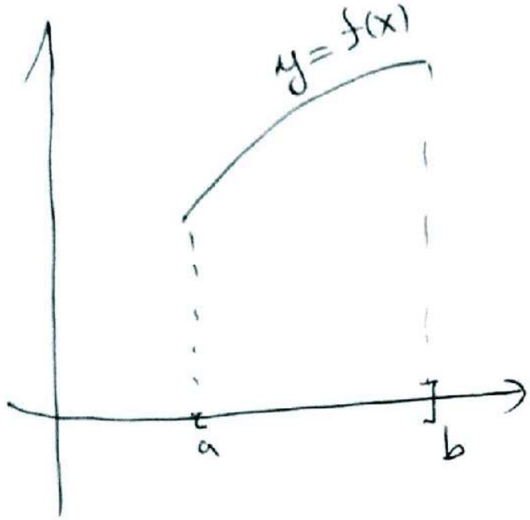
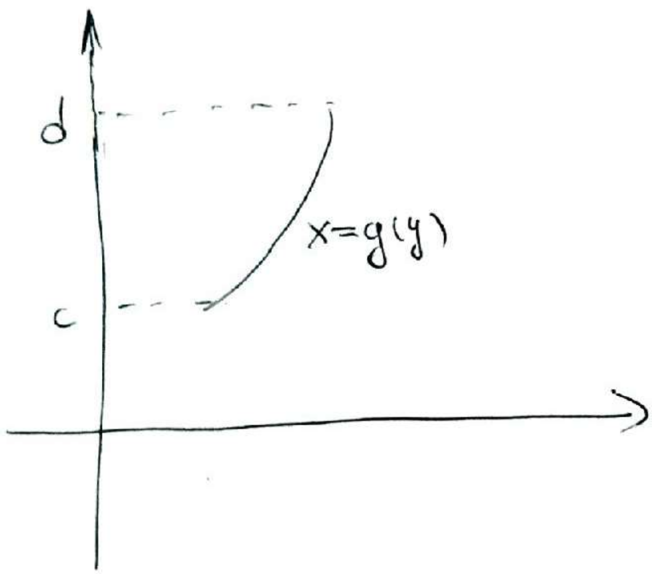
Dužina luka krive

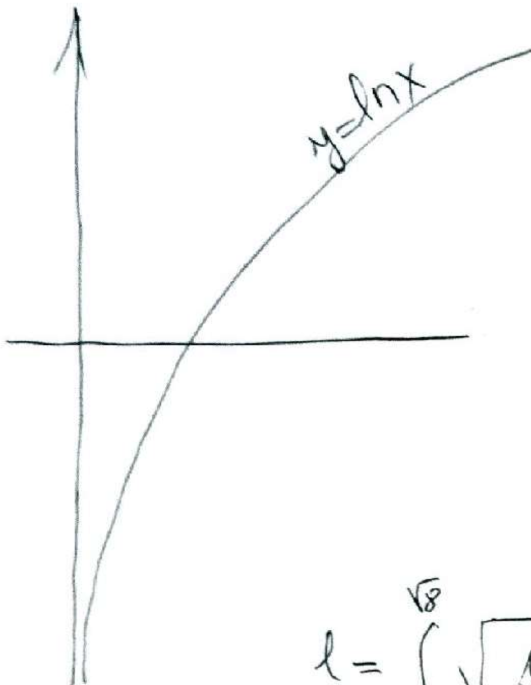


$$l = \int_a^b \sqrt{1+f'^2(x)}\,dx$$



$$l = \int_c^d \sqrt{1+g'^2(y)}\cdot dy$$

4 Izračunati duzinu luka krive $y=\ln x$ od tacke $x=\sqrt{3}$ do tacke $\sqrt{8}$.



$$f(x)=\ln x$$

$$l=\int_{\sqrt{3}}^{\sqrt{8}}\sqrt{1+f'^2(x)}\,dx$$

$$f'(x)=\frac{1}{x}$$

$$l=\int_{\sqrt{3}}^{\sqrt{8}}\sqrt{1+\frac{1}{x^2}}\,dx=\int_{\sqrt{3}}^{\sqrt{8}}\sqrt{\frac{x^2+1}{x^2}}\,dx=$$

$$=\int_{\sqrt{3}}^{\sqrt{8}}\frac{\sqrt{x^2+1}}{x}\,dx=\left[\begin{array}{l}\sqrt{x^2+1}=t\\ x^2+1=t^2\\ 2x\,dx=2t\,dt\\ x\,dx=t\,dt\end{array}\right]=\int_{\sqrt{3}}^{\sqrt{8}}\frac{\sqrt{x^2+1}\cdot x\,dx}{x^2}=$$

$$=\left[\begin{array}{c|c|c} x & \sqrt{3} & \sqrt{8}\\ \hline t & 2 & 3\end{array}\right]=\int_2^3\frac{t\cdot t\,dt}{t^2-1}=\int_2^3\frac{t^2-1+1}{t^2-1}\,dt=$$

$$=\int_2^3 dt+\int_2^3\frac{dt}{t^2-1}=t\Big|_2^3+\frac{1}{2}\ln\left|\frac{t-1}{t+1}\right|\Big|_2^3=$$

$$=3-2+\frac{1}{2}\left(\ln\frac{1}{2}-\ln\frac{1}{3}\right)=1+\frac{1}{2}\ln\frac{3}{2}=$$

(*) Naći dužinu luka krive $y=\arcsin e^{-x}$ od $x=0$ do $x=1$.

R: $l=\int_0^1 \sqrt{1+f'^2(x)}\cdot dx$, $f(x)=\arcsin e^{-x}$

$$f'(x)=\frac{1}{\sqrt{1-e^{-2x}}}\cdot e^{-x}\cdot(-1)=\frac{-e^{-x}}{\sqrt{1-e^{-2x}}}$$

$$l=\int_0^1\sqrt{1+\frac{e^{-2x}}{1-e^{-2x}}}\,dx=\int_0^1\sqrt{\frac{1-e^{-2x}+e^{-2x}}{1-e^{-2x}}}\,dx=$$

$$=\int_0^1\frac{1}{\sqrt{1-e^{-2x}}}\,dx=\int_0^1\frac{dx}{\sqrt{1-\frac{1}{e^{2x}}}}=\int_0^1\frac{dx}{\sqrt{\frac{e^{2x}-1}{e^{2x}}}}=$$

$$=\int_0^1\frac{dx}{\frac{\sqrt{e^{2x}-1}}{e^x}}=\int_0^1\frac{e^x\,dx}{\sqrt{e^{2x}-1}}=\left[\begin{array}{l}e^x=t\\ e^x dx=dt\end{array}\quad\begin{array}{c|c|c}x&0&1\\\hline t&1&e\end{array}\right]$$

$$=\int_1^e\frac{dt}{\sqrt{t^2-1}}=\ln\left|t+\sqrt{t^2-1}\right|\Big|_1^e=$$

$$=\ln(e+\sqrt{e^2-1})-\ln(1+\sqrt{1^2-1})=\ln(e+\sqrt{e^2-1})$$

(*) Odrediti dužinu luka krive zadate eksplicitno $y=\sqrt{x-x^2}+\arcsin\sqrt{x}$, od $x=0$ do $x=4$

$\underline{\underline{R}}$ $$l=\int_0^4\sqrt{1+f'^2(x)}\cdot dx$$

$$f(x)=\sqrt{x-x^2}+\arcsin\sqrt{x}$$

$$f'(x)=\frac{1}{2\sqrt{x-x^2}}\cdot(1-2x)+\frac{1}{\sqrt{1-x}}\cdot\frac{1}{2\sqrt{x}}=$$

$$=\frac{1-2x}{2\sqrt{x-x^2}}+\frac{1}{2\cdot\sqrt{x-x^2}}=\frac{2-2x}{2\sqrt{x-x^2}}=$$

$$=\frac{1-x}{\sqrt{x}\sqrt{1-x}}=\frac{\sqrt{1-x}}{\sqrt{x}}=\sqrt{\frac{1-x}{x}}$$

$$l=\int_0^4\sqrt{1+\frac{1-x}{x}}\,dx=\int_0^4\sqrt{\frac{x+1-x}{x}}\,dx=\int_0^4\sqrt{\frac{1}{x}}\,dx=\int_0^4\frac{dx}{\sqrt{x}}=$$

$$=2\sqrt{x}\Big|_0^4=4$$