

Aritmetični niz

$$a_n, a_1 + d, a_1 + 2d$$

$$a_n = a_1 + (n-1)d = a_1 + d$$

$$a_5 = a_1 + 4d \quad a_7 = a_1 + 6d \text{ (primjer)}$$

$$S_n = a_1 + a_2 + \dots + a_n \text{ parcijalna suma}$$

$$S_n = \frac{n}{2} (a_1 + a_n)$$

$$S_n = \frac{n}{2} (a_1 + a_1 + (n-1)d) = \frac{n}{2} (2a_1 + (n-1)d)$$

$$\lceil a_n = a_1 - (n-1)d \rceil$$

① Zadatak $1+2+3+\dots+n$. Izračunati sumu prvih n -članova

$$a_1 = 1 \quad d = 1$$

$$S_n = \frac{n}{2} (1+n) = \frac{n(n+1)}{2} \quad \text{suma prvih } n\text{-članova}$$

② Brojevi $3, x_1, x_2, x_3, x_4, 13$ su uzastopni članovi niza. Odredi niz

$$a_1 = 3 \quad a_6 = 13$$

$$a_6 = a_1 + 5d$$

$$13 = 3 + 5d$$

$$d = 2$$

$$3, 5, 7, 9, 11, 13$$

③ $4, 10, 16, 22, \dots$ $a_1 = 4$ $d = 6$

$$7, 5, 3, -1, -3 \dots \quad a_1 = 7 \quad d = -2$$

$$S_n = a_1 + a_2 + \dots + a_n$$

$$S_n = \frac{n}{2} (a_1 + a_n)$$

$$S_n = \frac{n}{2} \cdot (a_1 + a_1 + (n-1)d) = \frac{n}{2} (2a_1 + (n-1)d)$$

$$S_{57} = \frac{57}{2} \cdot (a_1 + a_{57}) \quad S_{57} = \frac{57}{2} (2 \cdot a_1 + 56d)$$

za prvi niz

$$S_{57} = \frac{57}{2} (2 \cdot 4 + 56 \cdot 6)$$

desni
$$S_{57} = \frac{57}{2} (2 \cdot 7 + 56 \cdot (-2))$$

Za prvi niz a_{1042}

$$a_{1042} = 4 + (1042-1) \cdot 6 = 4 + 1041 \cdot 6$$

$$a_{236} = 7 + (236-1) \cdot (-2) = 7 + 235 \cdot (-2)$$

$$S_{n+2} = a_1 + a_2 + \dots + a_{n+2}$$

$$S_{n+2} = \frac{n+2}{2} (2a_1 + (n+2-1)d)$$

$$S_{n+2} = \frac{n+2}{2} (2a_1 + (n+1)d)$$

$$S_{n+2} = \frac{n+2}{2} (a_1 + a_{n+2})$$

$$\textcircled{4} \quad 1+2+\dots+n-3$$

$$S_{n-3} = \frac{n-3}{2} (a_1 + a_{n-3})$$

$$S_{n-3} = \frac{n-3}{2} (1 + n-3)$$

$$S_{n-3} = \frac{n-3}{2} (n-2)$$

ili

$$S_{n-3} = \frac{n-3}{2} (2a_1 + (n-3-1)d)$$

$$S_{n-3} = \frac{n-3}{2} (2 + (n-4))$$

$$S_{n-3} = \frac{n-3}{2} (n-2)$$

Vježba 2. 01.10.2019. 11 sedmica

Geometrijski niz

$$a_1, a_1q, a_1q^2, a_1q^3, \dots, a_1q^n$$

$$a_1$$

$$a_2 = a_1 \cdot q$$

$$a_3 = a_1 \cdot q^2$$

\vdots

$$a_n = a_1 \cdot q^{n-1}$$

$$S_n = a_1 + a_2 + \dots + a_n$$

$$S_n = a_1 \cdot \frac{1 - q^n}{1 - q}$$

1) Izračunati sumu prvih 50 članova niza $\frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \dots$

$$a_1 = \frac{1}{3}$$

$$q = \frac{1}{3}$$

$$S_{50} = a_1 + a_2 + \dots + a_{50}$$

$$S_{50} = \frac{1}{3} \cdot \frac{1 - (\frac{1}{3})^{50}}{1 - \frac{1}{3}} = \frac{1}{3} \cdot \frac{3}{2} (1 - (\frac{1}{3})^{50}) = \frac{1}{2} (1 - (\frac{1}{3})^{50}) \approx \frac{1}{2} \cdot 1$$

$$\rightarrow \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \dots + \frac{1}{3^{n+1}}$$

$$S_{n+1} = \frac{1}{3} + \frac{1}{3^2} + \frac{1}{3^3} + \dots + \frac{1}{3^{n+1}}$$

$$S_{n+1} = \frac{1}{3} \cdot \frac{1 - (\frac{1}{3})^{n+1}}{1 - \frac{1}{3}}$$

$$\rightarrow 1 - \frac{1}{2} + \frac{1}{4} - \dots + \left(-\frac{1}{2}\right)^{n-2}$$

$$a_1 = 1$$

$$q = -\frac{1}{2}$$

$$1, -\frac{1}{2}, \frac{1}{2^2}, \dots, \frac{(-1)^{n-2}}{2^{n-2}} \quad (n-2) \text{ elementi}$$

$$S_{n-1} = 1 \cdot \frac{1 - \left(-\frac{1}{2}\right)^{n-1}}{1 - \left(-\frac{1}{2}\right)} = 2 \cdot \left(1 - \left(-\frac{1}{2}\right)^{n-1}\right)$$

② Naći 10-ti član geome. niza 1, 3, 9, 27

$$a_1 = 1$$

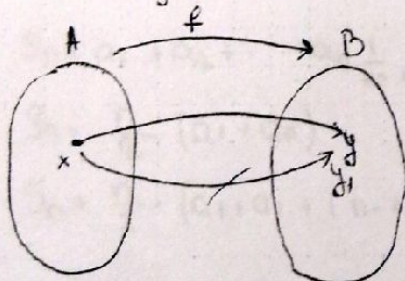
$$a_{10} = a_1 \cdot q^{10-1}$$

$$q = 3$$

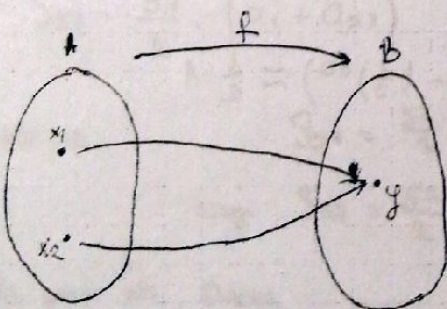
$$a_{10} = 1 \cdot 3^9 = 19\ 683$$

$$a_{10}$$

→ Funkcije ←



Ne može
A - domen
B - kodomen



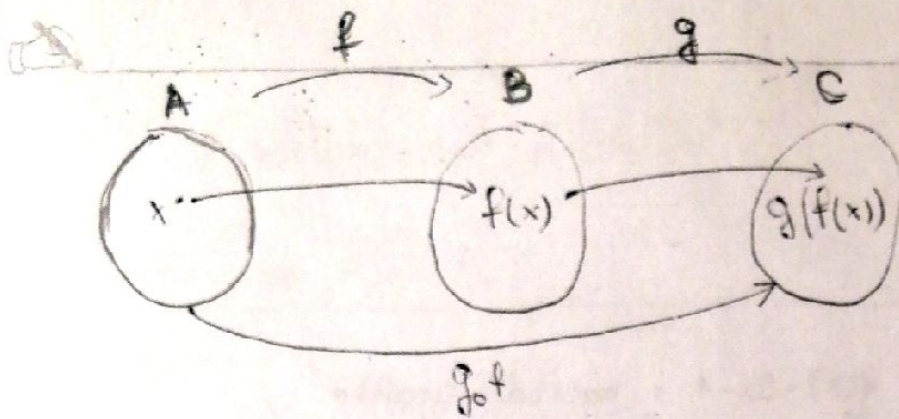
$$\begin{aligned} f(x) &= x^2 \\ f(2) &= 4 \\ f(2) &= 9 \quad \times \end{aligned}$$

Funkcija f je surjektivna preslikavanje ako važi $(\forall y \in B) (\exists x \in A) f(x) = y$

Funkcija f je injektivna ako važi $x_1 \neq x_2 \Rightarrow f(x_1) \neq f(x_2)$

$$f(x) = f(x_2) \Rightarrow x_1 = x_2$$

Ako je funkcija f surjektivna i injektivna onda kažemo da je bijektivna.



$$f: A \rightarrow B$$

$$g: B \rightarrow C$$

$$g \circ f: A \rightarrow C$$

$$(g \circ f)(x) = g(f(x))$$

Primer: Neka je $f(x) = 2x - 1$, $g(x) = 3x^2 - x$. Nađi $g \circ f$ i $f \circ g$

$$(g \circ f)(x) = g(f(x)) = g(2x - 1) = 3 \cdot (2x - 1)^2 - (2x - 1) = 3 \cdot (4x^2 - 4x + 1) - 2x + 1 = 12x^2 - 14x + 4$$

$$(f \circ g)(x) = f(g(x)) = f(3x^2 - x) = 2 \cdot (3x^2 - x) - 1 = 6x^2 - 2x - 1$$

Kažemo da je $f^*: B \rightarrow A$ inverzna funkcija funkcije $f: A \rightarrow B$ ako

važi:

$$f \circ f^* = i_B$$

$$i_B: B \rightarrow B, \quad i_B(x) = x$$

$$f^* \circ f = i_A$$

$$i_A: A \rightarrow A, \quad i_A(x) = x$$

Tada funkciju f^* označavamo sa f^{-1}

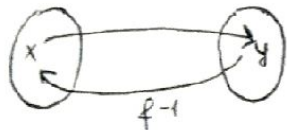
$$f \circ f^{-1} = i_B \quad f^{-1} \circ f = i_A$$

$$(f \circ f^{-1})(x) = x, \quad \forall x \in B$$

$$(f^{-1} \circ f)(x) = x, \quad \forall x \in A$$

Primer: Naci f^{-1} za funkciju $f(x) = 2x - 1$ i nacrtati grafike

I način



$$f(x) = y$$

$$f^{-1}(y) = x$$

$$f(x) = 2x - 1$$

$$y = 2x - 1$$

$$2x = y + 1$$

$$x = \frac{y + 1}{2}$$

$$f^{-1}(y) = \frac{y + 1}{2}$$

$$f^{-1}(x) = \frac{x + 1}{2}$$

II način

$$(f \circ f^{-1})(x) = x$$

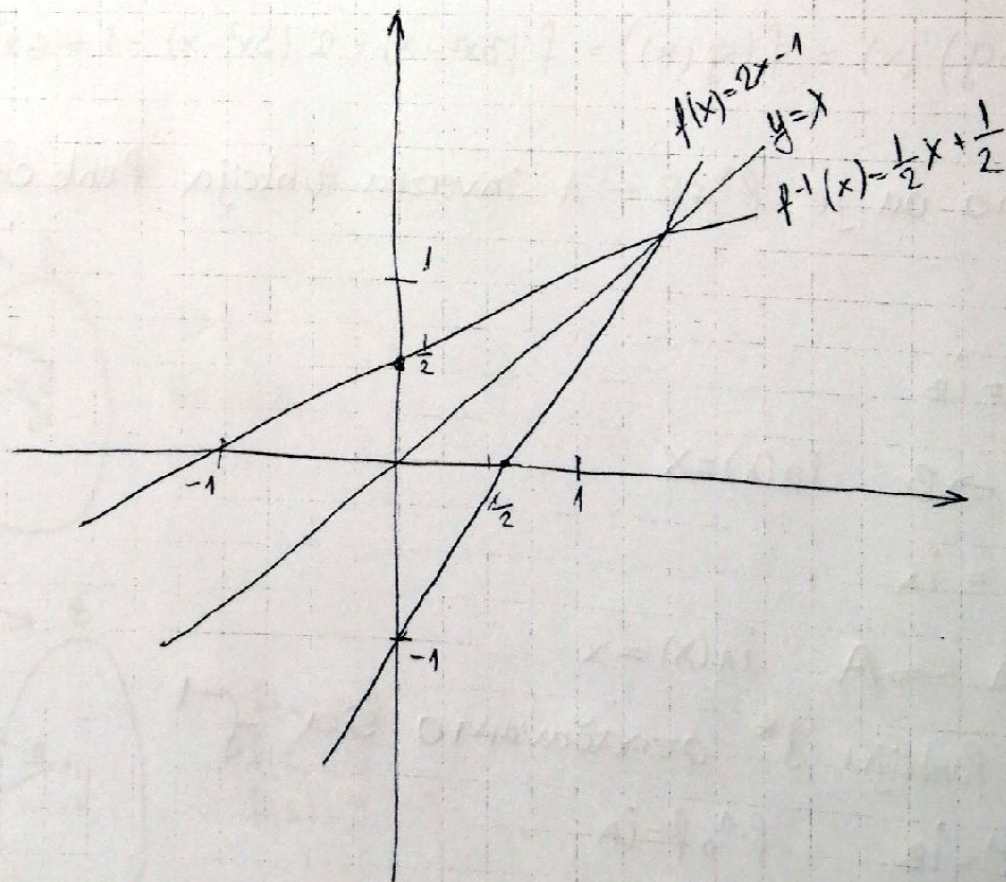
$$f(f^{-1}(x)) = x$$

$$2 \cdot f^{-1}(x) - 1 = x$$

$$2 \cdot f^{-1}(x) = x + 1$$

$$f^{-1}(x) = \frac{x + 1}{2}$$

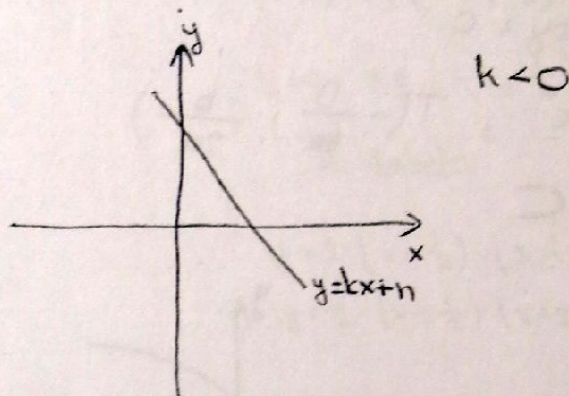
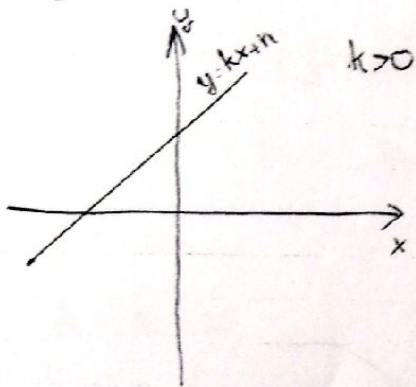
$$f^{-1}(x) = \frac{1}{2}x + \frac{1}{2}$$



Elementarne funkcije

① Linearna funkcija

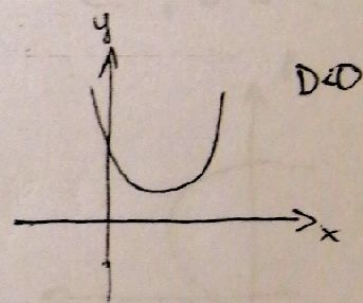
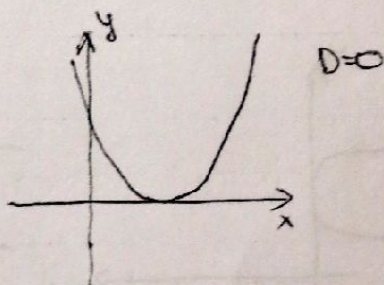
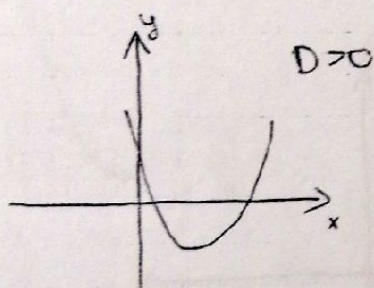
$$y = kx + n$$



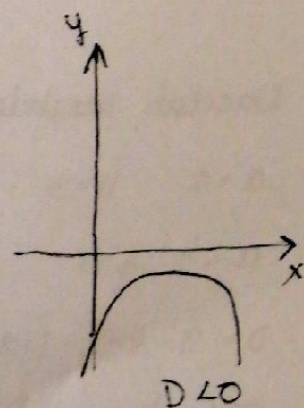
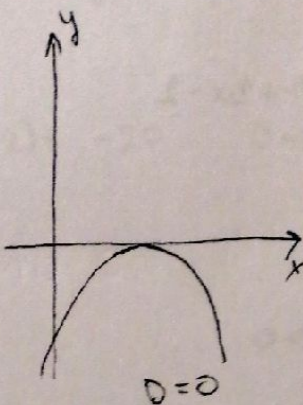
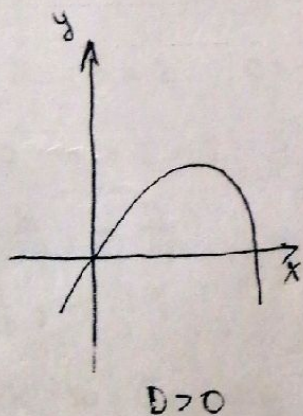
② Parabola

$$y = ax^2 + bx + c, \quad D = b^2 - 4ac \quad T\left(-\frac{b}{2a}, -\frac{D}{4a}\right)$$

I ako je $a > 0$, U



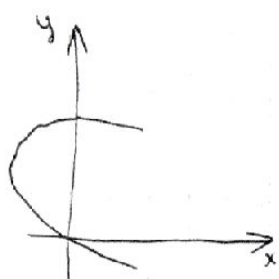
II ako je $a < 0$, N



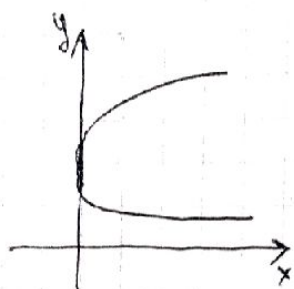
$$x = ay^2 + by + c$$

$$D = b^2 - 4ac, \quad T\left(-\frac{D}{4a}, \frac{-b}{2a}\right)$$

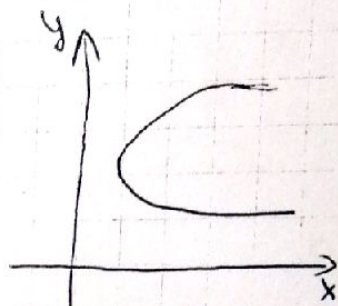
→ $a > 0, \subset$



$$D > 0$$

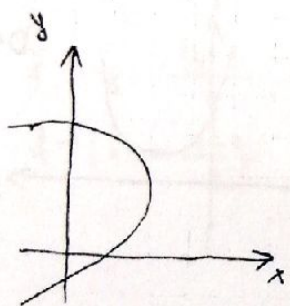


$$D = 0$$

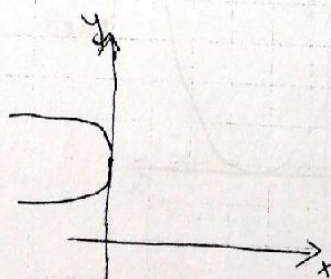


$$D < 0$$

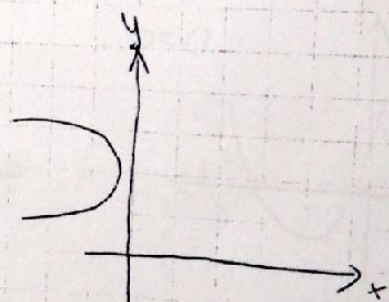
→ $a < 0, \supset$



$$D > 0$$



$$D = 0$$



$$D < 0$$

① Nacrtati parabolu, $y = 2x^2 + 3x - 2$

$$a = 2 \quad b = 3 \quad c = -2$$

$$a = 2 > 0 \quad \cup$$

$$D = 9 - 4 \cdot 2 \cdot (-2) = 25 > 0$$

$$T\left(-\frac{b}{2a}, -\frac{D}{4a}\right)$$

$$T\left(-\frac{3}{4}, -\frac{25}{8}\right)$$

Presjek sa OX-osom ($y = 0$)

$$2x^2 + 3x - 2 = 0$$

$$x_{1,2} = \frac{-3 \pm \sqrt{9 - 4 \cdot 2 \cdot (-2)}}{4}$$

$$x_{1,2} = \frac{-b \pm \sqrt{D}}{2a}$$

$$x_1 = \frac{2}{4} = \frac{1}{2} \quad x_2 = -\frac{8}{4} = -2$$

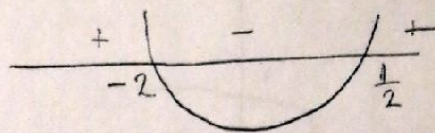
$$N_1(\frac{1}{2}, 0) \quad N_2(-2, 0)$$

Presjek sa Oxy-osom ($x=0$)

$$y = 2 \cdot 0 + 3 \cdot 0 - 2$$

$$y = -2$$

$$A(0, -2)$$



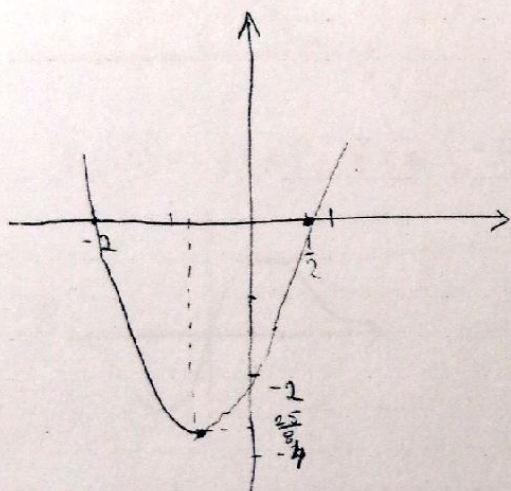
$$y > 0, x \in (-\infty, -2) \cup (\frac{1}{2}, +\infty)$$

$$y < 0, x \in (-2, \frac{1}{2})$$

ili tabela

$$y = 2 \cdot (x - \frac{1}{2}) \cdot (x - (-2))$$

$$y = 2 \cdot (x - \frac{1}{2}) \cdot (x + 2)$$



domaći :

Nacrtati parabolu $-3x^2 + 2x - 2 = y$

$$a = -3 \quad b = 2 \quad c = -2$$

$$D = b^2 - 4ac = 2^2 - 4(-3)(-2) = -20 \quad D < 0$$

$$T(-\frac{b}{2a}, -\frac{D}{4a})$$

$$T(\frac{1}{3}, -\frac{5}{3})$$

Presjek sa Ox-osom ($y=0$)

$$-3x^2 + 2x - 2 = 0$$

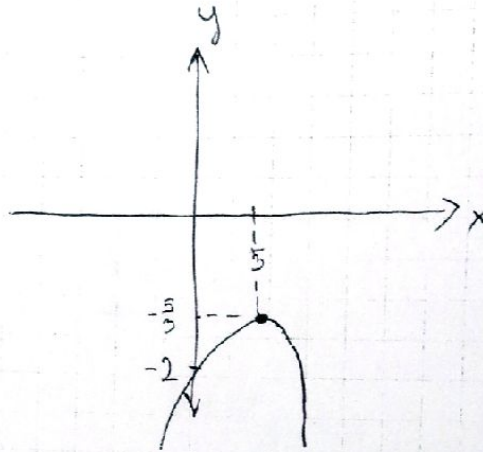
$$x_{1,2} = \frac{-2 \pm \sqrt{4 - 4 \cdot (-3) \cdot (-2)}}{-6}$$

Nema presjeka!

Przesiek sa Oy-osem ($x=0$)

$$y = -2$$

$$A(0, -2)$$



2. Nacrtati $x = -3y^2 + 4y - 1$

$$a = -3 \quad b = 4 \quad c = -1$$

∴

$>$ je je < 0

$$D = b^2 - 4ac = 16 - 4(-3)(-1) = 4 > 0$$

$$T\left(-\frac{D}{4a}, -\frac{b}{2a}\right)$$

$$T\left(\frac{1}{3}, \frac{2}{3}\right)$$

Przesiek sa Oy-osem ($x=0$)

$$-3y^2 + 4y - 1 = 0$$

$$y_{1,2} = \frac{-4 \pm \sqrt{16-12}}{-6}$$

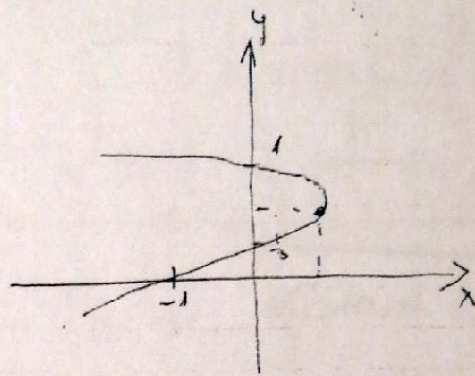
$$y_1 = \frac{1}{3} \quad y_2 = 1$$

$$M_1\left(0, \frac{1}{3}\right) \quad M_2(0, 1)$$

Presjek sa Ox-osom ($y=0$)

$$x = -1$$

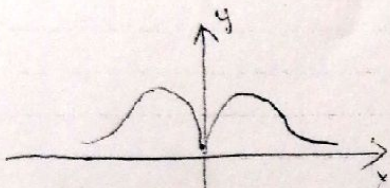
$$B(-1, 0)$$



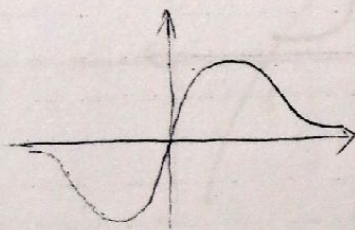
$x_1 < x_2 \Rightarrow f(x_1) < f(x_2)$, $\forall x_1, x_2 \in (a, b)$ f je rastuća

$x_1 < x_2 \Rightarrow f(x_1) > f(x_2)$, $\forall x_1, x_2 \in (a, b)$ f je opadajuća

$f(-x) = f(x)$, $\forall x \in (-a, a)$, f je parna na (a, b)



$f(-x) = -f(x)$, $\forall x \in (-a, a)$, f je neparna



Primeri: Ispitati parnost funkcije

a) $f(x) = 2x^4 - x^2 + 13$

$$f(-x) = 2 \cdot (-x)^4 - (-x)^2 + 13 = 2x^4 - x^2 + 13 = f(x) \rightarrow f \text{ je parna}$$

b) $f(x) = \frac{x^3 - 2x}{\cos x}$

$$f(-x) = \frac{(-x)^3 - 2 \cdot (-x)}{\cos(-x)} = \frac{-x^3 - 2x}{\cos x} = -\frac{x^3 - 2x}{\cos x} = -f(x) \rightarrow f \text{ je neparna}$$