

$$= 3\sqrt{10} + \sqrt{2} - \int_{-1}^3 \frac{y^2 + 1 - 1}{\sqrt{1+y^2}} dy = 3\sqrt{10} + \sqrt{2} \int_{-1}^3 \frac{dy}{\sqrt{1+y^2}} + \int_{-1}^3 \frac{dy}{\sqrt{1+y^2}}$$

$$l = 3\sqrt{10} + \sqrt{2} - l + \ln |y + \sqrt{1+y^2}| \Big|_{-1}^3 \Rightarrow$$

$$2l = 3\sqrt{10} + \sqrt{2} + \ln(3 + \sqrt{10}) - \ln(\sqrt{2} - 1)$$

$$l = \frac{3\sqrt{10} + \sqrt{2}}{2} + \frac{1}{2} \ln \frac{3 + \sqrt{10}}{\sqrt{2} - 1}$$

⊗ Odrediti dužinu luka krive između tačaka sa apscisama $x=1$ i $x=2$, ako je $y = \frac{1}{4}x^2 - \frac{1}{2}\ln x$.

$$\underline{\underline{l}} = \int_1^2 \sqrt{1 + f'^2(x)} \cdot dx$$

$$f'(x) = \frac{1}{2}x - \frac{1}{2} \cdot \frac{1}{x} = \frac{x^2 - 1}{2x}$$

$$l = \int_1^2 \sqrt{1 + \left(\frac{x^2 - 1}{2x}\right)^2} dx = \int_1^2 \sqrt{1 + \frac{x^4 - 2x^2 + 1}{4x^2}} dx =$$

$$= \int_1^2 \sqrt{\frac{x^4 + 2x^2 + 1}{4x^2}} dx = \int_1^2 \sqrt{\frac{(x^2 + 1)^2}{4x^2}} dx = \int_1^2 \frac{x^2 + 1}{2x} dx =$$

$$= \frac{1}{2} \int_1^2 x dx + \frac{1}{2} \int_1^2 \frac{1}{x} dx = \frac{1}{2} \cdot \frac{x^2}{2} \Big|_1^2 + \frac{1}{2} \ln |x| \Big|_1^2 =$$

$$= \frac{1}{4}(2^2 - 1^2) + \frac{1}{2}(\ln 2 - \ln 1) = \frac{3}{4} + \frac{1}{2} \ln 2 =$$

$$= \frac{3}{4} + \ln \sqrt{2}$$