Aritmetički niz

$a_n, a_1 + d, a_1 + 2d$

$a_n = a_1 + (n - nd) = a_1 + d$

$a_5 = a_1 + 4d \qquad a_7 = a_1 + 6d$ (primjer)

$S_n = a_1 + a_2 + \dots a_n$ parcijalna suma

$S_n = \frac{n}{2}(a_1 + a_n)$

$S_n = \frac{n}{2}(a_1 + a_2 + (n-1)d) = \frac{n}{2}(2a_1 + (n-1)d)$

$a_n = a_1 - (n-1)d$

① Zadato $1+2+3+\dots n$. Izračunati sumu prvih n-članova

$a_1=1 \quad d=1$

$S_n=\frac{n}{2}(1+n)=\frac{n(n+1)}{2}$ suma prvih n-članova

② Brojevi $3, x_1, x_2, x_3, x_4, 13$ su uzastopni članovi niza. Odredi niz

$a_1=3 \quad a_6=13$

$a_6=a_1+5d$

$13=3+5d$ $\quad$ $3, 5, 7, 9, 11, 13$

$d=2$

③ $4, 10, 16, 22, \dots \quad a_1=4 \quad d=6$

$7, 5, 3, -1, -3 \quad a_1=7 \quad d=-2$

$S_n=a_1+a_2+\dots+a_n$

$S_n=\frac{n}{2}(a_1+a_n)$

$S_n=\frac{n}{2}\cdot(a_1+a_1+(n-1)d)=\frac{n}{2}(2a_1+(n-1)d)$

$S_{57}=\frac{57}{2}\cdot(a_1+a_{57}) \quad S_{57}=\frac{57}{2}(2\cdot a_1+56d)$

za prvi niz $\quad S_{57}=\frac{57}{2}(2\cdot 4+56\cdot 6)$

drugi $\quad S_{57}=\frac{57}{2}(2\cdot 7+56\cdot(-2))$

Za prvi niz a_{1042}

$a_{1042}=4+(1042-1)\cdot 6=4+1041\cdot 6$

$a_{236}=7+(236-1)\cdot(-2)=7+735(-2)$

$S_{n+2}=a_1+a_2+\dots+a_{n+2}$

$S_{n+2}=\frac{n+2}{2}(2a_1+(n+2-1)d)$

$S_{n+2}=\frac{n+2}{2}(2a_1+(n+1)d) \quad S_{n+2}=\frac{n+2}{2}(a_1+a_{n+2})$

4. $1+2+\dots n-3$

$S_{n-3} = \frac{n-3}{2}(a_1 + a_{n-3})$

$S_{n-3} = \frac{n-3}{2}(1 + n-3)$

$S_{n-3} = \frac{n-3}{2}(n-2)$

ili

$S_{n-3} = \frac{n-3}{2}(2a_1 + (n-3-1)d)$

$S_{n-3} = \frac{n-3}{2}(2 + (n-4).\ )$

$S_{n-3} = \frac{n-3}{2}(n-2)$

Vježba 2. 01.10.2019 II sedmica

Geometrijski niz

$a_1, a_1q, a_1q^2, a_1q^3 \cdots a_1 q^n$

a_1

$a_2 = a_1 q$

$a_3 = a_1 \cdot q^2$

$\vdots$

$a_n = a_1 \cdot q^{n-1}$

$S_n = a_1 + a_2 + \dots + a_n$

$$\boxed{S_n = a_1 \cdot \frac{1-q^n}{1-q}}$$

1. Izračunati sumu prvih 50 članova niza $\frac{1}{3}, \frac{1}{9}, \frac{1}{27} \dots$

$a_1 = \frac{1}{3}$

$q = \frac{1}{3}$

$S_{50} = a_1 + a_2 + \dots a_{50}$

$S_{50} = \frac{1}{3} \cdot \frac{1-(\frac{1}{3})^{50}}{1-\frac{1}{3}} = \frac{1}{3} \cdot \frac{3}{2}\left(1-(\frac{1}{3})^{50}\right) = \frac{1}{2}\left(1-(\frac{1}{3})^{50}\right) \approx \frac{1}{2} \cdot 1$

$\rightarrow \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \dots + \frac{1}{3^{n+1}}$

$S_{n+1} = \frac{1}{3} + \frac{1}{3^2} + \frac{1}{3^3} + \cdot \ \frac{1}{3^{n+1}}$

$S_{n+1} = \frac{1}{3} \cdot \frac{1-(\frac{1}{3})^{n+1}}{1-\frac{1}{3}}$

$\rightarrow 1 - \frac{1}{2} + \frac{1}{4} - \dots + \left(-\frac{1}{2}\right)^{n-2}$

$a_1 = 1$

$q = -\frac{1}{2}$

$1, -\frac{1}{2}, \frac{1}{2^2}, \dots \frac{(-1)^{n-2}}{2^{n-2}}$ $(n-2)$ elementi

$S_{n-1} = 1 \cdot \frac{1-\left(-\frac{1}{2}\right)^{n-1}}{1-\frac{1}{2}} = 2 \cdot \left(1 - \left(-\frac{1}{2}\right)^{n-1}\right)$

(2) Naći 10-ti član geome. niza 1, 3, 9, 27

$a_1 = 1$

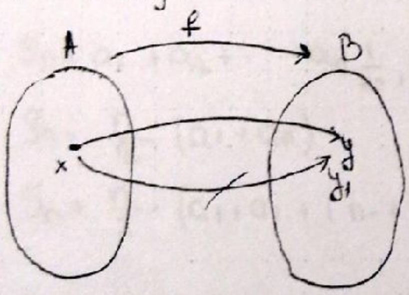
$q = 3$

a_{10}

$a_{10} = a_1 \cdot q^{10-1}$

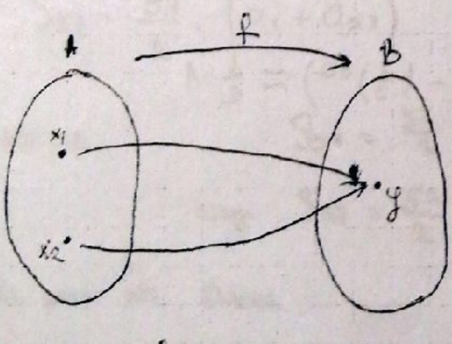
$a_{10} = 1 \cdot 3^9 = 19\ 683$

→ Funkcije ←



Ne može

A - domen
B - kodomen



$f(x) = x^2$
$f(2) = 4$
$f(2) = 9$ X

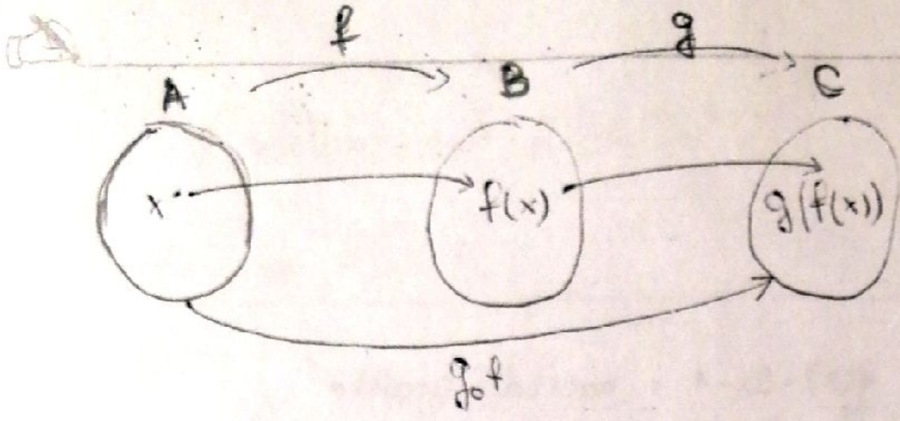
funkcija ne može jednu tačku preslikati u dvije različite, ali može dvije različite preslikati u istu

Funkcija f je sirjektivno preslikavanje ako važi $(\forall y \in B)(\exists x \in A)\, f(x) = y$

Funkcija f je injektivna ako važi $x_1 \neq x_2 \Rightarrow f(x_1) \neq f(x_2)$

$(f(x_1) = f(x_2) \Rightarrow x_1 = x_2)$

Ako je **funkcija** f **sirjektivna i injektivna** onda kažemo da je bijektivna.



$f : A \to B$

$g : B \to C$

$g \circ f : A \to C$

$(g \circ f)(x) = g(f(x))$

Primjer: Neka je $f(x) = 2x - 1$, $g(x) = 3x^2 - x$. Naći $g \circ f$ i $f \circ g$

$$(g \circ f)(x) = g(f(x)) = g(2x-1) = 3 \cdot (2x-1)^2 - (2x-1) = 3 \cdot (4x^2 - 4x + 1) - 2x + 1 = 12x^2 - 14x + 4$$

$$(f \circ g)(x) = f(g(x)) = f(3x^2 - x) = 2 \cdot (3x^2 - x) - 1 = 6x^2 - 2x - 1$$

Kažemo da je $f^* : B \to A$ inverzna funkcija funkcije $f : A \to B$ ako

važi:

$f \circ f^* = i_B$

$i_B : B \to B$, $i_B(x) = x$

$f^* \circ f = i_A$

$i_A : A \to A$ $i_A(x) = x$

Tada funkciju f^* označavamo sa f^{-1}

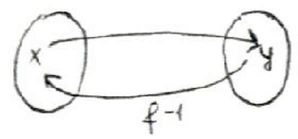
$f \circ f^{-1} = i_B$ $\quad f^{-1} \circ f = i_A$

$(f \circ f^{-1})(x) = x, \ \forall x \in B$

$(f^{-1} \circ f)(x) = x, \ \forall x \in A$

Primjer: Naći f^{-1} za funkciju $f(x)=2x-1$ i nacrtati grafike

I način



$f(x)=y$

$f^{-1}(y)=x$

$f(x)=2x-1$

$y=2x-1$

$2x=y+1$

$x=\frac{y+1}{2}$

$f^{-1}(y)=\frac{y+1}{2}$

$f^{-1}(x)=\frac{x+1}{2}$

II način

$(f\circ f^{-1})(x)=x$

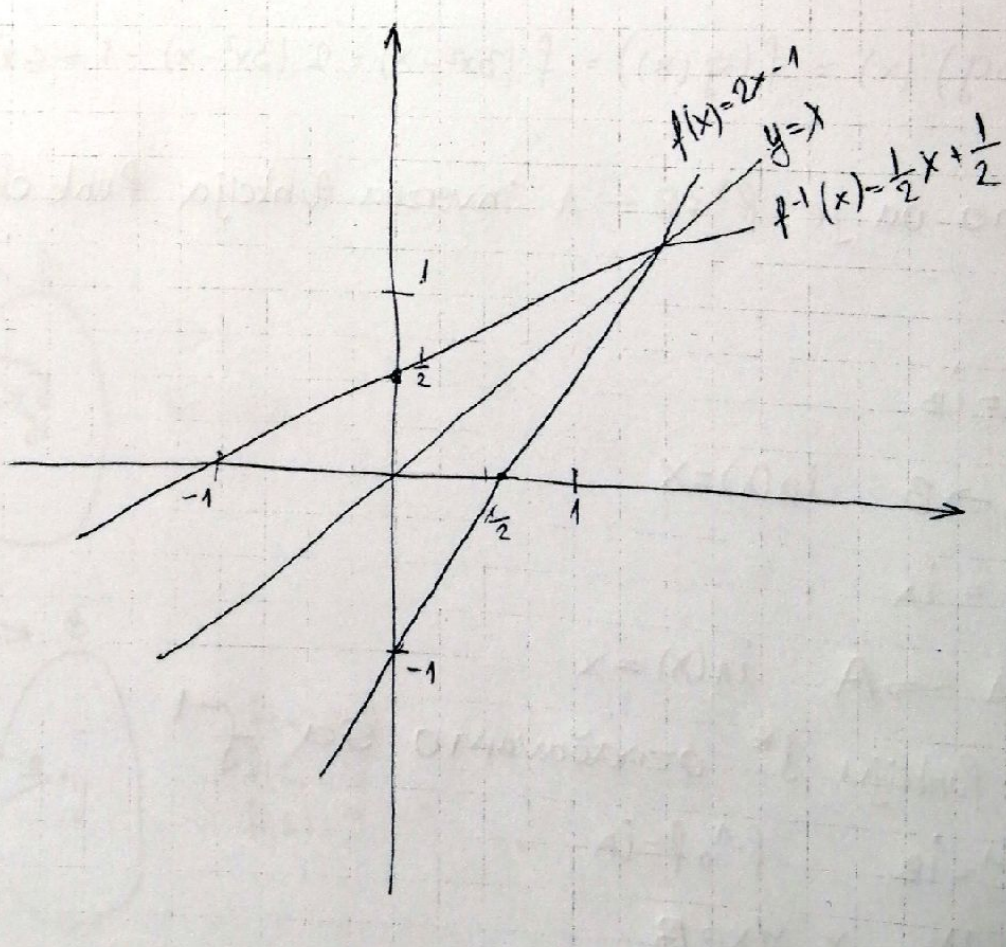
$f(f^{-1}(x))=x$

$2\cdot f^{-1}(x)-1=x$

$2\cdot f^{-1}(x)=x+1$

$f^{-1}(x)=\frac{x+1}{2}$

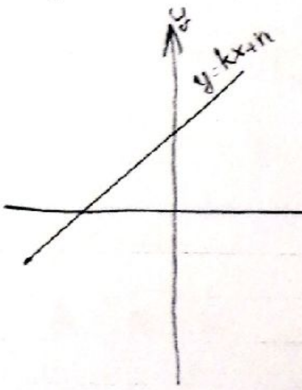
$f^{-1}(x)=\frac{1}{2}x+\frac{1}{2}$



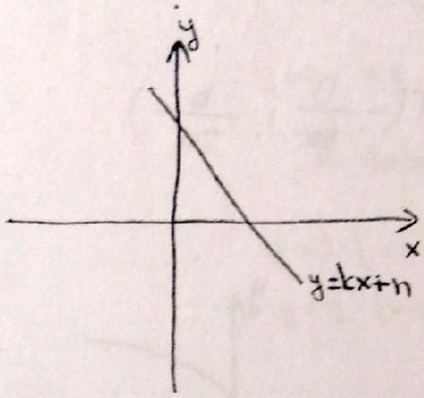
Elementarne funkcije

① Linearna funkcija

$y = kx + n$



$k > 0$

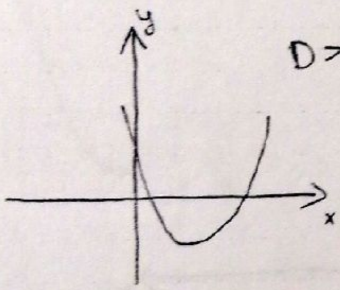


$k < 0$

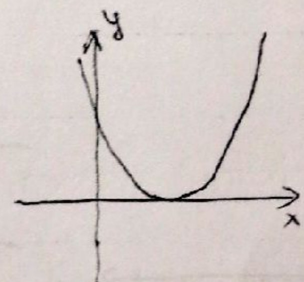
② Parabola

$y = ax^2 + bx + c$, $D = b^2 - 4ac$ $T\left(-\frac{b}{2a}, -\frac{D}{4a}\right)$

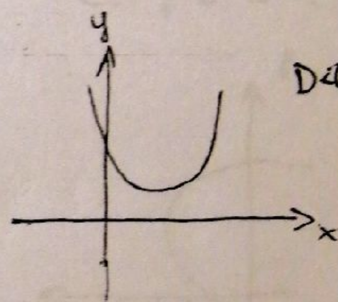
I ako je $a > 0$, $\cup$



$D > 0$

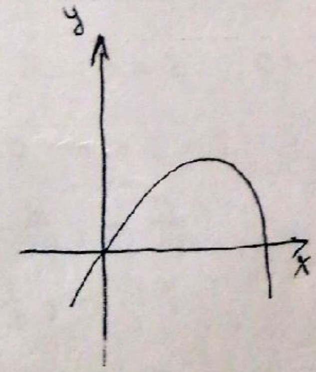


$D = 0$

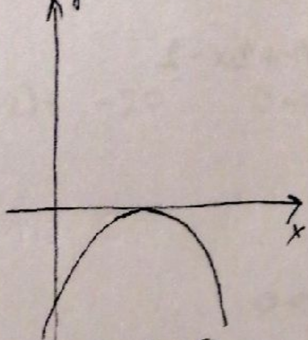


$D < 0$

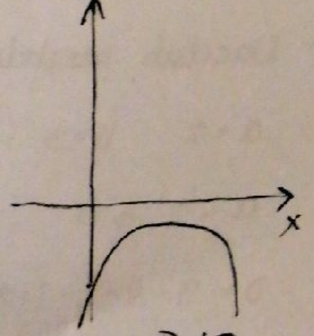
II ako je $a < 0$, $\cap$



$D > 0$



$D = 0$

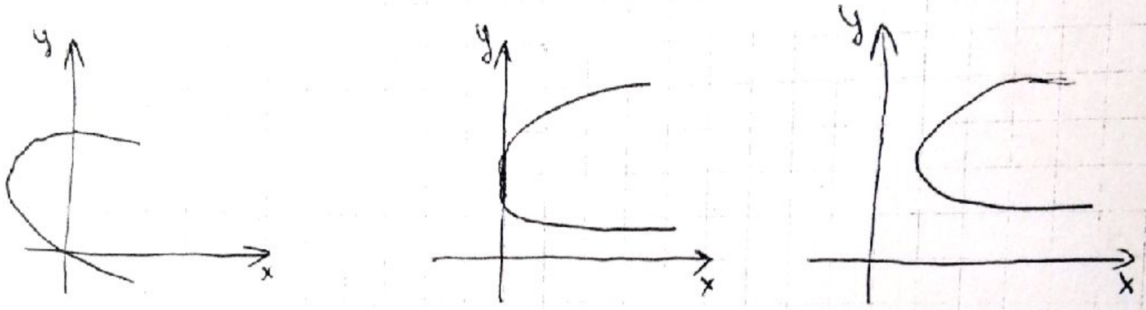


$D < 0$

$x = ay^2 + by + c$

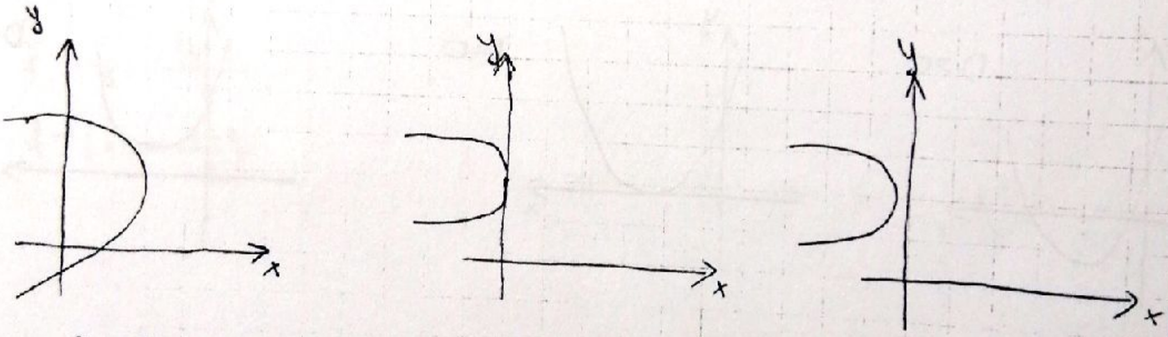
$D = b^2 - 4ac$, $T\left(-\frac{D}{4a}, \frac{-b}{2a}\right)$

$\longrightarrow$ $a > 0$, $\subset$



$D > 0$ $\quad$ $D = 0$ $\quad$ $D < 0$

$\longrightarrow$ $a < 0$, $\supset$



$D > 0$ $\quad$ $D = 0$ $\quad$ $D < 0$

① Nacrtati parabolu , $y = 2x^2 + 3x - 2$

$a = 2 \quad b = 3 \quad c = -2$

$a = 2 > 0 \quad \cup$

$D = 9 - 4 \cdot 2 \cdot (-2) = 25 \quad > 0$

$T\left(-\frac{b}{2a}, -\frac{D}{4a}\right)$

$T\left(-\frac{3}{4}, -\frac{25}{8}\right)$

Presjek sa OX-osom ($y = 0$)

$2x^2 + 3x - 2 = 0$

$x_{1,2} = \frac{-3 \pm \sqrt{9 - 4 \cdot 2 \cdot (-2)}}{4}$

$x_{1,2} = \frac{-3 \pm 5}{4}$

$x_1 = \frac{2}{4} = \frac{1}{2}$ $\quad x_2 = -\frac{8}{4} = -2$

$N_1\left(\frac{1}{2}, 0\right)$ $\quad N_2(-2, 0)$

Presjek sa Oxy-osom $(x=0)$

$y = 2\cdot 0 + 3\,0 - 2$

$y = -2$

$A(0, -2)$

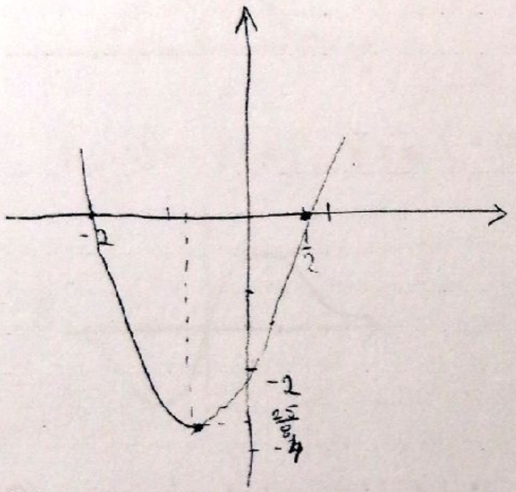
$y>0,\ x \in (-\infty, -2) \cup \left(\frac{1}{2}, +\infty\right)$

$y<0,\ x \in \left(-2, \frac{1}{2}\right)$

ili tabela

$y = 2\cdot\left(x-\frac{1}{2}\right)\cdot(x\cdot(-2))$

$y = 2\cdot\left(x-\frac{1}{2}\right)(x+2)$



domaći:

Nacrtati parabolu $-3x^2+2x-2=y$

$a=-3 \quad b=2 \quad c=-2$

$D = b^2 - 4ac = 2^2 - 4\,(-3)(-2) = -20 \qquad D<0$

$T\left(-\frac{b}{2a}, -\frac{D}{4a}\right)$

$T\left(\frac{1}{3}, -\frac{5}{3}\right)$

Presjek sa Ox-osom $(y=0)$

$-3x^2+2x-2=0$

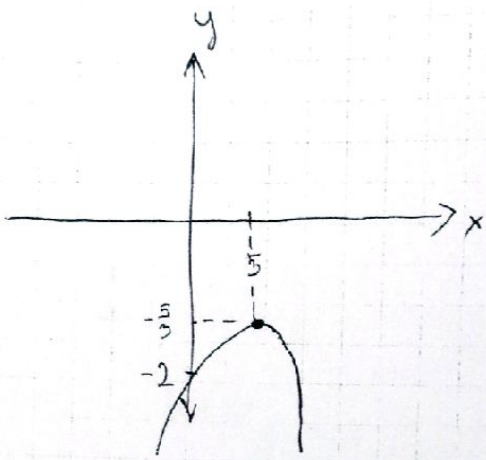
$x_{1,2} = \frac{-2 \pm \sqrt{4-4\cdot(-3)(-2)}}{-6}$

Nema presjeka!

Presjek sa Oy-osom $(x=0)$

$y=-2$

$A(0,-2)$



② Nacrtati $x=-3y^2+4y-1$

$a=-3 \quad b=4 \quad c=-1$

$\vdots$

$\supset$ jer je <0

$D=b^2-4ac=16-4(-3)(-1)=4>0$

$T\left(-\frac{D}{4a}, -\frac{b}{2a}\right)$

$T\left(\frac{1}{3}, \frac{2}{3}\right)$

Presjek sa Oy-osom $(x=0)$

$-3y^2+4y-1=0$

$y_{1,2}=\frac{-4\pm\sqrt{16-12}}{-6}$

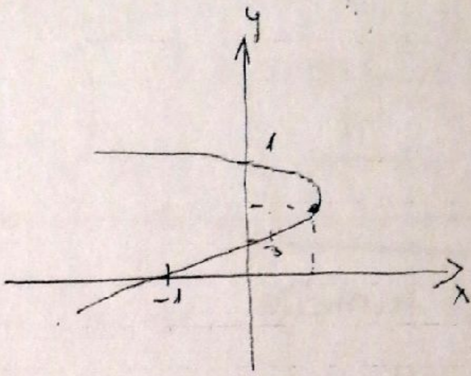
$y_1=\frac{1}{3} \quad y_2=1$

$M(0,\frac{1}{3}) \quad M_2(0,1)$

Presjek sa Ox-osom ($y=0$)

$x = -1$

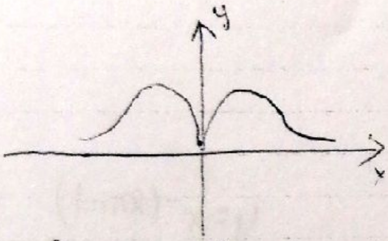
$B(-1,0)$



$x_1 < x_2 \Rightarrow f(x_1) < f(x_2)$, $\forall x_1, x_2 \in (a,b)$ f - je rastuća

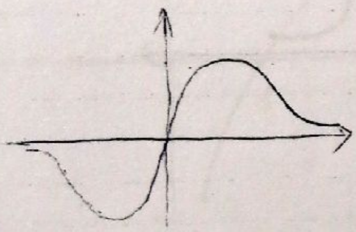
$x_1 < x_2 \Rightarrow f(x_1) > f(x_2)$, $\forall x_1, x_2 \in (a,b)$ f - je opadajuća

$f(-x) = f(x)$, $\forall x \in (-a,a)$, f je parna na (a,b)



grafik parne funkcije je simetričan u odnosu na Oy osu

$f(-x) = -f(x)$, $\forall x \in (-a,a)$, f je neparna



grafik neparne funkcije je simetričan u odnosu na koordinatni početak

Primjer: Ispitati parnost funkcije

a) $f(x) = 2x^4 - x^2 + 13$

$f(-x) = 2\cdot(-x)^4 - (-x)^2 + 13 = 2x^4 - x^2 + 13 = f(x) \rightarrow$ f-ja je parna

b) $f(x) = \dfrac{x^3 - 2x}{\cos x}$

$f(-x) = \dfrac{(-x)^3 - 2\cdot(-x)}{\cos\cdot(-x)} = \dfrac{-x^3 - 2x}{\cos x} = -\dfrac{x^3 - 2x}{\cos x} = -f(x) \rightarrow$ f - je neparna