

Vježba 1. Obnavljanje

1. Riješiti nejednačinu:

$$\frac{x-1}{(x+2)(3-x)} < 0$$

	$-\infty$	-2	1	3	$+\infty$
$x-1$		-	-	+	+
$x+2$	-	-	+	+	+
$3-x$	+	+	+	-	-
	+	-	+	-	-

$$\begin{aligned} x-1 &< 0 \\ x &< 1 \\ x+2 &< 0 \\ x &< -2 \\ 3-x &< 0 \\ -x &< -3 \quad | \cdot (-1) \\ x &> 3 \end{aligned}$$

$$x \in (-2, 1) \cup (3, +\infty)$$

$$D: (-\infty, -2) \cup (-2, 3) \cup (3, +\infty)$$

2. $\frac{x+3}{x^2-5x+6} \geq 0$

$$x^2-5x+6=0$$

$$x_{1,2} = \frac{5 \pm \sqrt{25-24}}{2}$$

$$x_{1,2} = \frac{5 \pm 1}{2}$$

$$x_1 = 3 \quad x_2 = 2$$

$$x^2-5x+6 = (x-3)(x-2)$$

$$\frac{x+3}{(x-3)(x-2)} \geq 0$$

	-3	2	3	$+\infty$
$x+3$	-	+	+	+
$x-3$	-	-	-	+
$x-2$	-	-	+	+
R	-	+	-	+

$$x \in [-3, 2) \cup (3, +\infty)$$

$$\begin{aligned} ax^2+bx+c &= 0 & \text{diskriminanta} \\ & \downarrow \\ x_{1,2} &= \frac{-b \pm \sqrt{b^2-4ac}}{2a} & D = b^2-4ac \\ ax^2+bx+c &= a \cdot (x-x_1)(x-x_2) \end{aligned}$$

$$\textcircled{3} \quad \frac{6-x}{3-x} < -2 \quad x \neq 3$$

$$\frac{6-x}{3-x} + 2 < 0$$

$$\frac{6-x+6-2x}{3-x} < 0$$

$$\frac{-3x+12}{3-x} < 0$$

$$\frac{3(4-x)}{3-x} < 0$$

	$-\infty$	3	4	$+\infty$
$4-x$		+	+	-
$3-x$		+	-	-
R		+	-	+

$x \in (3, 4)$

Apsolutna vrijednost

$$|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

$$|x| = |-x|, \quad |x| \geq 0$$

$$|x-a| = \begin{cases} x-a, & x-a \geq 0, \quad x \geq a \\ -(x-a), & x-a < 0, \quad x < a \end{cases}$$

$$|x| \leq a \Leftrightarrow -a \leq x \leq a \quad x \in [-a, a]$$

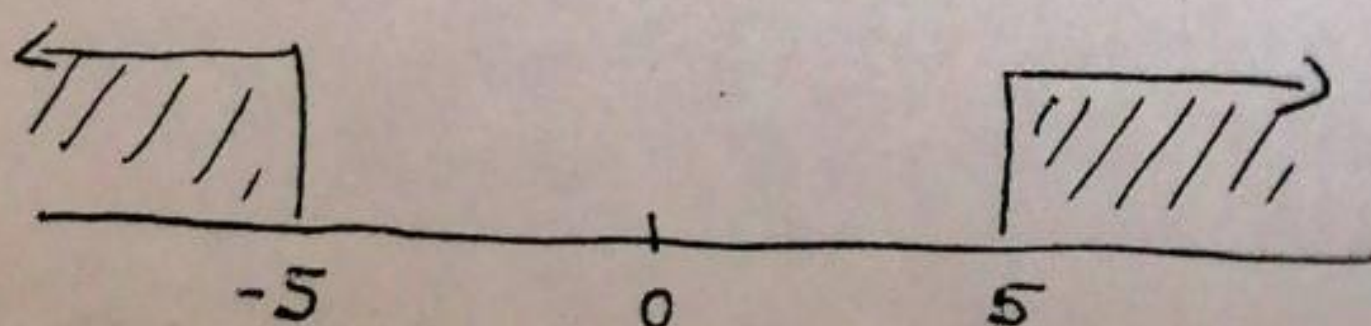
$$|x| < a \Leftrightarrow -a < x < a \quad x \in (-a, a) \quad |x+y| \leq |x| + |y|$$

$$\textcircled{1} \quad |x-2| = \begin{cases} x-2, & x-2 \geq 0, (x \geq 2) \\ -(x-2), & x-2 < 0 (x < 2) \end{cases}$$

$$\textcircled{2} \quad |x| < 4 \Leftrightarrow -4 < x < 4 \quad x \in (-4, 4)$$

$$\textcircled{3} \quad |x| \leq 3 \Leftrightarrow -3 \leq x \leq 3 \quad x \in [-3, 3]$$

$$\textcircled{4} \quad |x| > 5 \Leftrightarrow \begin{matrix} x < -5 & \vee & x > 5 \\ x < -a & \vee & x > a \end{matrix} \quad \begin{matrix} x \in (-\infty, -a) \cup (a, +\infty) \\ x \in (-\infty, -5) \cup (5, +\infty) \end{matrix}$$



$$\textcircled{5} \quad |x-3| < 4 \quad |x-3| = \begin{cases} x-3, & x-3 \geq 0 \\ -(x-3), & x-3 < 0 \end{cases} \quad \begin{matrix} x \geq 3 \\ x < 3 \end{matrix}$$

1. način

$$\begin{aligned} 1. \quad & (x-3 < 4 \wedge x \geq 3) \quad \wedge \quad (-x+3 < 4 \wedge x < 3) \\ & 3 \leq x < 7 \quad \wedge \quad (-x < +1 \wedge x < 3) \\ & \quad \quad \quad (x > -1 \wedge x < 3) \end{aligned} \quad \Bigg\} \Rightarrow x \in (-1, 7)$$

2. način

$$\begin{aligned} -4 &< x-3 < 4 \\ -1 &< x < 7 \end{aligned} \quad x \in (-1, 7)$$

$$|x| \geq a \text{ akko } x \leq -a \quad \vee \quad x \geq a$$

$$x \in (-\infty, -a] \cup [a, +\infty)$$

$$\textcircled{6} \quad |2x-1| \geq 7$$

$$\begin{aligned} 2x-1 &\leq -7 & \wedge & \quad 2x-1 \geq 7 \\ 2x &\leq -6 & & \quad 2x \geq 8 \\ x &\leq -3 & & \quad x \geq 4 \end{aligned}$$

$$x \in (-\infty, -3] \cup [4, +\infty)$$

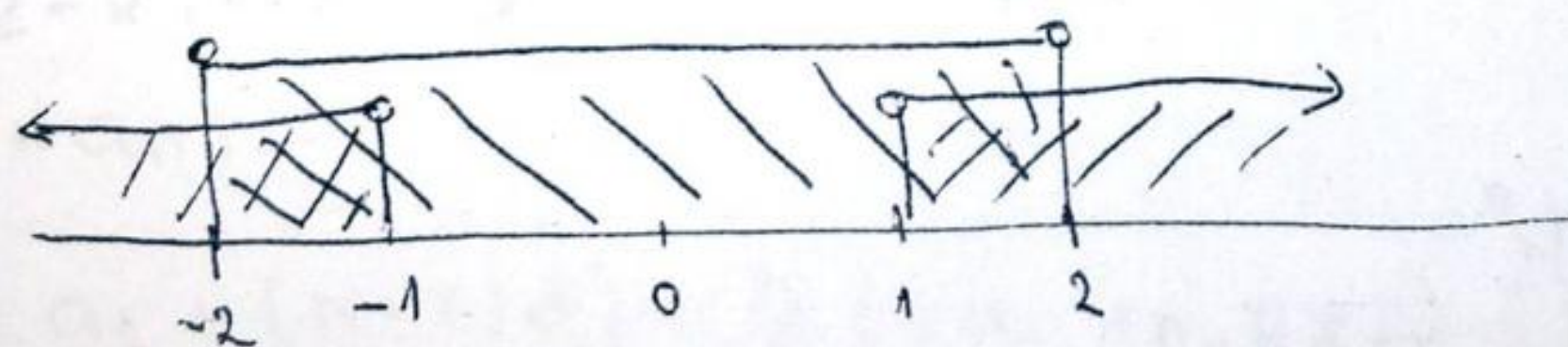
$$\textcircled{7} \quad 1 < |x| < 2$$

$$(|x| > 1 \Rightarrow x < -1 \quad \vee \quad x > 1)$$

$$(x \in (-\infty, -1) \cup (1, +\infty))$$

$$\wedge \quad |x| < 2$$

$$\begin{aligned} (-2 < x < 2) \\ x \in (-2, 2) \end{aligned}$$



$$x \in (-2, -1) \cup (1, 2)$$

$$\textcircled{8} \quad |x| + x = 0$$

$$|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

$$1. \quad x \geq 0$$

$$x + x = 0$$

$$2x = 0$$

$$x = 0$$

$$2. \quad x < 0$$

$$-x + x = 0$$

$$0 = 0$$

$$\text{Rješenje } x \in (-\infty, 0)$$

$$\text{Konačno: } x \in (-\infty, 0]$$

$$\textcircled{9} \quad |x-3| = 2$$

$$|x-3| = \begin{cases} x-3, & x-3 \geq 0 \\ -(x-3), & x-3 < 0 \end{cases}$$

$$|x-3| = \begin{cases} x-3, & x \geq 3 \\ 3-x, & x < 3 \end{cases}$$

$$1. \quad x \geq 3$$

$$x-3 = 2$$

$$x = 5$$

$$2. \quad x < 3$$

$$3-x = 2$$

$$x = 1$$

$$\mathbb{R}: x = 1, x = 5$$

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$$x-3 = 2$$

$$x = 5$$

$$x-3 = -2$$

$$x = 1$$

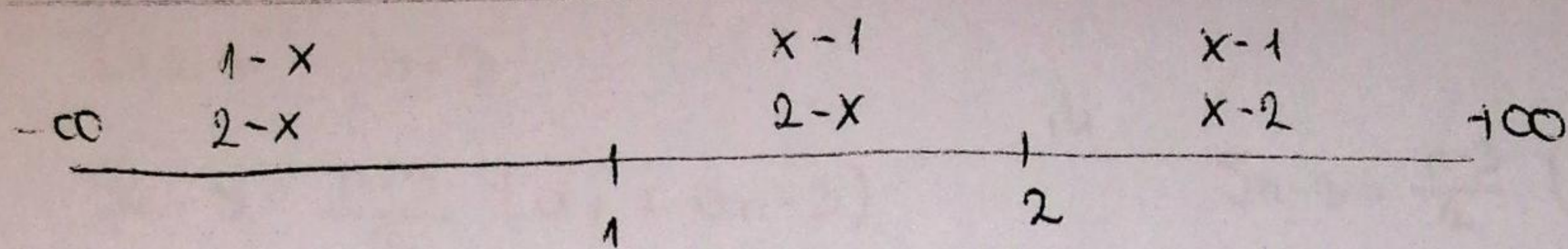
$$\textcircled{10} \quad |x-1| + |2-x| = 3x-6$$

$$|x-1| = \begin{cases} x-1, & x-1 \geq 0 \\ -(x-1), & x-1 < 0 \end{cases}$$

$$= \begin{cases} x-1, & x \geq 1 \\ 1-x, & x < 1 \end{cases}$$

$$|2-x| = \begin{cases} 2-x, & 2-x \geq 0 \\ -(2-x), & 2-x < 0 \end{cases}$$

$$= \begin{cases} 2-x, & x \leq 2 \\ x-2, & x > 2 \end{cases}$$



1. $x < 1$

jedn. glasi

$$1-x+2-x=3x-6$$

$$3-2x=3x-6$$

$$5x=9$$

$$x = \frac{9}{5} \in (-\infty, 1) \quad \frac{9}{5} > 1 \quad (\text{što nije uslov})$$

nema rj.

2. $1 \leq x \leq 2$

$$x-1+2-x=3x-6$$

$$3x=7$$

$$x = \frac{7}{3} \notin [1, 2]$$

nema rj.

3. $x > 2$

$$x-1+x-2=3x-6$$

$$2x-3=3x-6$$

$$x=3 \in (2, +\infty) \rightarrow \text{Rjesenje j-ne.}$$