University of Montenegro
Faculty of Electrical Engineering

Compressive sensing: Theory, Algorithms and Applications

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About CS group

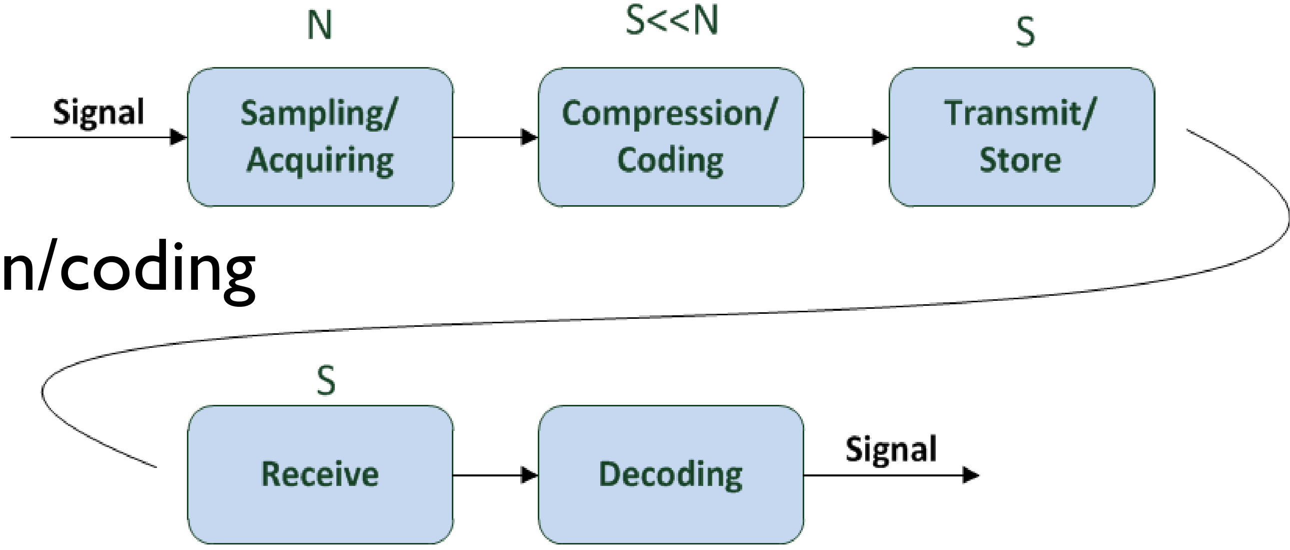
- Project CS-ICT supported by the Ministry of Science of Montenegro
- 12 Researchers
  - 2 Full Professors
  - 3 Associate Professors
  - 1 Assistant Professor
  - 6 PhD students
- Partners:
  - INP Grenoble, FEE Ljubljana, University of Pittsburgh, Nanyang Technological University, Technical faculty Split, Zhejiang University, Zhejiang University of Technology, Hangzhou Normal University

Shannon-Nyquist sampling
Standard acquisition approach

- **Sampling frequency - at least twice higher than the maximal signal frequency ($2f_{max}$)**

- Standard digital data acquisition approach

  - sampling
  - compression/coding
  - decoding



Audio, Image, Video Examples

- **Audio signal:**
- sampling frequency 44,1 KHz
- 16 bits/sample

Uncompressed:

86.133 KB/s

MPEG 1 – compression ratio 4:

21.53 KB/s

- **Color image:**

-256x256 dimension

– 24 bits/pixel

- 3 color channels

Uncompressed:

576 KB

JPEG – quality 30% :

7.72 KB

- **Video:**

- CIF format (352x288)

-NTSC standard (25 frame/s)

-4:4:4 sampling scheme (24 bits/pixel)

Uncompressed:

60.8 Mb/s

MPEG 1- common bitrate 1.5 Kb/s

MPEG 4

28-1024 Kb/s

Compressive Sensing / Sampling

- Is it always necessary to sample the signals **according to the Shannon-Nyquist criterion**?

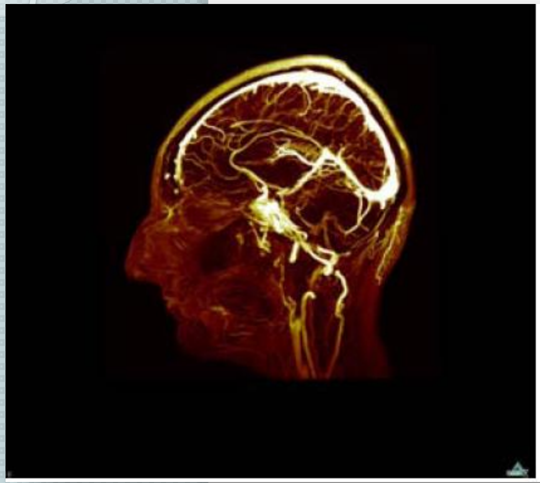
- Is it possible to apply the **compression during** the acquisition process?

↓

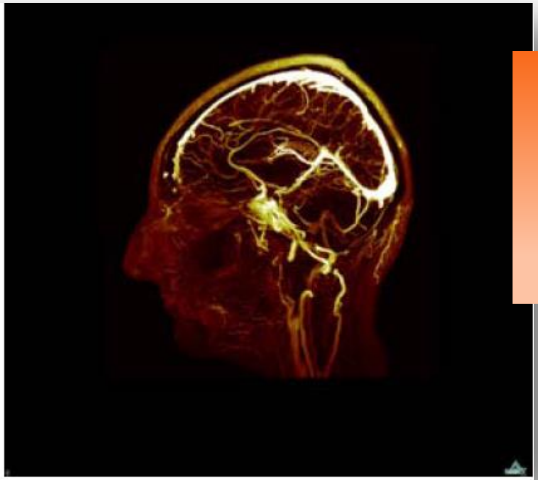
Compressive Sensing:

- **overcomes constraints** of the traditional sampling theory
- applies a concept of **compression during the sensing** procedure

CS Applications



Standard sampling



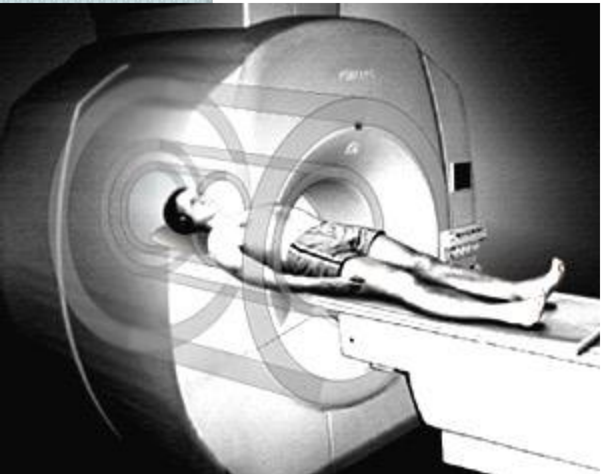
CS reconstruction using 1/6 samples

Biomedical Appl. MRI

Reconstruction from undersampled data
Standard (left)
CS (right)

Make entire "puzzle" having just a few pieces:

Reconstruct entire information from just few measurements/pixels/data

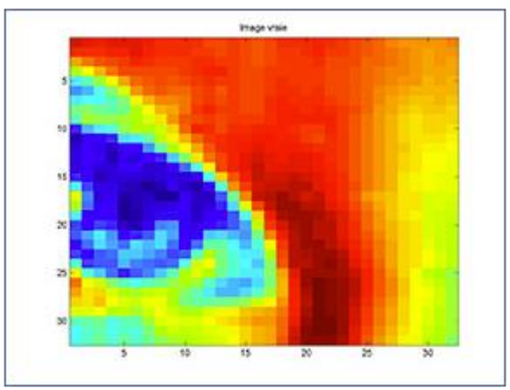


CS promises SMART acquisition and processing and SMART ENERGY consumption

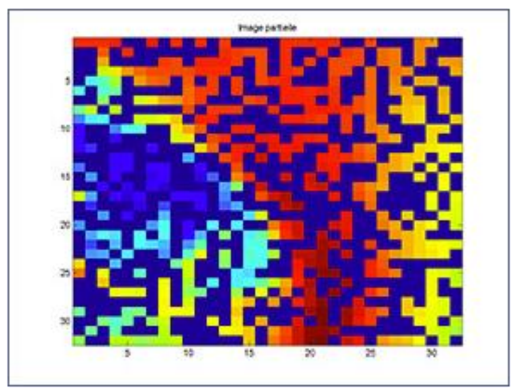
Compressive sensing is useful in the applications where people used to make a large number of measurements

CS Applications

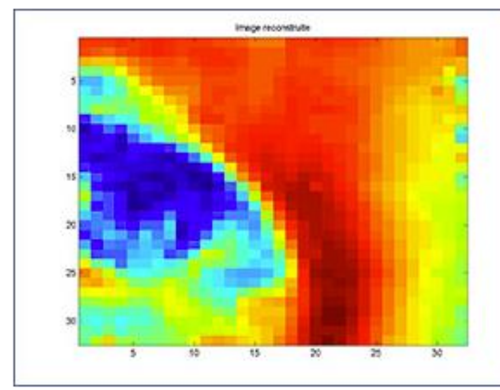
Reconstruction of Lenna's eye



Starting Image



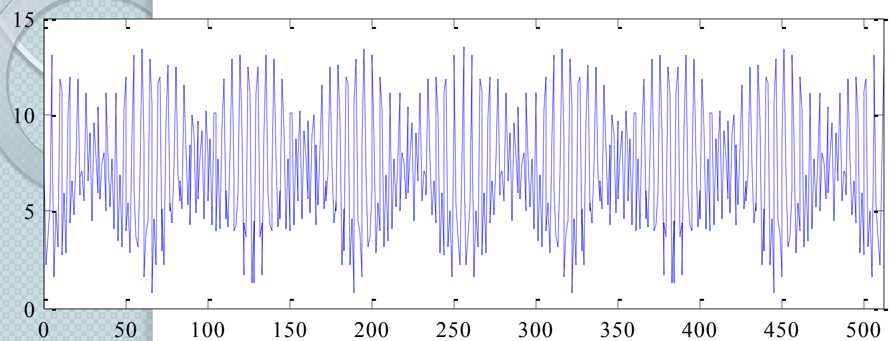
Partial Image

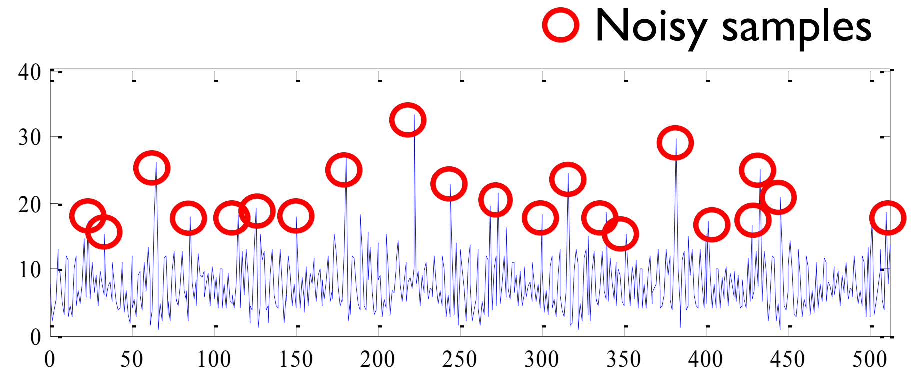


Reconstructed Image

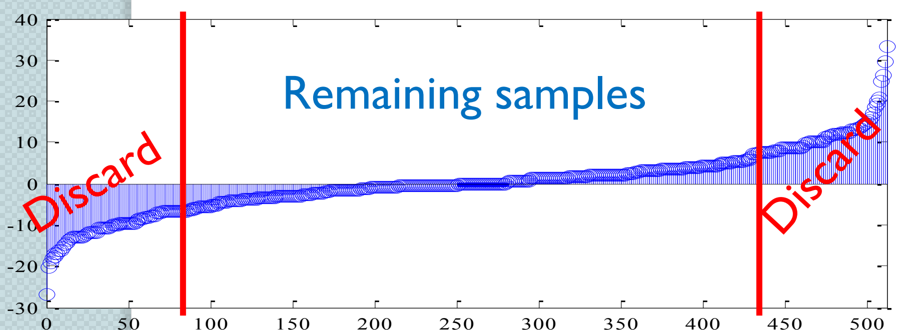
L-statistics based Signal Denoising

Non-noisy signal





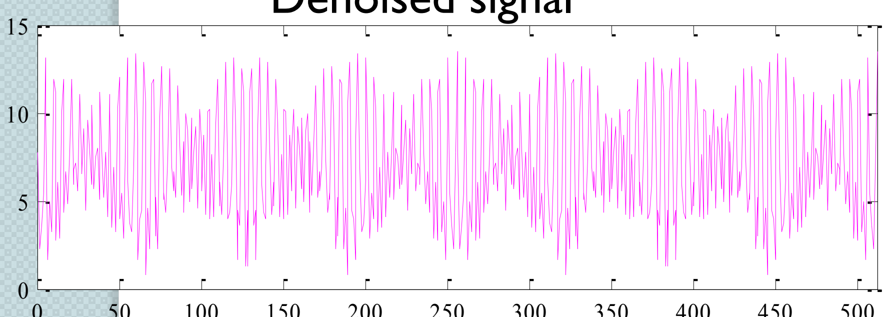
Sorted samples - Removing the extreme values



Discarded samples are declared as “missing samples” on the corresponding original positions in non-sorted sequence

This corresponds to CS formulation

Denoised signal



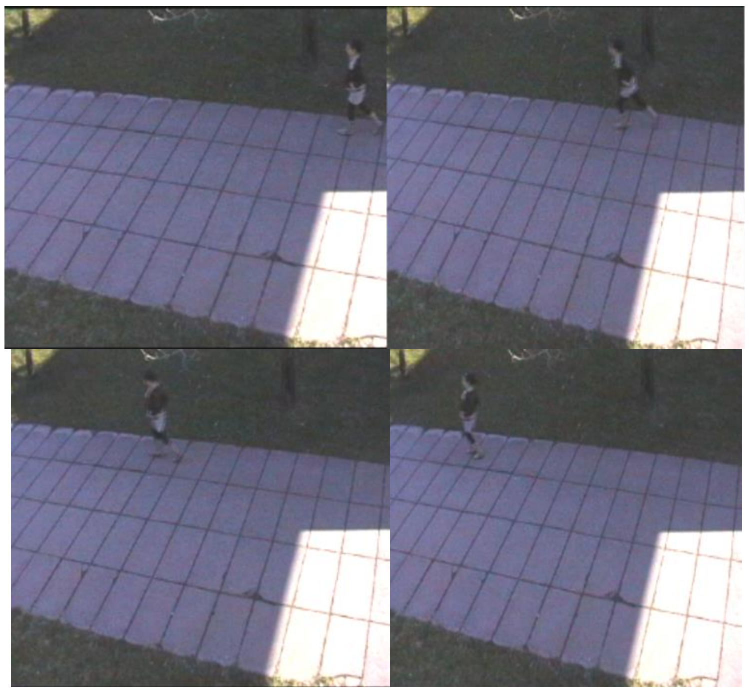
After reconstructing “missing samples” the denoised version of signal is obtained

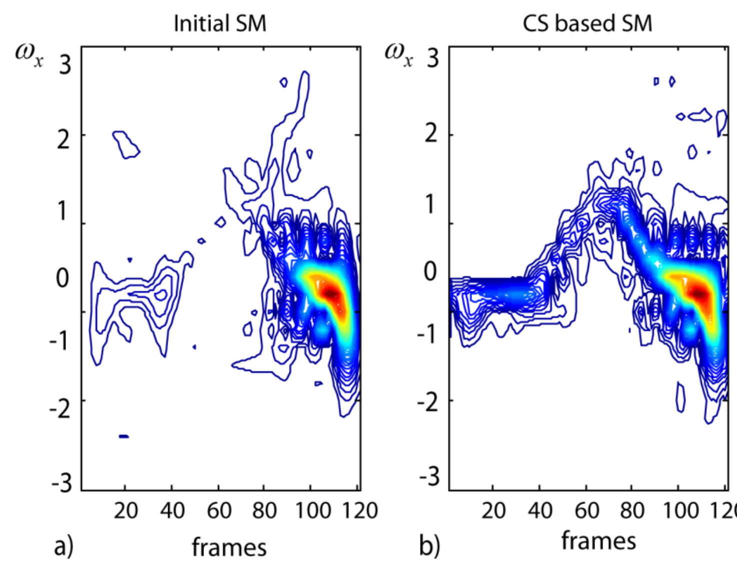
Video sequences

*Video Object Tracking

*Velocity Estimation

*Video Surveillance



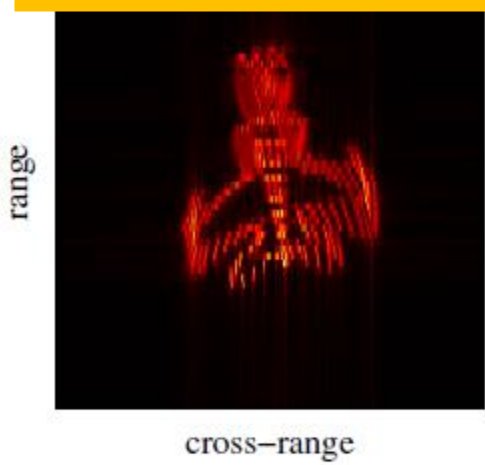


CS Applications

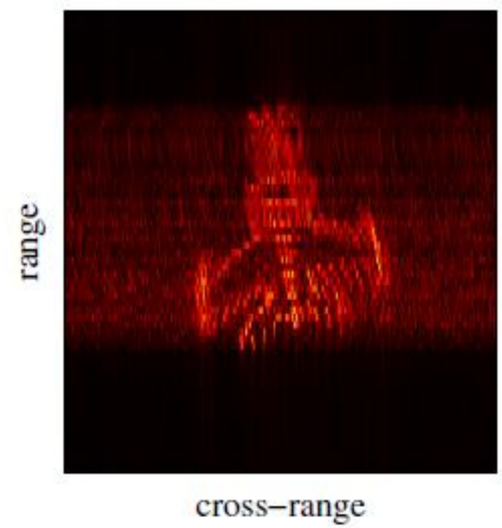
Reconstruction of the radar images

Mig 25 example

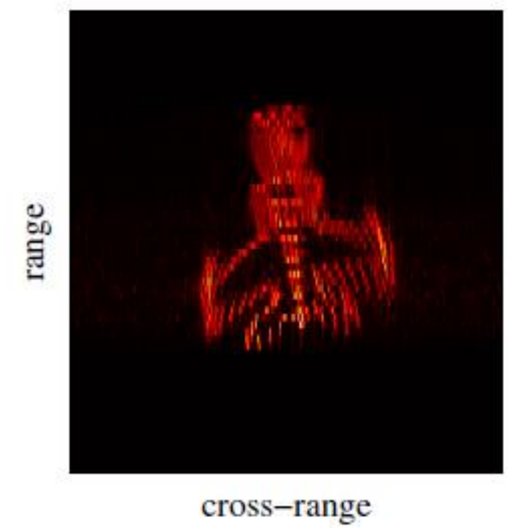
ISAR image with full data set



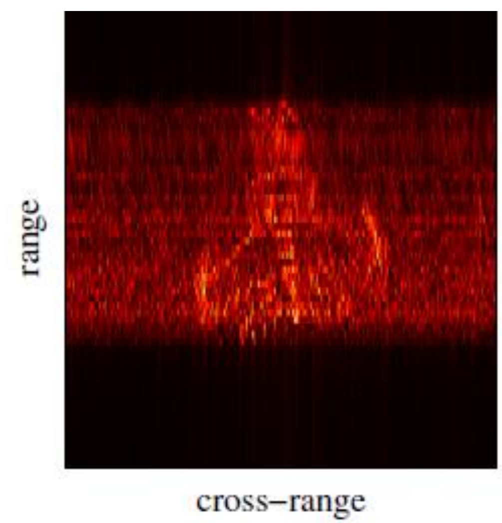
50% available pulses



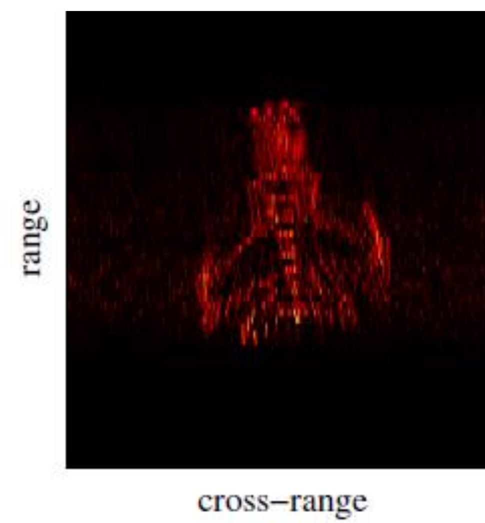
reconstructed image



30% available pulses



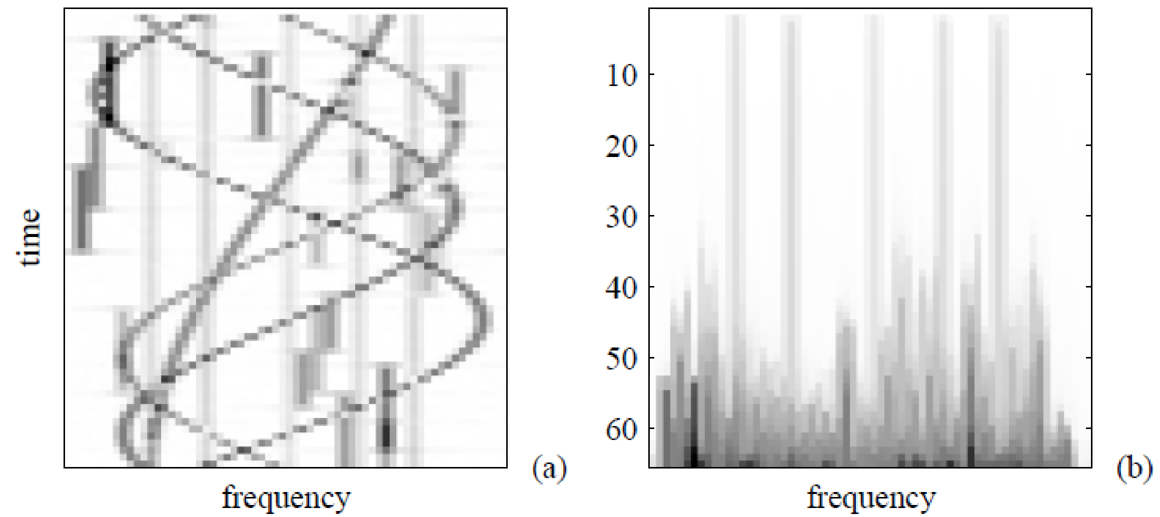
reconstructed image



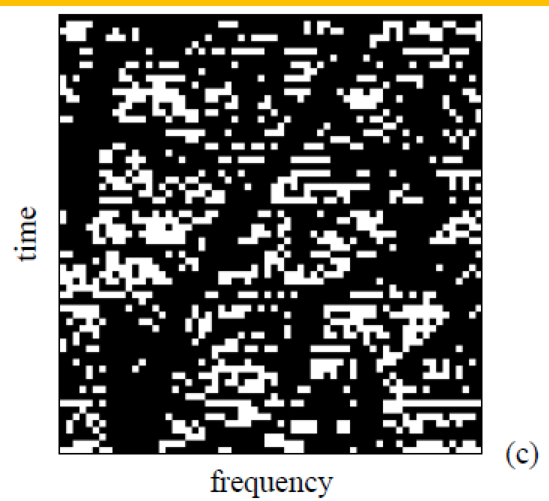
Compressive Sensing Based Separation of Non-stationary and Stationary Signals

Absolute STFT values

Sorted values



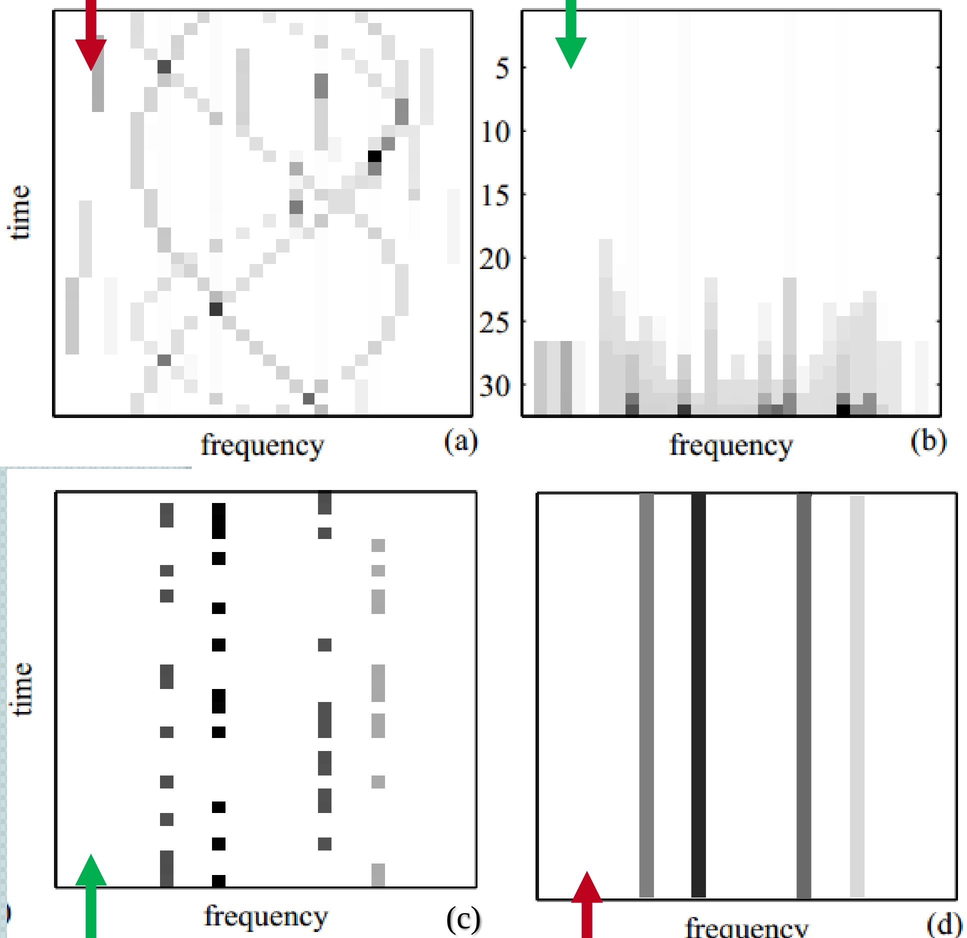
CS mask in TF

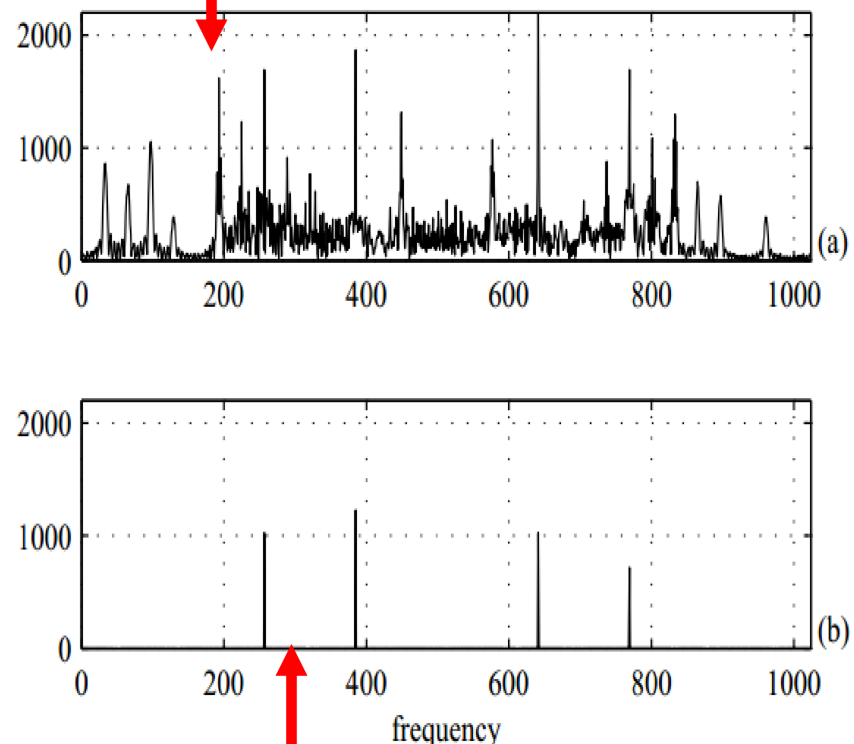


STFT of the composite signal

STFT sorted values

Fourier transform of the original composite signal





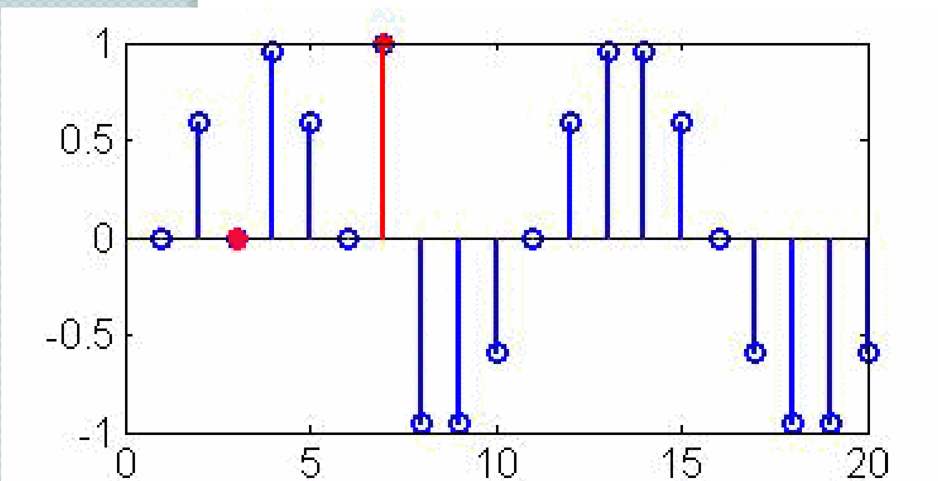
STFT values that remain after the L-statistics

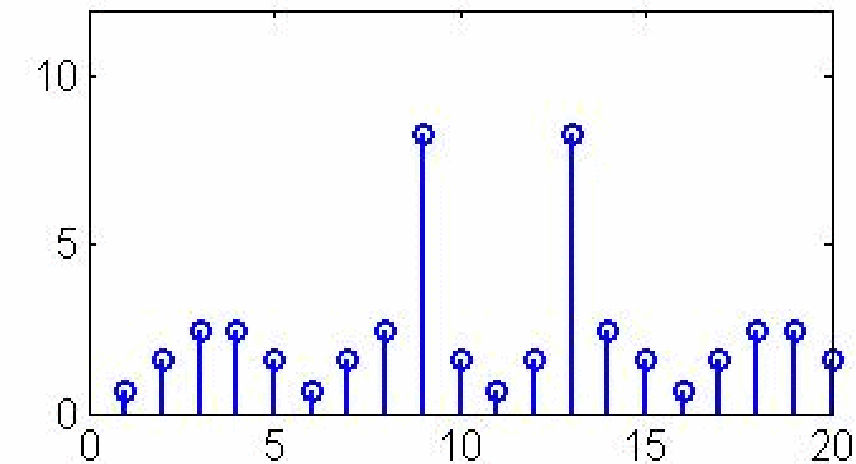
Reconstructed STFT values

The reconstructed Fourier transform by using the CS values of the STFT

CS Applications

- **Simplified case: Direct search reconstruction of two missing samples (marked with red)**





Time domain

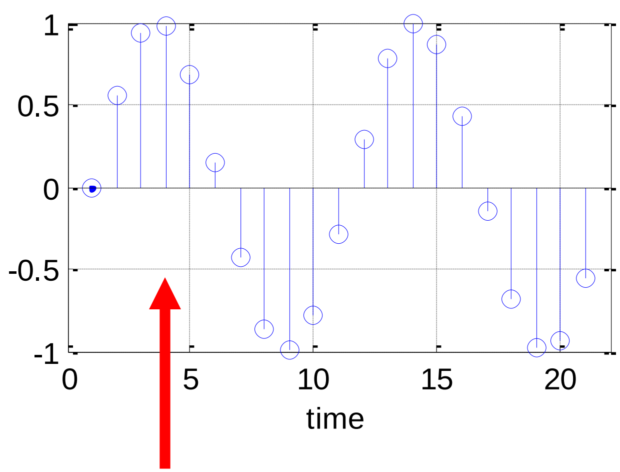
Frequency domain

If we have more missing samples, the direct search would be practically useless

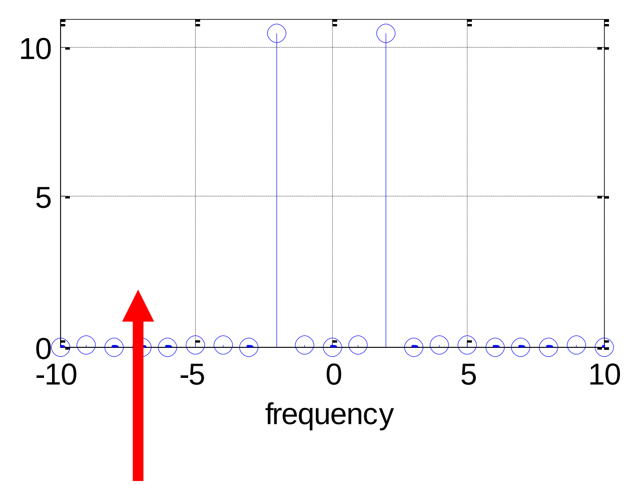
CS Applications-Example

- Let us consider a signal: $f_x(n) = \sin\left(2\cdot\pi\cdot\left(2/N\right)\cdot n\right) \quad for \;\; n = 0,..,20$
- The signal is **sparse in DFT**, and vector of DFT values is:

$$\mathbf{F}_x = [0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 10.5i \;\; 0 \;\; 0 \;\; 0 \; -10.5i \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0];$$



Signal *fx*



DFT values of the signal *fx*

- **CS reconstruction using small set of samples:**

1. Consider the elements of inverse and direct Fourier transform matrices, denoted by $\boldsymbol{\Psi}$ and $\boldsymbol{\Psi}^{-1}$ respectively (relation $\mathbf{f}_x = \boldsymbol{\Psi}\mathbf{F}_x$ holds)

$$\boldsymbol{\Psi} = \frac{1}{21}\begin{bmatrix} 1 & 1 & 1 & 1 & \dots & 1 \\ 1 & e^{j\frac{2\pi}{21}} & e^{2j\frac{2\pi}{21}} & e^{3j\frac{2\pi}{21}} & \dots & e^{20j\frac{2\pi}{21}} \\ 1 & e^{2j\frac{2\pi}{21}} & e^{4j\frac{2\pi}{21}} & e^{6j\frac{2\pi}{21}} & \dots & e^{40j\frac{2\pi}{21}} \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 1 & e^{19j\frac{2\pi}{21}} & e^{38j\frac{2\pi}{21}} & e^{57j\frac{2\pi}{21}} & \dots & e^{380j\frac{2\pi}{21}} \\ 1 & e^{20j\frac{2\pi}{21}} & e^{40j\frac{2\pi}{21}} & e^{60j\frac{2\pi}{21}} & \dots & e^{400j\frac{2\pi}{21}} \end{bmatrix}$$

$$\boldsymbol{\Psi}^{-1} = \begin{bmatrix} 1 & 1 & 1 & 1 & \dots & 1 \\ 1 & e^{-j\frac{2\pi}{21}} & e^{-2j\frac{2\pi}{21}} & e^{-3j\frac{2\pi}{21}} & \dots & e^{-20j\frac{2\pi}{21}} \\ 1 & e^{-2j\frac{2\pi}{21}} & e^{-4j\frac{2\pi}{21}} & e^{-6j\frac{2\pi}{21}} & \dots & e^{-40j\frac{2\pi}{21}} \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 1 & e^{-19j\frac{2\pi}{21}} & e^{-38j\frac{2\pi}{21}} & e^{-57j\frac{2\pi}{21}} & \dots & e^{-380j\frac{2\pi}{21}} \\ 1 & e^{-20j\frac{2\pi}{21}} & e^{-40j\frac{2\pi}{21}} & e^{-60j\frac{2\pi}{21}} & \dots & e^{-400j\frac{2\pi}{21}} \end{bmatrix}$$

2. Take *M* random samples/measurements in the time domain

It can be modeled by using **matrix $\boldsymbol{\Phi}$:**

$$\mathbf{y} = \boldsymbol{\Phi}\mathbf{f}_x$$

- $\boldsymbol{\Phi}$ is defined as a **random permutation matrix**
- $\mathbf{y}$ is obtained by **taking *M* random** elements of $\mathbf{f}_x$

- Taking 8 random samples (out of 21) on the positions:

$$[5 \quad 9 \quad 10 \quad 12 \quad 13 \quad 15 \quad 18 \quad 20]$$

$$\mathbf{y} = \mathbf{\Phi\Psi F}_x = \mathbf{AF}_x$$

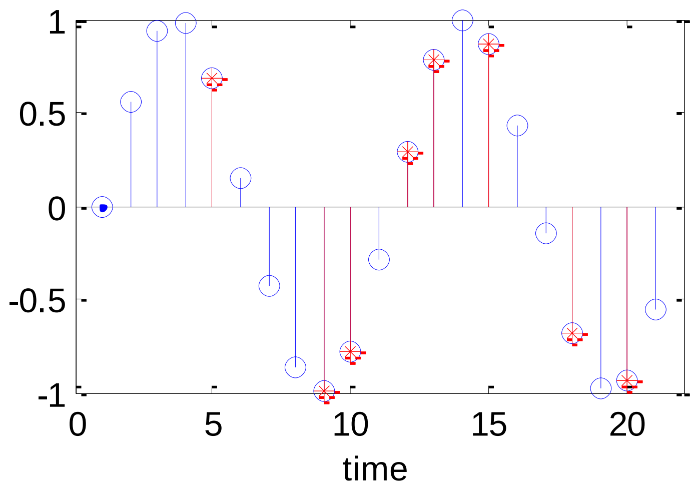
$$\mathbf{A} = \mathbf{\Phi\Psi}$$

obtained by using the **8** randomly chosen rows in $\mathbf{\Psi}$

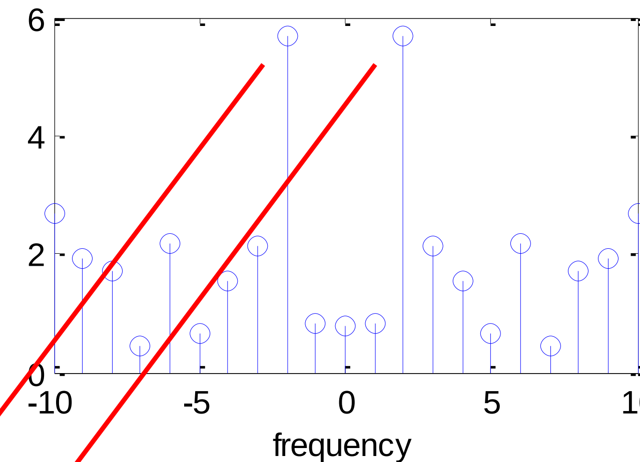
$$\mathbf{A}_{MxN} = \mathbf{\Phi\Psi} = \frac{1}{21}\begin{bmatrix} 1 & e^{4j\frac{2\pi}{21}} & e^{8j\frac{2\pi}{21}} & \dots & e^{80j\frac{2\pi}{21}} \\ 1 & e^{8j\frac{2\pi}{21}} & e^{16j\frac{2\pi}{21}} & \dots & e^{160j\frac{2\pi}{21}} \\ 1 & e^{9j\frac{2\pi}{21}} & e^{18j\frac{2\pi}{21}} & \dots & e^{180j\frac{2\pi}{21}} \\ 1 & e^{11j\frac{2\pi}{21}} & e^{22j\frac{2\pi}{21}} & \dots & e^{220j\frac{2\pi}{21}} \\ 1 & e^{12j\frac{2\pi}{21}} & e^{24j\frac{2\pi}{21}} & \dots & e^{240j\frac{2\pi}{21}} \\ 1 & e^{14j\frac{2\pi}{21}} & e^{28j\frac{2\pi}{21}} & \dots & e^{280j\frac{2\pi}{21}} \\ 1 & e^{17j\frac{2\pi}{21}} & e^{34j\frac{2\pi}{21}} & \dots & e^{340j\frac{2\pi}{21}} \\ 1 & e^{19j\frac{2\pi}{21}} & e^{38j\frac{2\pi}{21}} & \dots & e^{380j\frac{2\pi}{21}} \end{bmatrix}$$

$\mathbf{A}_\Omega$

- The system with 8 equations and 21 unknowns is obtained



Blue dots – missing samples
Red dots – available samples



The initial Fourier transform

Components are on the positions -2 and 2 (center-shifted spectrum), which corresponds to 19 and 2 in nonshifted spectrum

$$\mathbf{A}_\Omega = \frac{1}{21}\begin{bmatrix} e^{4j\frac{2\pi}{21}} & e^{72j\frac{2\pi}{21}} \\ e^{8j\frac{2\pi}{21}} & e^{144j\frac{2\pi}{21}} \\ e^{9j\frac{2\pi}{21}} & e^{162j\frac{2\pi}{21}} \\ e^{11j\frac{2\pi}{21}} & e^{198j\frac{2\pi}{21}} \\ e^{12j\frac{2\pi}{21}} & e^{216j\frac{2\pi}{21}} \\ e^{14j\frac{2\pi}{21}} & e^{253j\frac{2\pi}{21}} \\ e^{17j\frac{2\pi}{21}} & e^{306j\frac{2\pi}{21}} \\ e^{19j\frac{2\pi}{21}} & e^{342j\frac{2\pi}{21}} \end{bmatrix}$$

$\mathbf{A}_\Omega$ is obtained by taking the 2nd and the 19th column of $\mathbf{A}$

$$\Omega=\{2,19\}$$

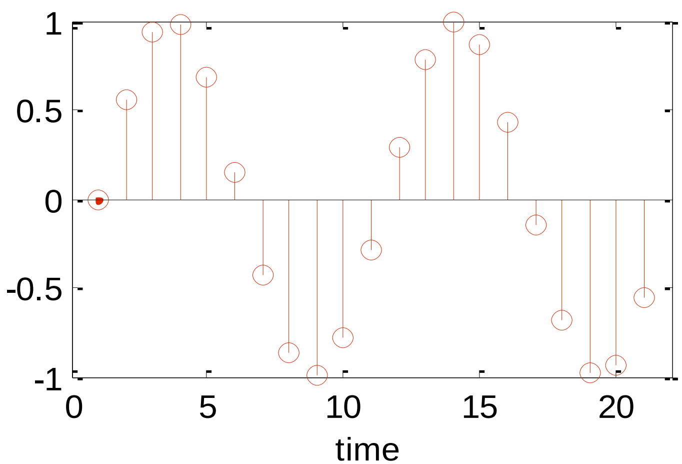
Least square solution

Problem formulation:

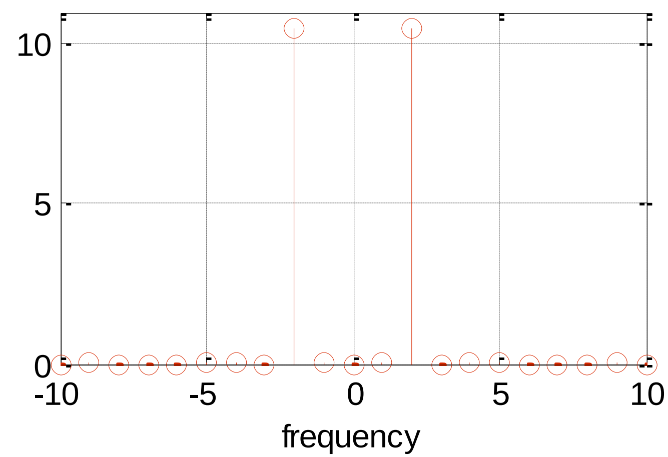
$$\mathbf{A}_{\Omega}\tilde{\mathbf{F}}_{x} = \mathbf{y} \quad \Longrightarrow \quad \mathbf{A}_{\Omega}^{T}\mathbf{A}_{\Omega}\tilde{\mathbf{F}}_{x} = \mathbf{A}_{\Omega}^{T}\mathbf{y}$$

$$\tilde{\mathbf{F}}_{x} = (\mathbf{A}_{\Omega}^{T}\mathbf{A}_{\Omega})^{-1}\mathbf{A}_{\Omega}^{T}\mathbf{y}$$

$$\mathbf{A}_{\Omega} = \frac{1}{21}\begin{bmatrix} e^{12j\frac{2\pi}{21}} & e^{216j\frac{2\pi}{21}} \\ e^{9j\frac{2\pi}{21}} & e^{162j\frac{2\pi}{21}} \\ e^{14j\frac{2\pi}{21}} & e^{252j\frac{2\pi}{21}} \\ e^{11j\frac{2\pi}{21}} & e^{198j\frac{2\pi}{21}} \\ e^{8j\frac{2\pi}{21}} & e^{144j\frac{2\pi}{21}} \\ e^{17j\frac{2\pi}{21}} & e^{306j\frac{2\pi}{21}} \\ e^{4j\frac{2\pi}{21}} & e^{72j\frac{2\pi}{21}} \\ e^{19j\frac{2\pi}{21}} & e^{342j\frac{2\pi}{21}} \end{bmatrix}$$

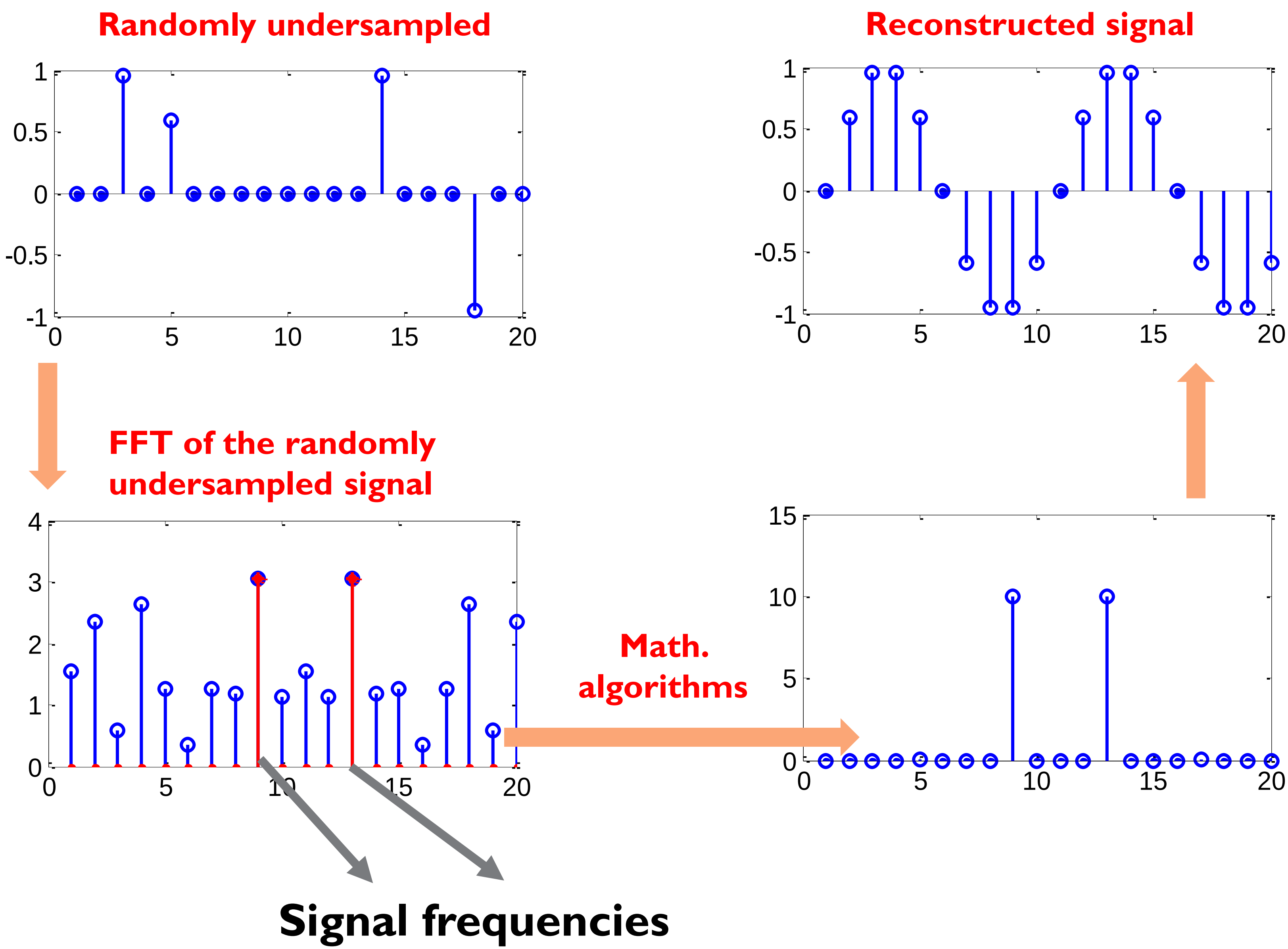


Reconstructed



Reconstructed

CS Applications



CS problem formulation

- The method of solving the undetermined system of equations $\mathbf{y}=\boldsymbol{\Phi}\boldsymbol{\Psi}\mathbf{x}=\mathbf{A}\mathbf{x}$, by searching for the sparsest solution can be described as:

$$\min \|\mathbf{x}\|_0 \quad subject\ to\ \mathbf{y}=\mathbf{A}\mathbf{x}$$

$\|\mathbf{x}\|_0$ $\quad l_0$ - norm

- We need to search over all possible sparse vectors **x** with *K* entries, where the subset of *K*-positions of entries are from the set {1,…,*N*}. The total number of possible *K*-position subsets is

$$\binom{N}{K}$$

CS problem formulation

- A more efficient approach uses the near optimal solution based on the l1-norm, defined as:

$$\min \|\mathbf{x}\|_1 \ \ subject\ to\ \mathbf{y} = \mathbf{A}\mathbf{x}$$

- In real applications, we deal with noisy signals.
- Thus, the previous relation should be modified to include the influence of noise:

$$\min \|\mathbf{x}\|_1 \ \ subject\ to\ \|\mathbf{y} - \mathbf{A}\mathbf{x}\|_2 \leq \varepsilon$$

$$\|\mathbf{e}\|_2 = \varepsilon$$

L2-norm cannot be used because the minimization problem solution in this case is reduced to minimum energy solution, which means that all missing samples are zeros

CS conditions

- CS relies on the following conditions:

Sparsity – related to the signal nature;

- Signal needs to have concise representation when expressed in a proper basis (*K*<<*N*)

Incoherence – related to the sensing modality; It should provide a linearly independent measurements (matrix rows)
Random undersampling is crucial

Restricted **I**sometry **P**roperty – is important for preserving signal isometry by selecting an appropriate transformation

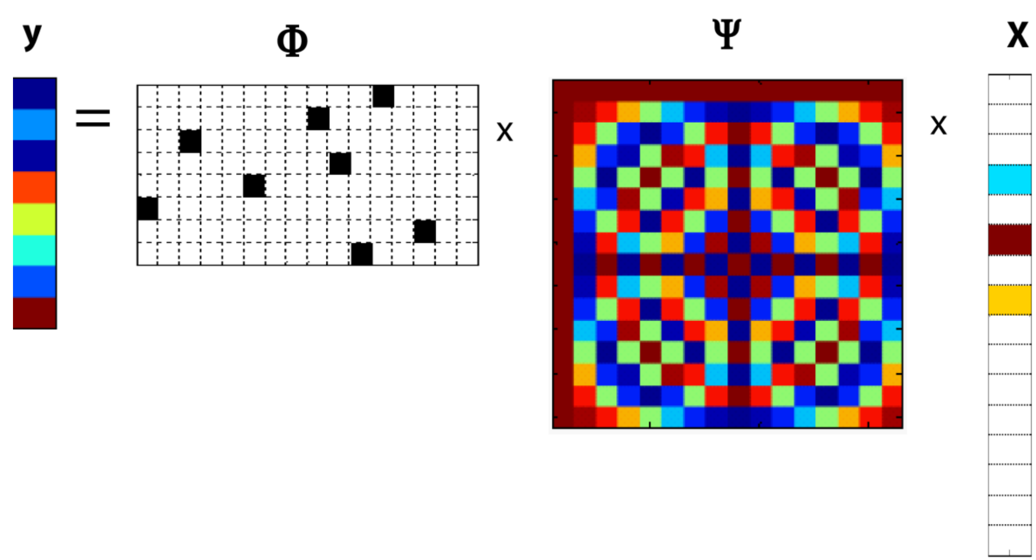
Summary of CS problem formulation

Signal **f** linear combination of the orthonormal basis vectors

$$f(t)=\sum_{i=1}^{N} x_i \psi_i(t), \quad or: \quad \mathbf{f}=\mathbf{\Psi x}.$$

Set of random measurements: $\rightarrow$ $\mathbf{y}=\mathbf{\Phi f}$

random measurement matrix **transform matrix** **transform domain vector**



CS conditions

- Restricted isometry property
  - Successful reconstruction for a **wider range of sparsity** level
  - Matrix **A** satisfies **Isometry Property** if it preserves the vector intensity in the *N*-dimensional space:

$$\|\mathbf{Ax}\|_2^2 = \|\mathbf{x}\|_2^2$$

  - If **A** is a full Fourier transform matrix, i.e. :

$$\mathbf{A} = N\mathbf{\Psi}$$

$$\frac{N\|\mathbf{\Psi x}\|_2^2 - \|\mathbf{x}\|_2^2}{\|\mathbf{x}\|_2^2} = 1$$

CS conditions

- RIP

- For each integer number K the isometry constant δ_K of the matrix **A** is the smallest number for which the relation holds:

$$(1-\delta_K)\|\mathbf{x}\|_2^2 \leq \|\mathbf{Ax}\|_2^2 \leq (1+\delta_K)\|\mathbf{x}\|_2^2$$

$$\left|\frac{\|\mathbf{Ax}\|_2^2 - \|\mathbf{x}\|_2^2}{\|\mathbf{x}\|_2^2}\right| \leq \delta_K$$

$0<\delta_K<1$ - **restricted isometry constant**

CS conditions

- Matrix **A** satisfies RIP → the **Euclidian length** of sparse vectors is preserved

- For the RIP matrix **A** with $(2K, \delta_K)$ and $\delta_K < 1$, all subsets of $2K$ columns are **linearly independent**

⬇

$$spark(\mathbf{A}) > 2K$$

spark - the smallest number of dependent columns

CS conditions

A (MXN) →

$$2 \le spark(\mathbf{A}) \le M+1$$

$spark(\mathbf{A}) = 1$ - one of the columns has all zero values

$spark(\mathbf{A}) = M+1$ - no dependent columns

↓

$$K < \frac{1}{2} spark(\mathbf{A}) \le \frac{1}{2}(M+1)$$

the number of measurements should be at least twice the number of components *K*:

$$M \ge 2K$$

Incoherence

- Signals sparse in the transform domain Ψ, should be dense in the domain where the acquisition is performed
- Number of nonzero samples in the transform domain Ψ and the number of measurements (required to reconstruct the signal) depends on the **coherence** between the matrices Ψ and $\mathbf{\Phi}$.
- Ψ and $\mathbf{\Phi}$ are **maximally coherent** - **all** coefficients would be required for signal reconstruction

Mutual coherence: the maximal absolute value of correlation between two elements from Ψ and $\mathbf{\Phi}$ →

$$\mu(\Phi,\Psi)=\max_{i\neq j}\left|\frac{\langle \phi_i,\psi_j\rangle}{\|\phi_i\|^2\,\|\psi_j\|^2}\right|$$

Incoherence

Mutual coherence:

$$\mu(\mathbf{A}) = \max_{i \neq j, 1 \leq i, j \leq M} \left| \frac{\langle A_i, A_j \rangle}{\|A_i\|^2 \|A_j\|^2} \right|, \quad \mathbf{A} = \mathbf{\Phi\Psi}$$

maximum absolute value of normalized inner product between all columns in A

$\boldsymbol{A_i}$ and $\boldsymbol{A_j}$ - columns of matrix **A**

- The maximal mutual coherence will have the value **1** in the case when certain pair of columns coincides
- If the number of measurements is: $M \geq C \cdot K \cdot \mu(\mathbf{\Phi}, \mathbf{\Psi}) \cdot \log N$

then the sparsest solution is exact with a high probability (*C* is a constant)

Reconstruction approaches

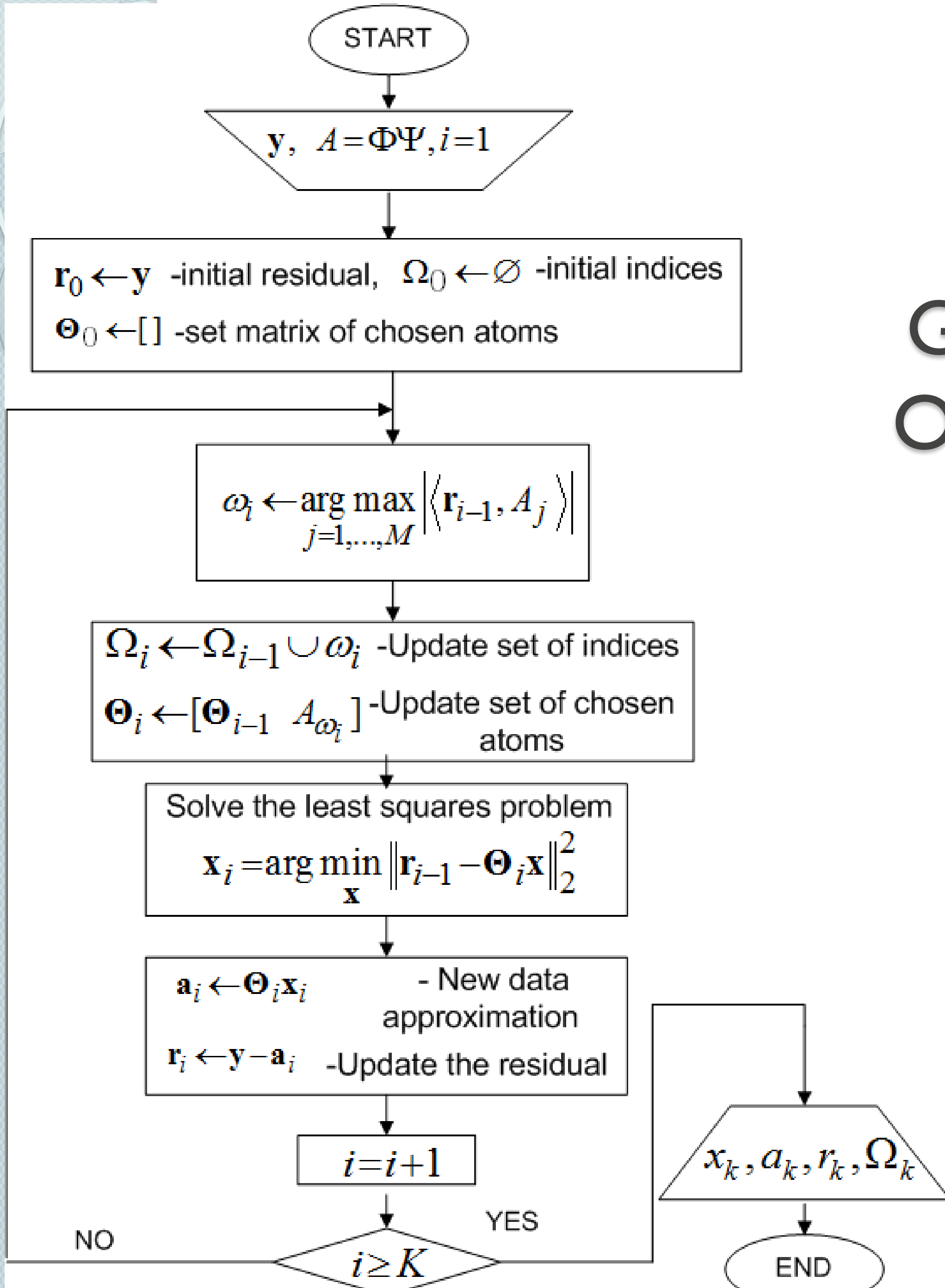
- The main challenge of the CS reconstruction: solving an **underdetermined system of linear equations** using sparsity assumption

ℓ_1 - optimization, based on linear programming methods, provide efficient signal reconstruction with high accuracy

- Linear programming techniques (e.g. convex optimization) may require a **vast number of iterations** in practical applications

- **Greedy and threshold based algorithms** are fast solutions, but in general less stable

Greedy algorithms – Orthogonal Matching Pursuit (OMP)



Ψ – *Transform matrix*

Φ – *Measurement matrix*

$y = \Phi f$ – *Measurement vector*

Influence of missing samples to the spectral representation

- **Missing samples produce noise in the spectral domain. The variance of noise in the DFT case depends on M, N and amplitudes *Ai*:**

$$\sigma^2_{MS} = \mathrm{var}\{F_{k \neq k_i}\} = M\frac{N-M}{N-1}\sum_{i=1}^{K} A_i^2$$

- **The probability that all (*N-K*) non-signal components are below a certain threshold value defined by T is (only K signal components are above T):**

$$P(T) = \left(1 - \exp\left(-\frac{T^2}{\sigma_{MS}^2}\right)\right)^{N-K}$$

Consequently, for a fixed value of *P*(*T*) (e.g. *P*(*T*)=0.99), threshold is calculated as:

$$T = \sqrt{-\sigma_{MS}^2 \log\left(1 - P(T)^{\frac{1}{N-K}}\right)} \approx \sqrt{-\sigma_{MS}^2 \log\left(1 - P(T)^{\frac{1}{N}}\right)}$$

When ALL signal components are above the noise level in DFT, the reconstruction is done using a Single-Iteration Reconstruction algorithm using threshold T

Optimal number of available samples M

- **How can we determine the number of available samples *M*, which will ensure detection of all signal components?**
- Assuming that the DFT of the *i*-th signal component (with the lowest amplitude) is equal to Ma_i, then the approximate expression for the probability of error is obtained as:

$$P_{err} = 1 - P_i \cong 1 - \left(1 - \exp\left(-\frac{M^2 a_i^2}{\sigma_{MS}^2}\right)\right)^{N-K}$$

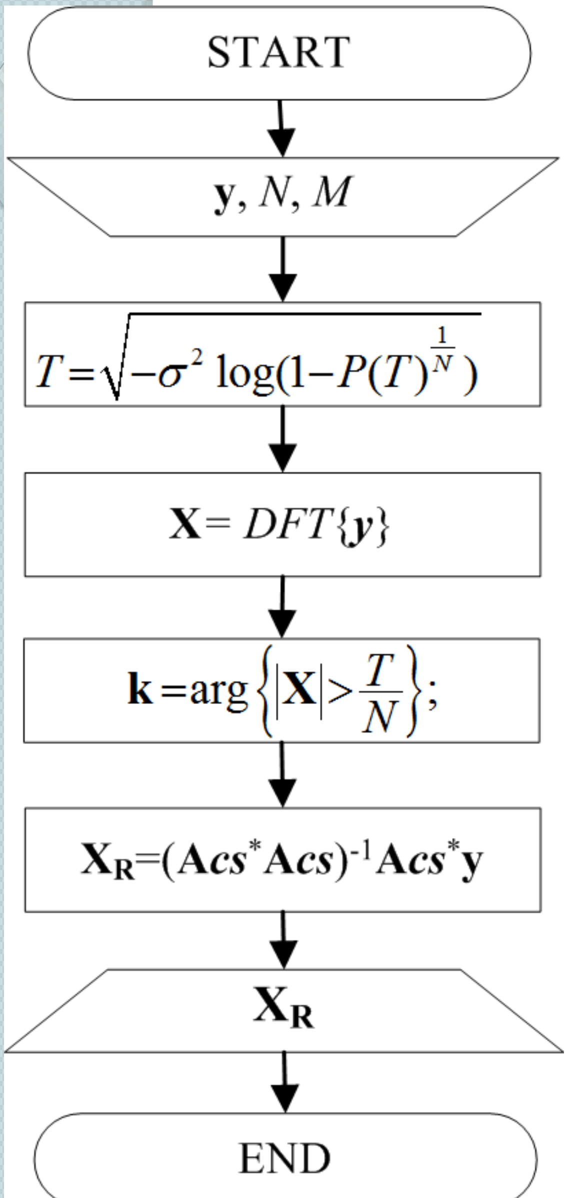
- For a fixed Perr, the optimal value of M (that allows to detect all signal components) can be obtained as a solution of the minimization problem:

$$M_{opt} \geq \arg\min_{M}\{P_{err}\}$$

For chosen value of P_{err} and expected value of minimal amplitude a_i, there is an optimal value of M that will assure components detection.

Algorithms for CS reconstruction of sparse signals

Single-Iteration Reconstruction Algorithm in DFT domain



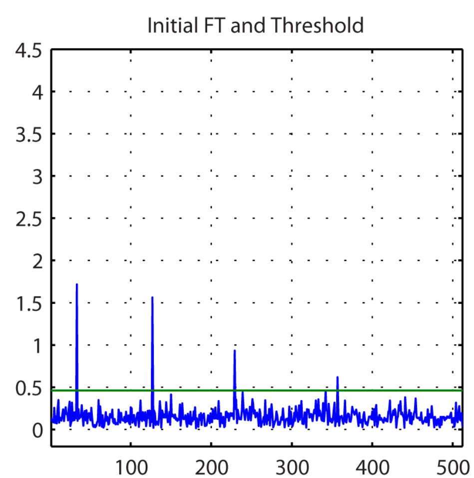
y – measurements

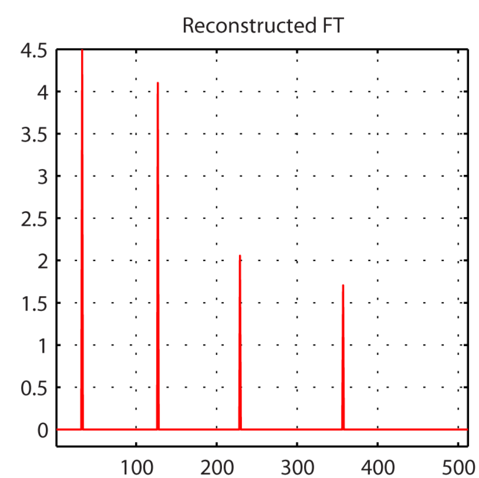
M - number of measurements

N – signal length

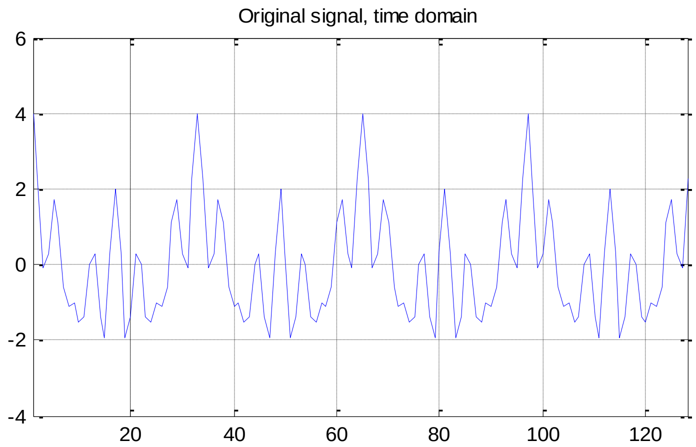
T - Threshold

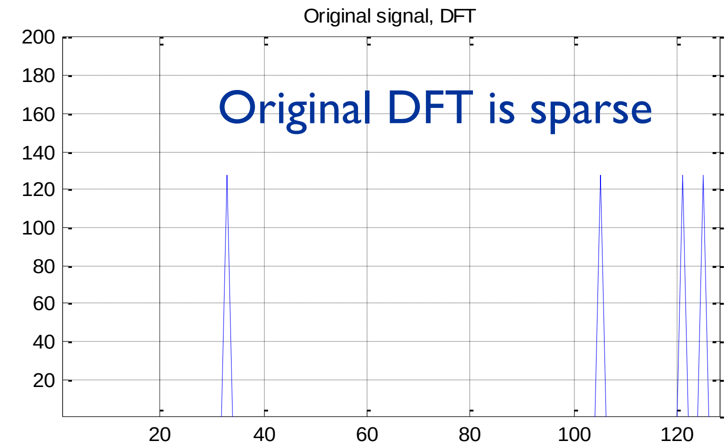
- **DFT domain is assumed as sparsity domain**
- **Apply threshold to initial DFT components (determine the frequency support)**
- **Perform reconstruction using identified support**

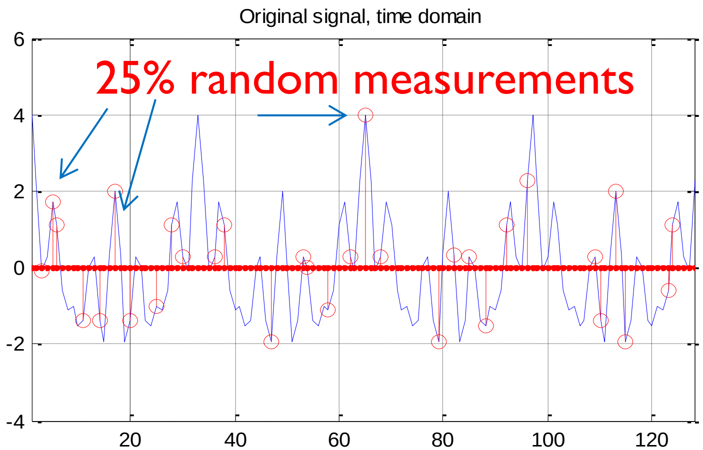


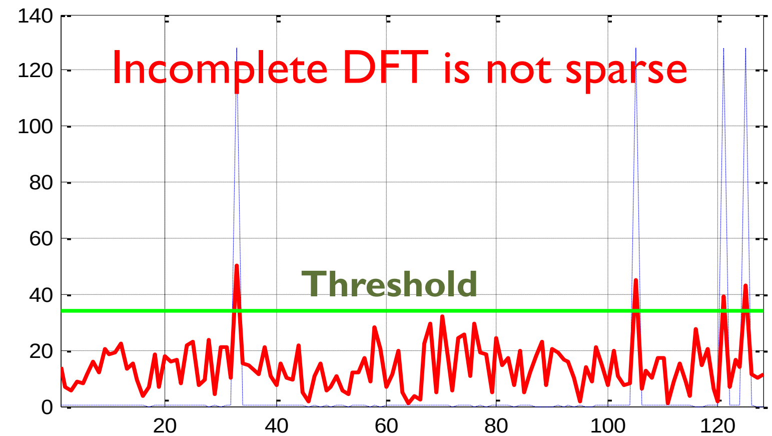


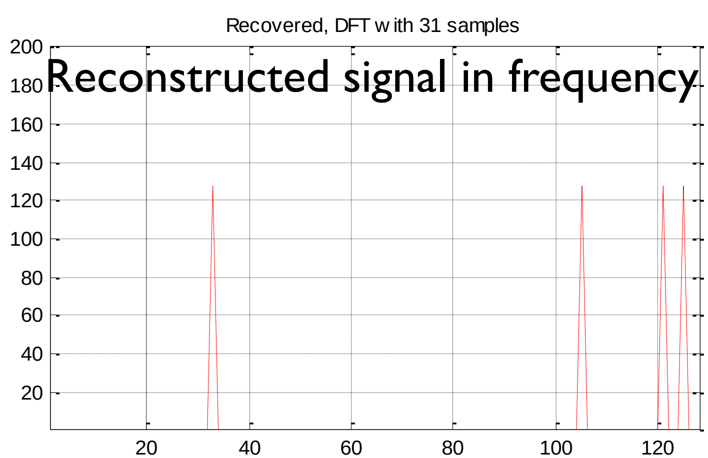
Example I: Single iteration

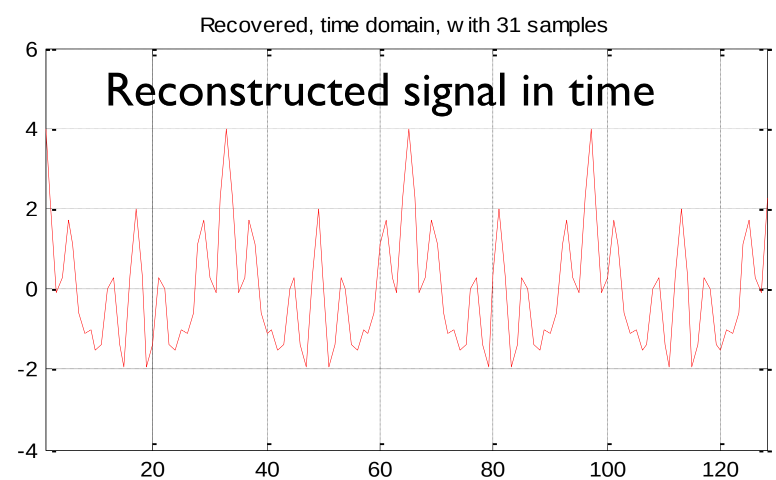






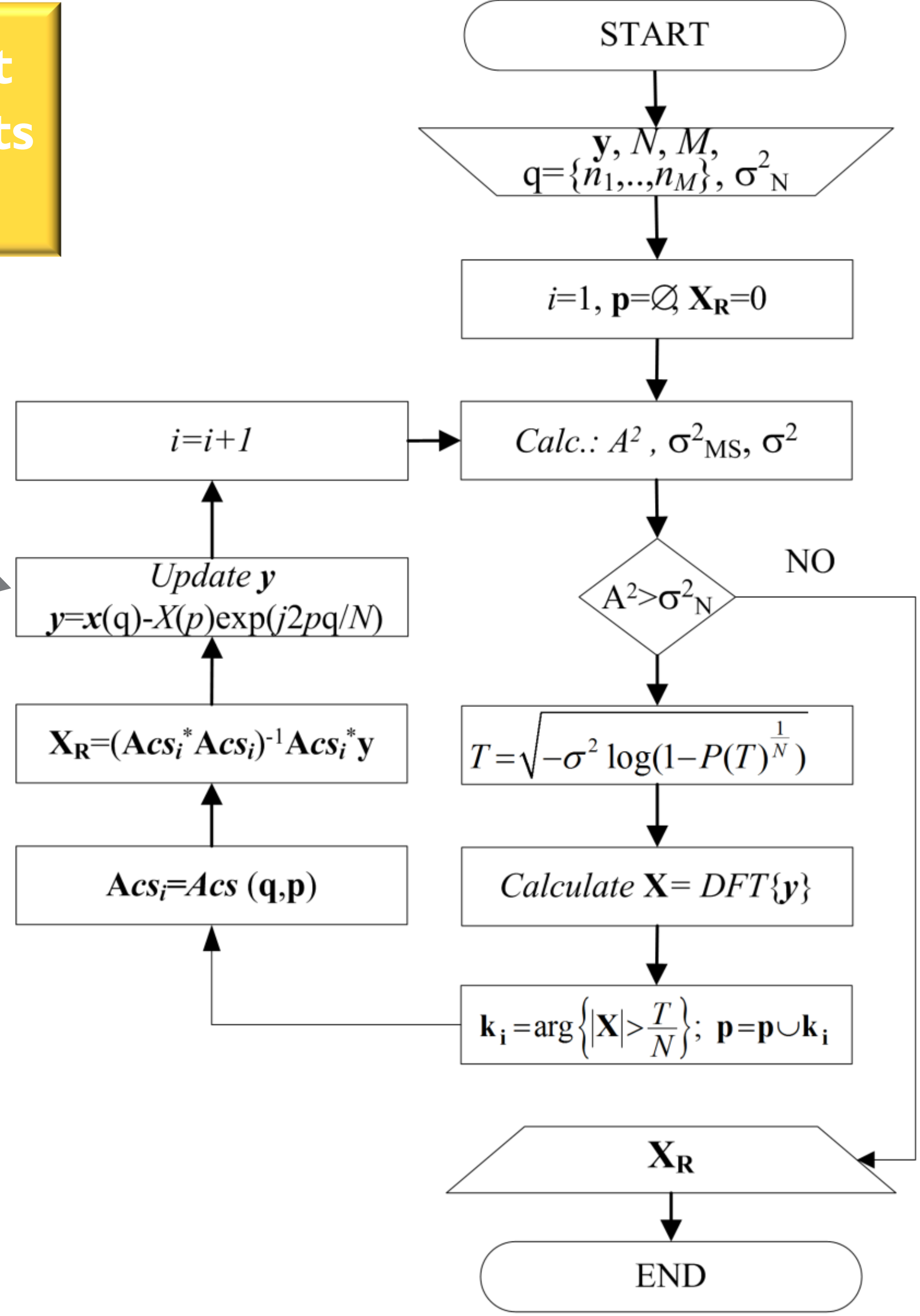






Case 2: Threshold cannot select desired components – iterative solution

In each iteration we need **to remove the influence of previously detected components** and to **update** the value of threshold

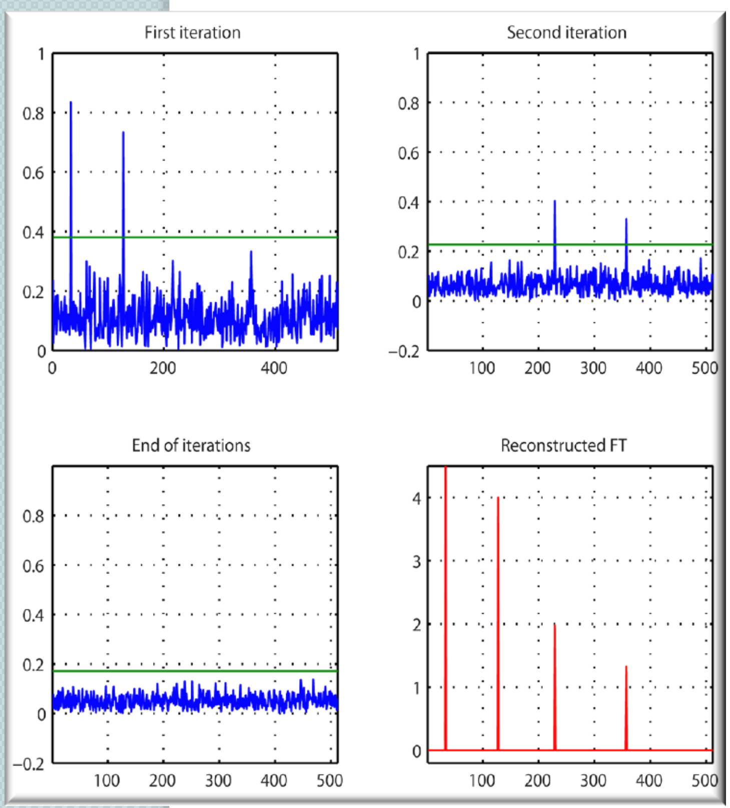


Case 3: External noise

- **External noise + noise caused by missing samples**

$$\sigma^2 = \sigma_{MS}^2 + M\sigma_N^2 = M\frac{N-M}{N-1}\sum_{i=1}^{K} A_i^2 + M\sigma_N^2$$

$$T = \sqrt{-\sigma^2 \log(1-P(T)^{\frac{1}{N}})}$$



- **To ensure the same probability of error as in the noiseless case we need to increase the number of measurements M such that:**

$$\frac{\sigma_{MS}^2}{\sigma^2} = \frac{M\frac{N-M}{N-1}(A_1^2 + A_2^2 + \ldots + A_K^2)}{M_N\frac{N-M_N}{N-1}(A_1^2 + A_2^2 + \ldots + A_K^2) + M_N\sigma_N^2} = 1$$

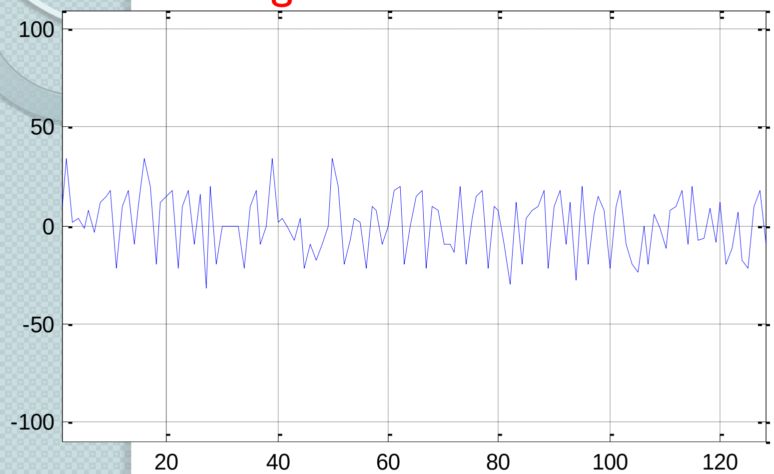
$$\frac{M(N-M)}{M_N}\frac{SNR}{SNR(N-M_N)+(N-1)} = 1$$

Solve the equation:

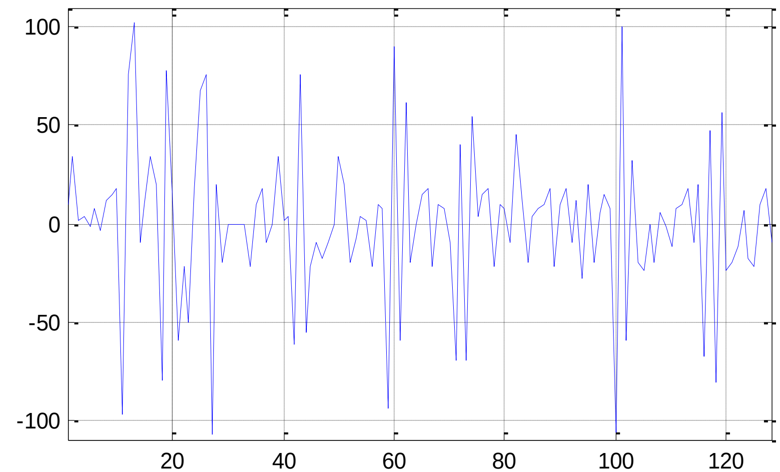
$$M_N^{\,2}\cdot SNR - M_N(SNR\cdot N + N - 1) + SNR\cdot(MN - M^2) = 0$$

Dealing with a set of noisy data – L-estimation approach

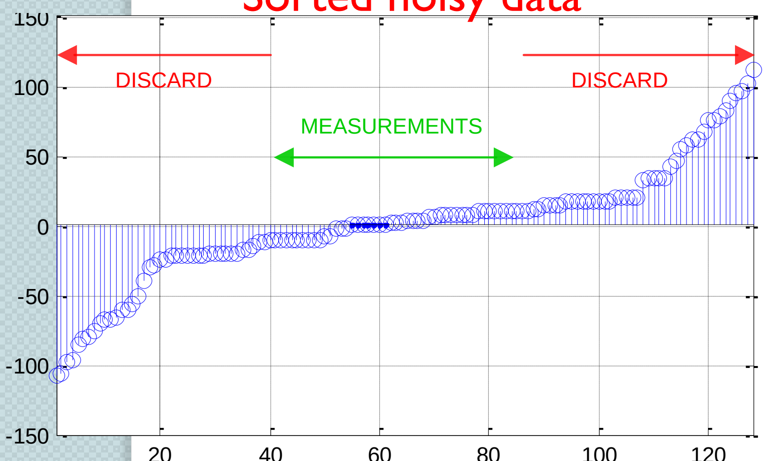
Original data- DESIRED



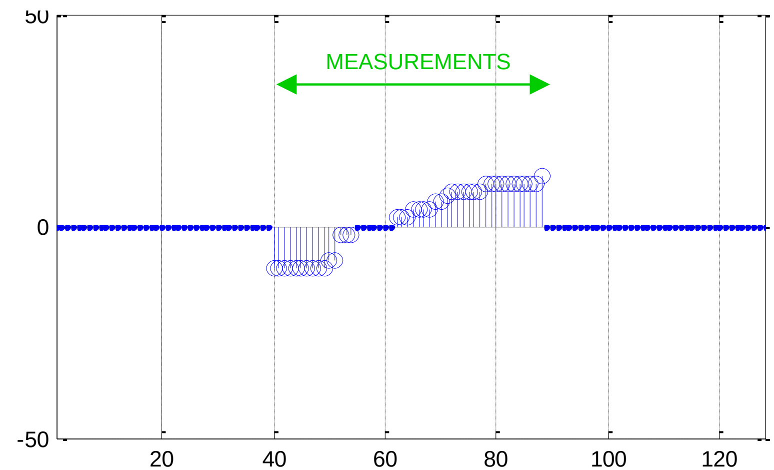
Noisy data - AVAILABLE



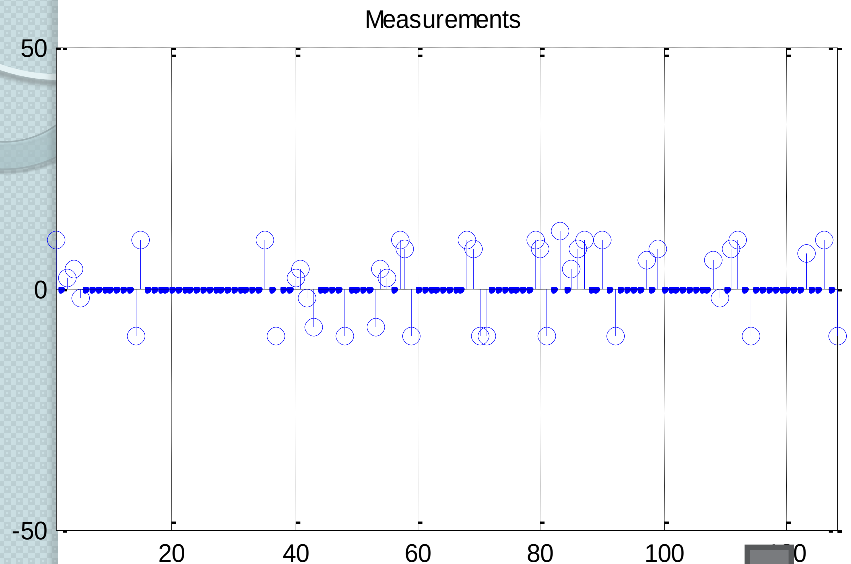
Sorted noisy data



Denoised data

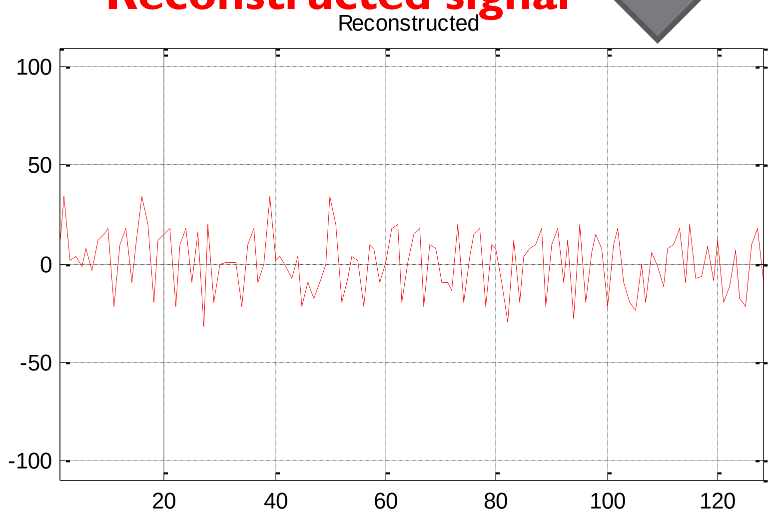


Dealing with a set of noisy data – L-estimation approach

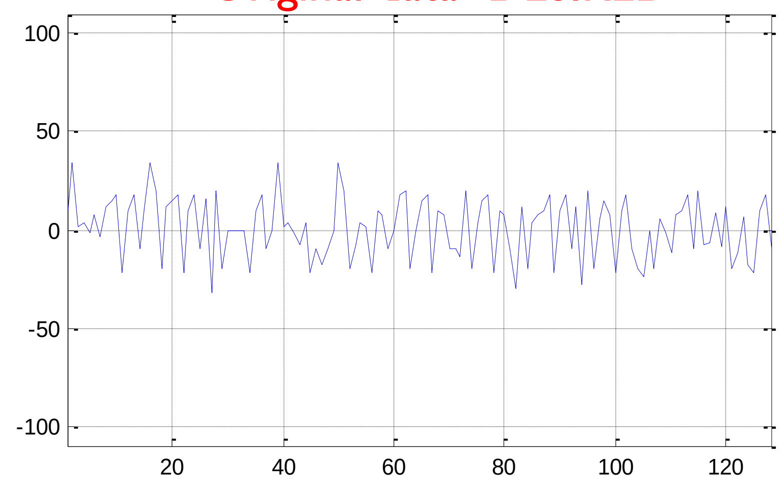


We end up with a random incomplete set of samples that need to be recovered

Reconstructed signal



Original data- DESIRED



General deviations-based approach

$x(n)$-K-sparse in DFT domain

- N_{avail} – positions of the available samples
- M-number of available samples

$$F\{e(n,k)\} = F\{\left|x(n)e^{-j2\pi kn/N} - X(k)\right|\}$$

Loss function

$$F\{e\} = \begin{cases} |e|^2 - \text{standard form} \\ |e| - \text{robust form} \\ |e|^L - \text{general form} \end{cases}$$

Robust statistics

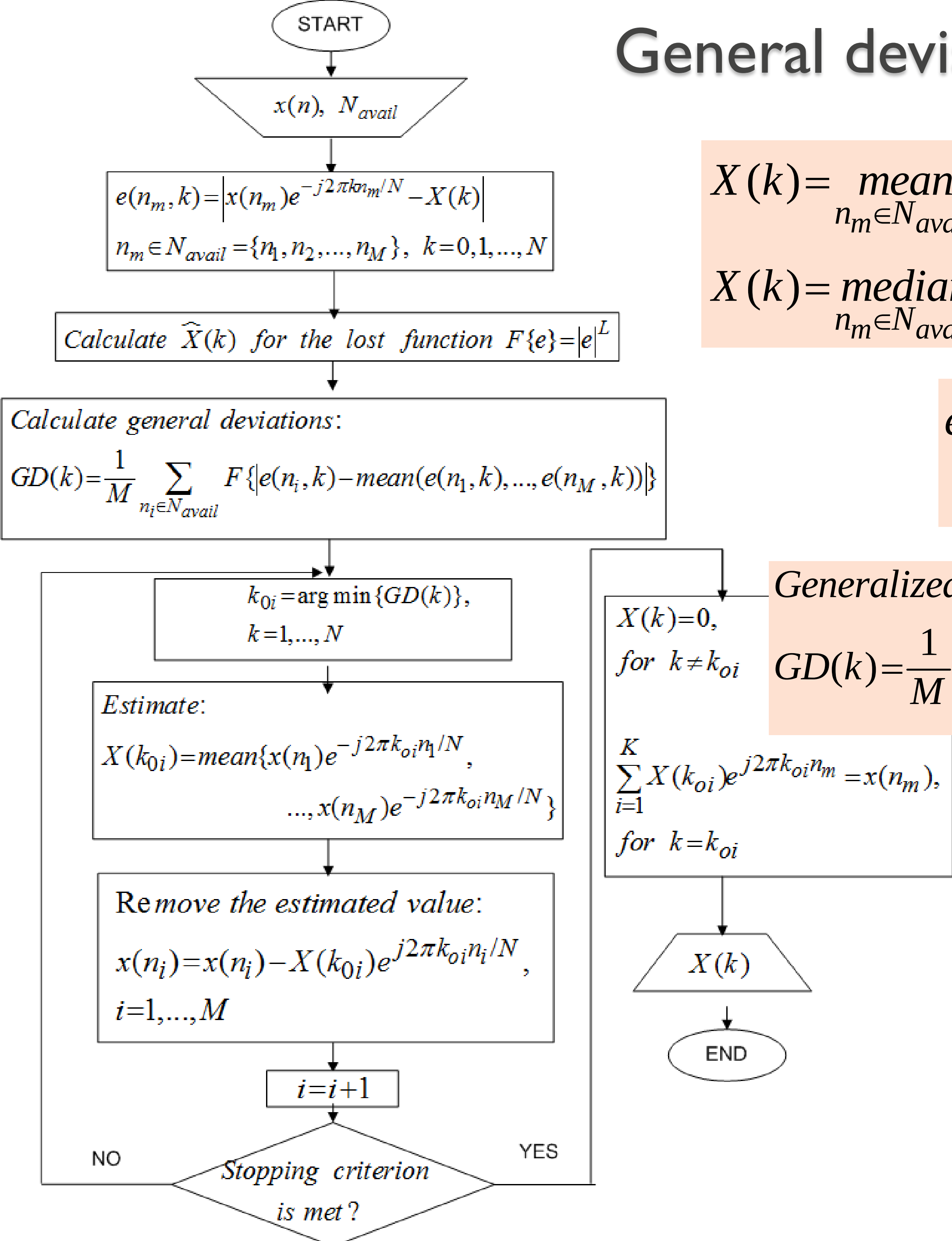
$$X(k) = \underset{n_m \in N_{avail}}{mean}\{x(n_1)e^{-j2\pi kn_1/N}, \ldots, x(n_M)e^{-j2\pi kn_M/N}\}$$

$$X(k) = \underset{n_m \in N_{avail}}{median}\{x(n_1)e^{-j2\pi kn_1/N}, \ldots, x(n_M)e^{-j2\pi kn_M/N}\}$$

An incomplete set of samples causes random deviations of the DFT outside the signal frequencies.

The DFT values at the frequencies corresponding to signal components are characterized by non-random behavior: the sum of generalized deviations of the values at non-signal frequencies is constant and higher than at the signal components positions.

General deviations-based approach



$$X(k)=\underset{n_m\in N_{avail}}{mean}\{x(n_1)e^{-j2\pi kn_1/N},\ldots,x(n_M)e^{-j2\pi kn_M/N}\}$$

$$X(k)=\underset{n_m\in N_{avail}}{median}\{x(n_1)e^{-j2\pi kn_1/N},\ldots,x(n_M)e^{-j2\pi kn_M/N}\}$$

$$e(n_m,k)=\left|x(n_m)e^{-j2\pi kn/N}-X(k)\right|$$

for all available samples

Generalized deviations:

$$GD(k)=\frac{1}{M}\sum_{n_i\in N_{avail}}F\ \{|e(n_i,k)-mean\left(e(n_1,k),\ldots,e(n_M,k)\right)|\}$$

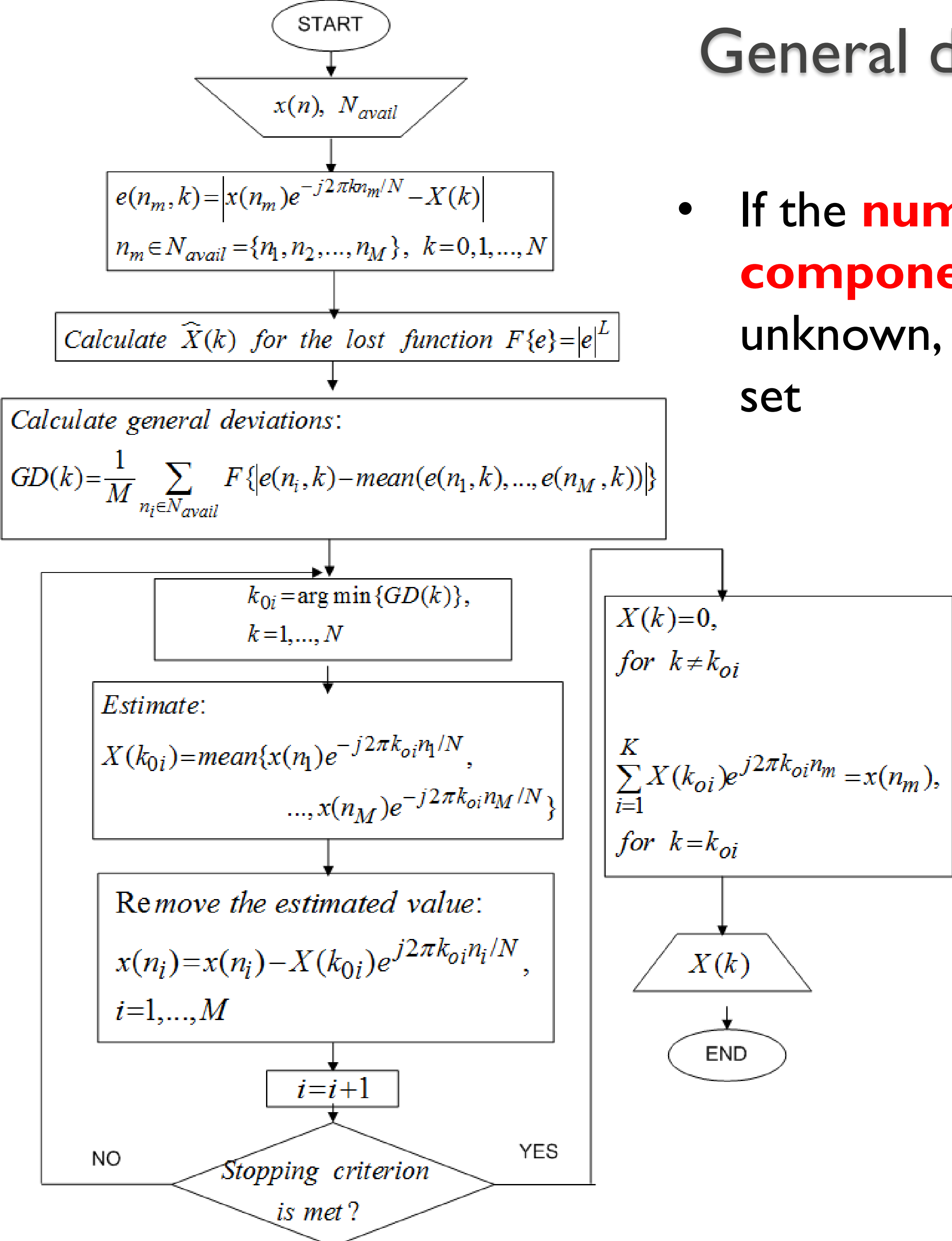
k_p – signal frequency

$$GD(k_p)=\frac{M(N-M)}{N-1}\sum_{i=1,i\neq p}^{K}A_i^L$$

$$GD(k_q)=\frac{M(N-M)}{N-1}\sum_{i=1}^{K}A_i^L\approx\text{const}$$

k_q – non-signal frequency

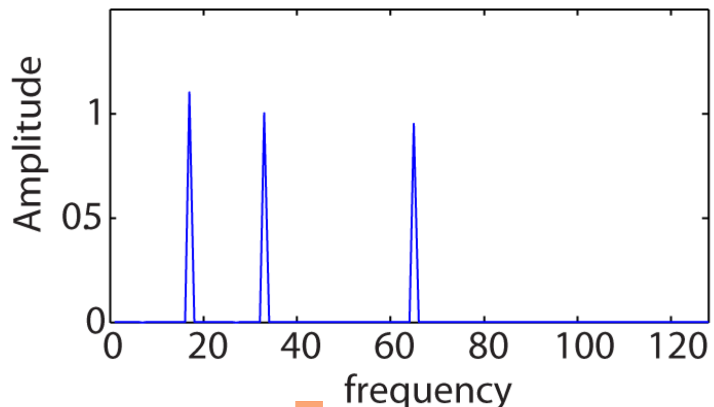
General deviations-based approach



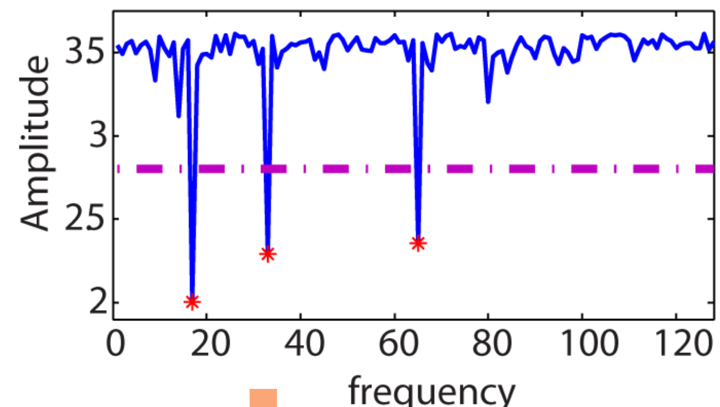
- If the number of components/number of iterations is unknown, the **stopping criterion** can be set

- **Stopping criterion:** Adjusted based on the l2-norm bounded residue that remains after removing previously detected components

General deviations-based approach
Example

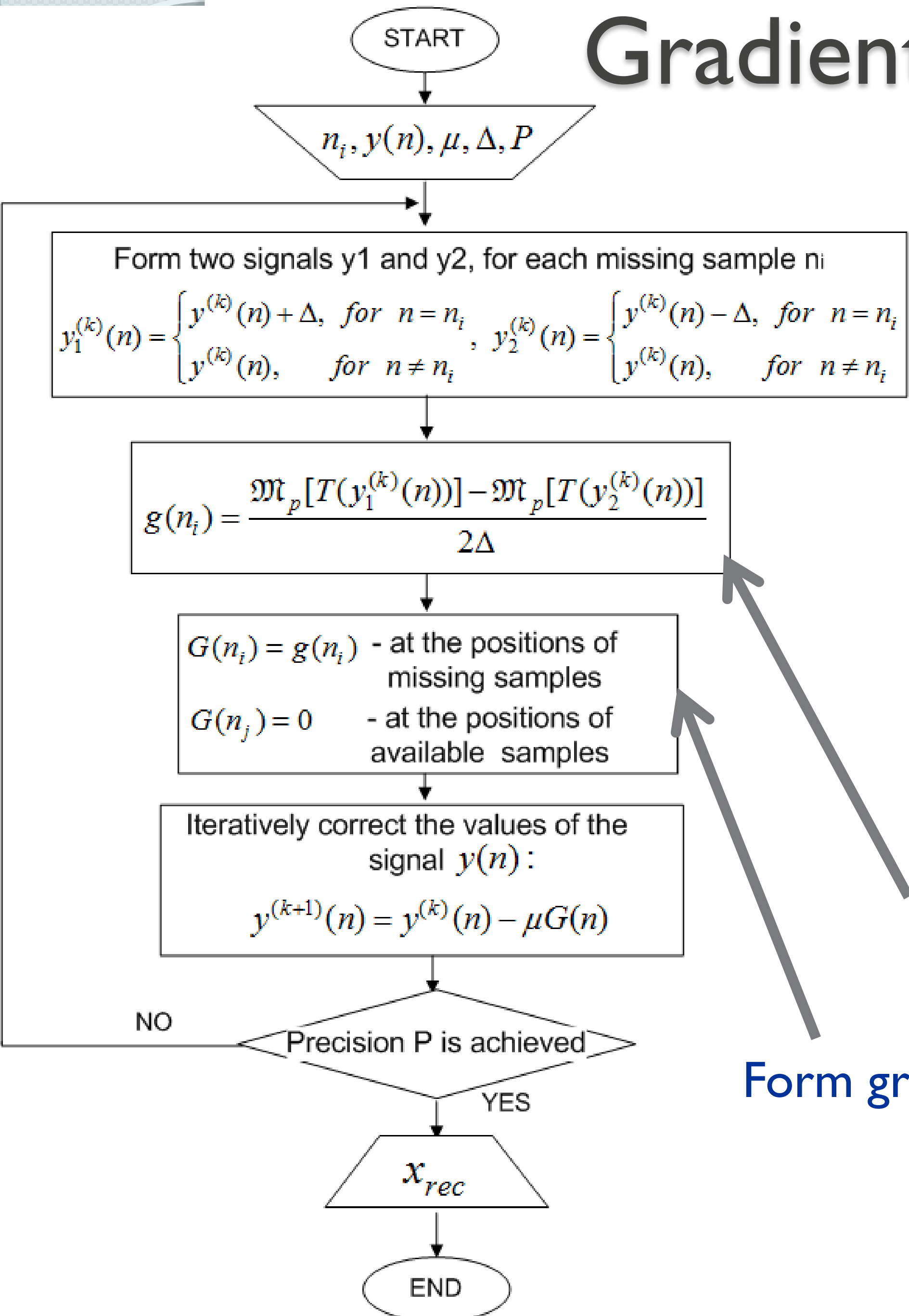


Fourier transform of original signal



Variances at signal (marked by red symbols) and non-signal positions

Gradient algorithm



n_i - missing samples positions

n_j - Available samples positions

$y(n)$ - Available signal samples

Δ - Constant; determines whether sample should be decreased or increased

μ - Constant that affect algorithm performance

P - Precision

Estimate the differential of the signal transform measure

Form gradient vector G

$$\mathfrak{M}_p[T(x(n))] = \frac{1}{N}\sum_k |X(k)|^{1/p},$$

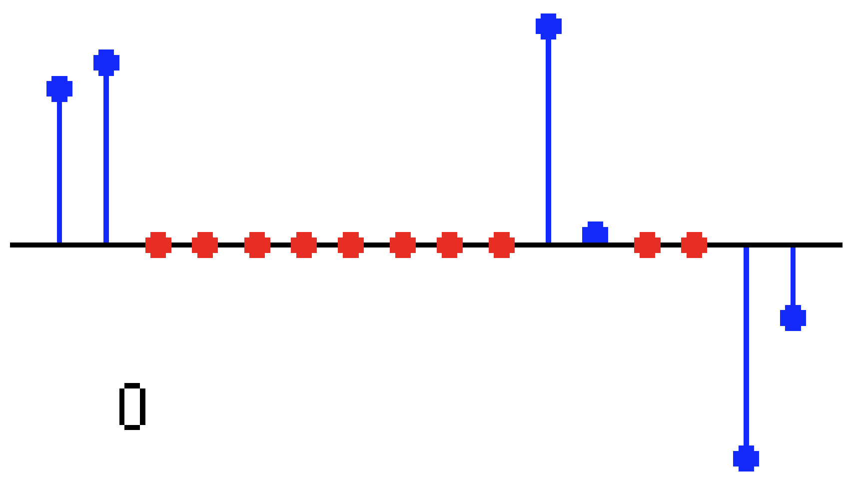
$$1 \leq p < \infty$$

Gradient algorithm - Example

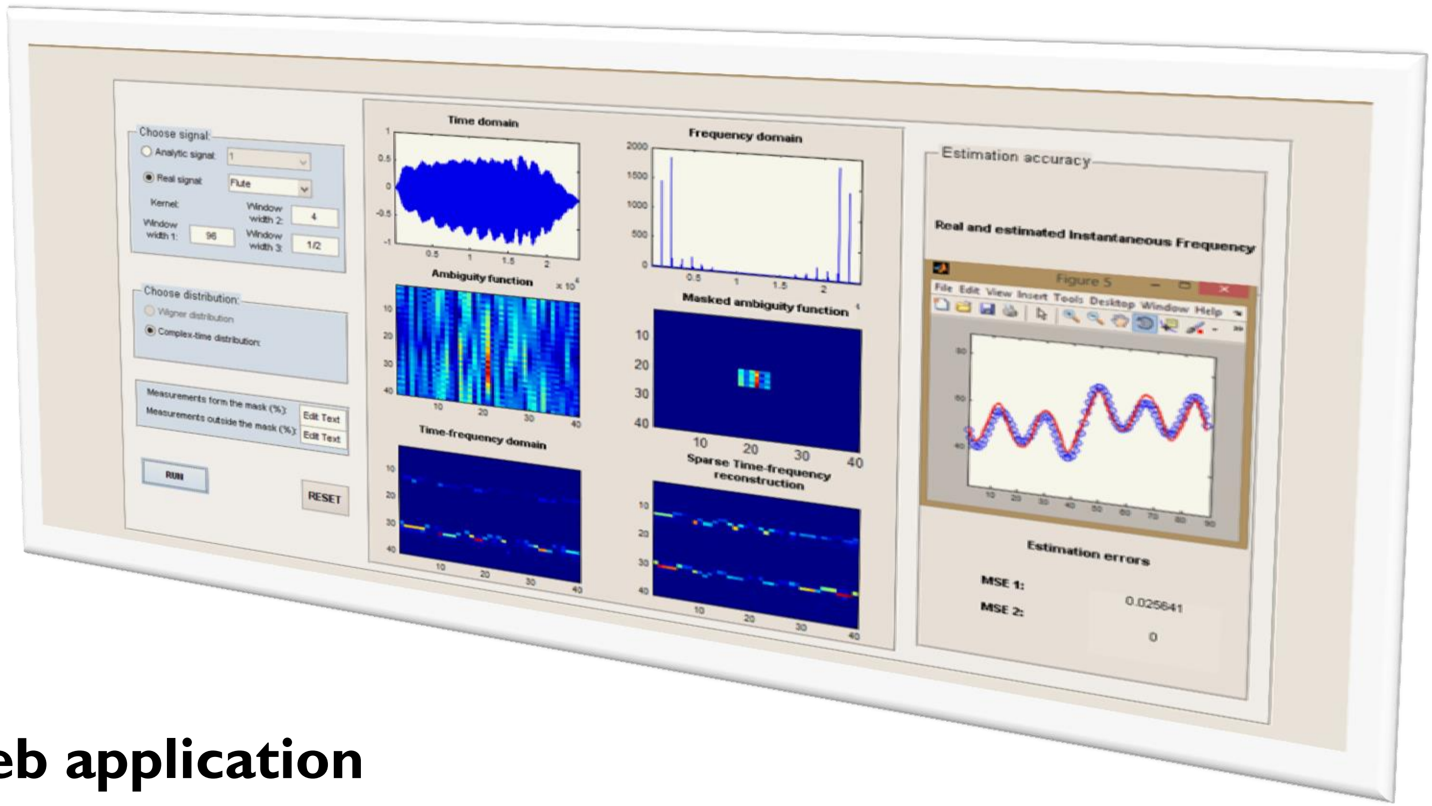
Signal contains 16 samples

Missing – 10 samples (marked with red)

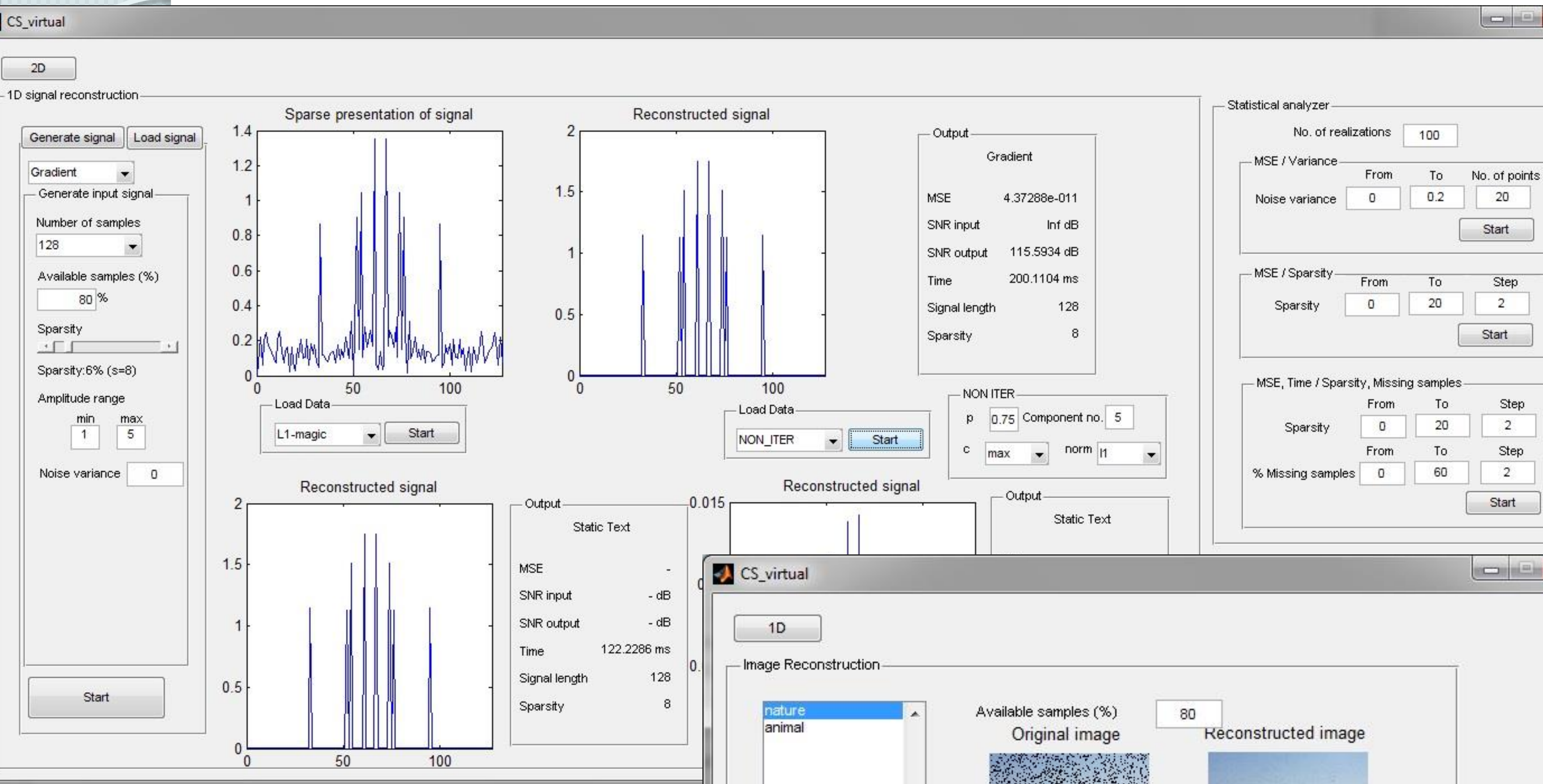
Signal is iteratively reconstructed using **Gradient algorithm**

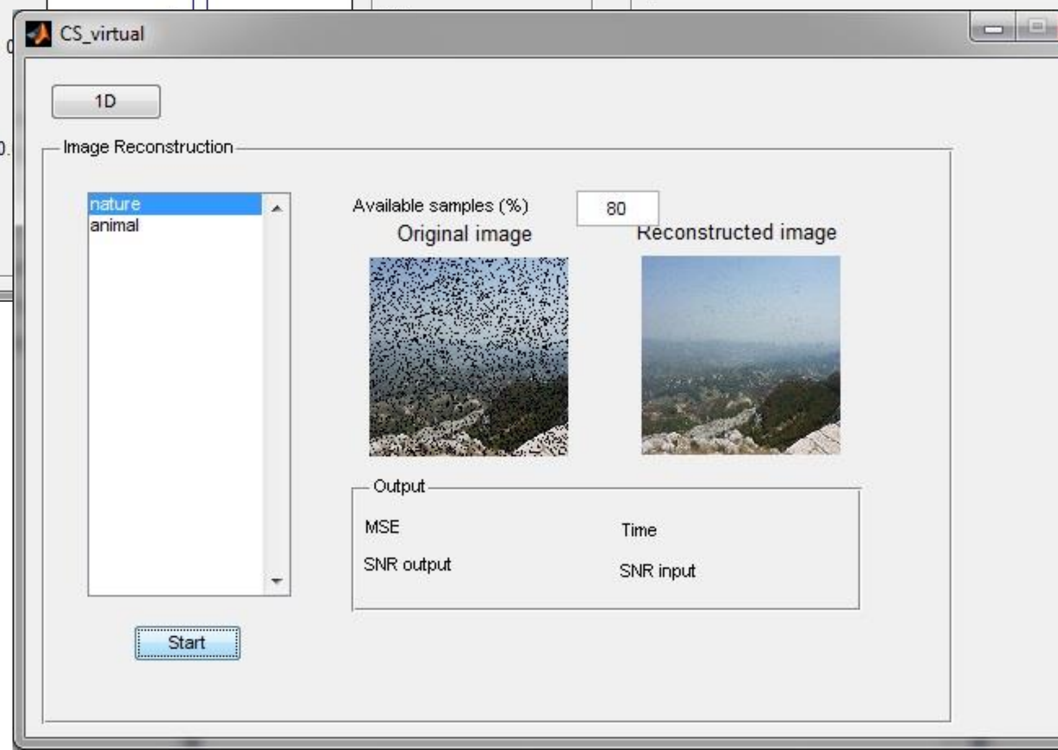


Some Developments



Web application

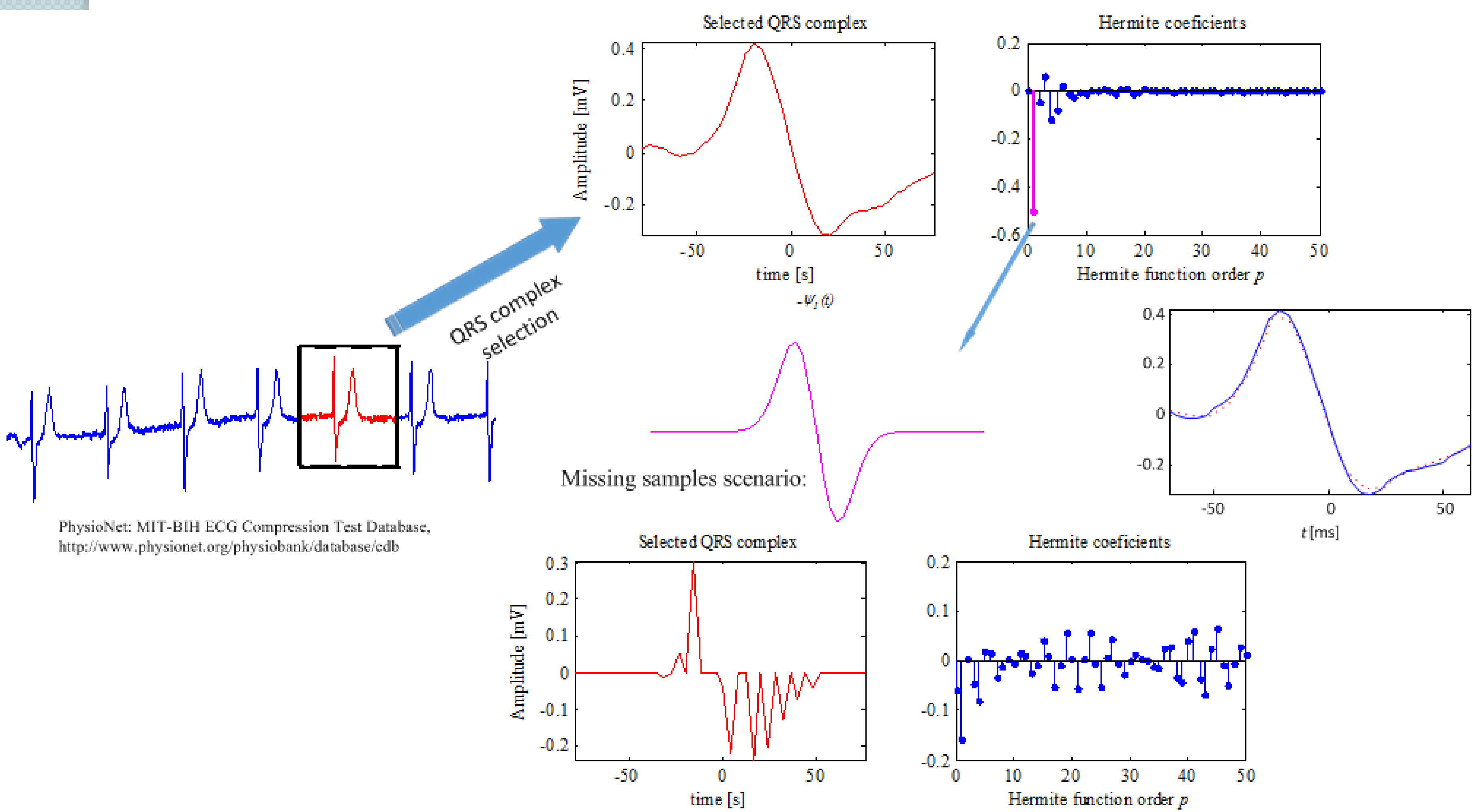




Virtual instrument for Compressive sensing

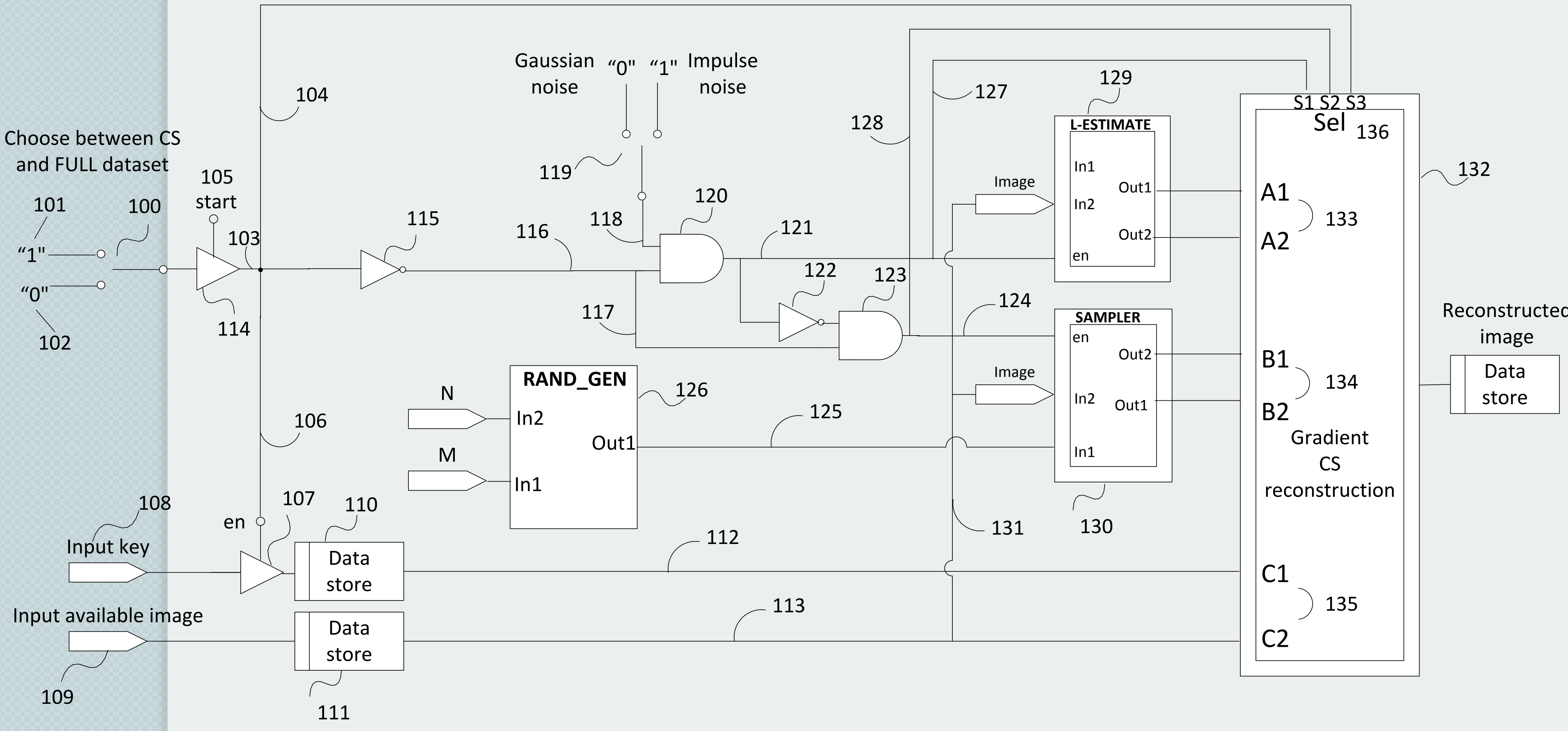
EEG signals: QRS complex is sparse in Hermite transform domain, meaning that it can be represented using just a few Hermite functions and corresponding coeffs.

CS of QRS complexes in the Hermite transform domain

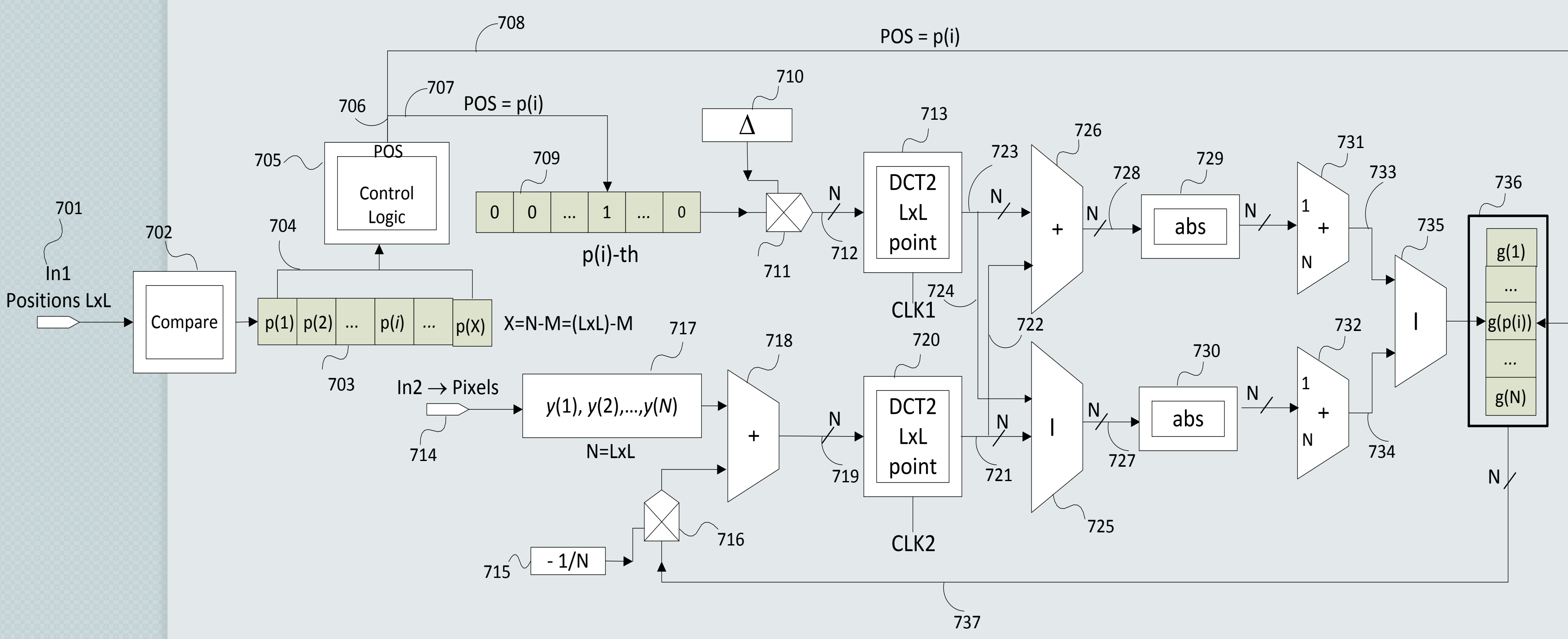


PhysioNet: MIT-BIH ECG Compression Test Database,
http://www.physionet.org/physiobank/database/cdb

Compressive sensing based image filtering and reconstruction



Realization of the adaptive gradient-based image reconstruction algorithm



THANK YOU

FOR YOUR ATTENTION!