \log\left(1 - (P_{NN}(T))^{\frac{2}{M-1}}\right)$ and $a \approx 0.147$

Illustration of the threshold for one-component signal case



Probability of the detection error in multicomponent signal case



Multicomponent signal case

- Illustration of the detection of the strongest signal component, first three strongest signal components, first four signal components and the detection of all signal components.