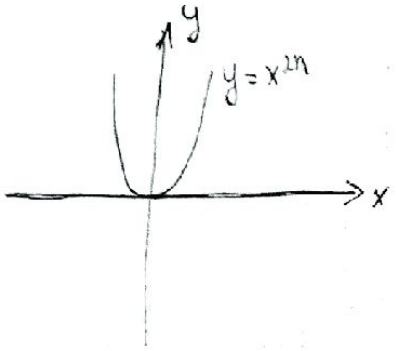
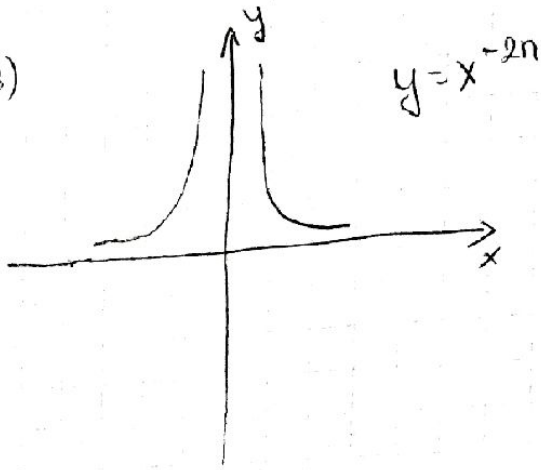
③ Stepene funkcije

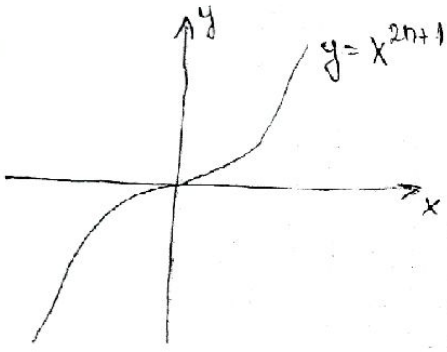
a)

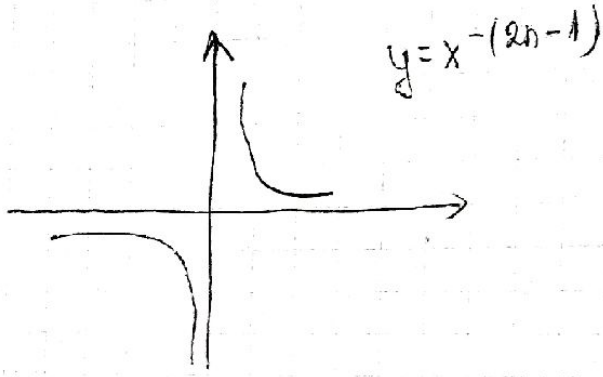


b)

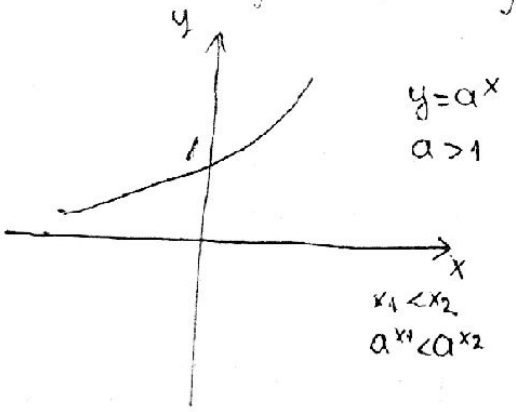


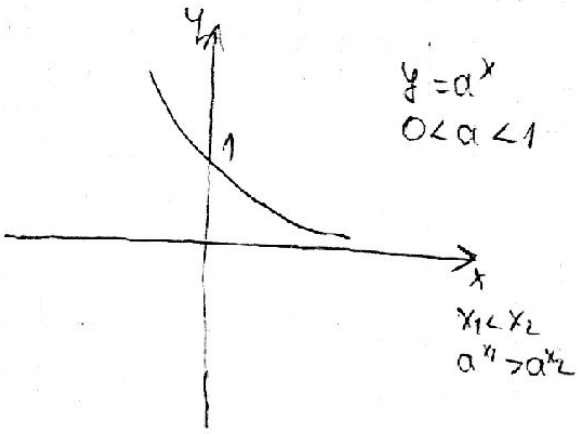
c)

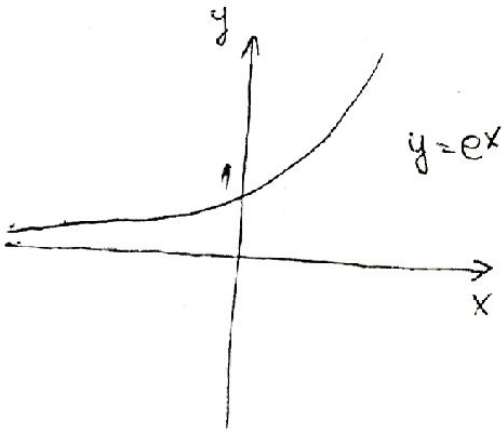


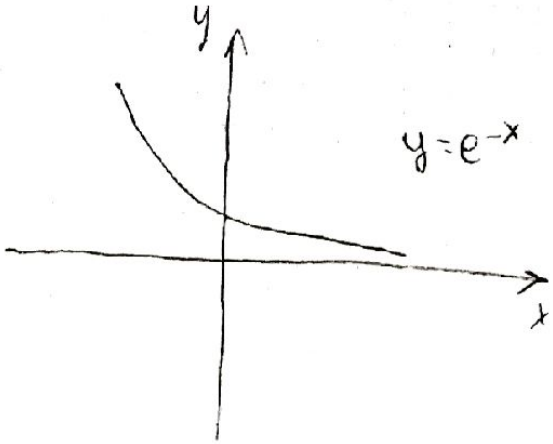


④ Eksponencijalne funkcije



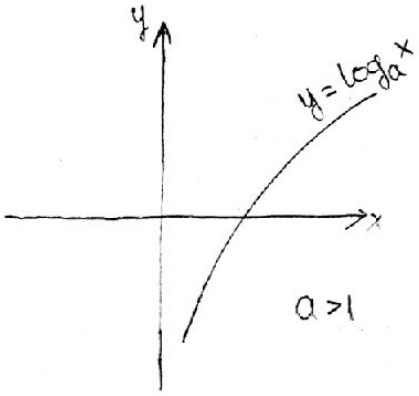


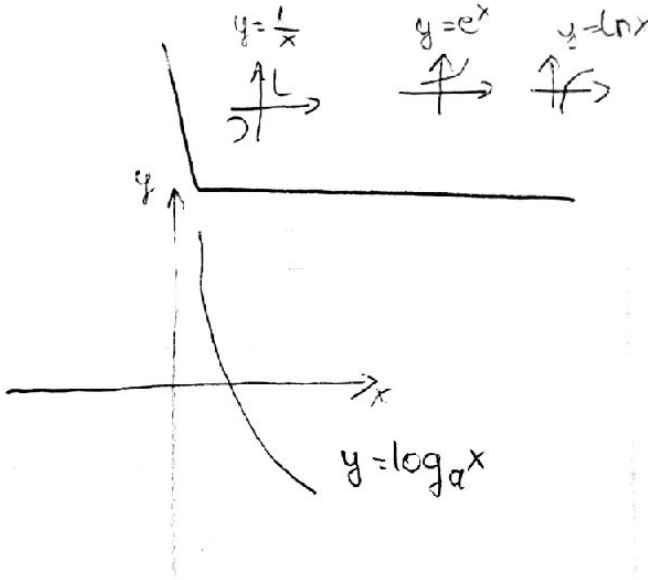


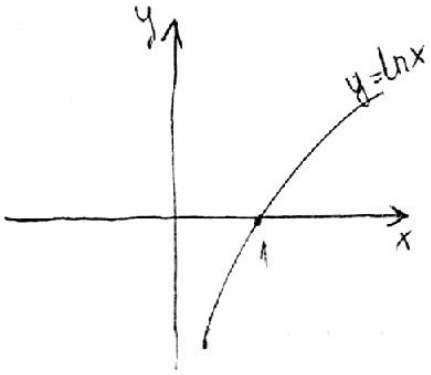


Vježba 5. 08.10.2019 III sedmica

⑤ Logaritamske funkcije





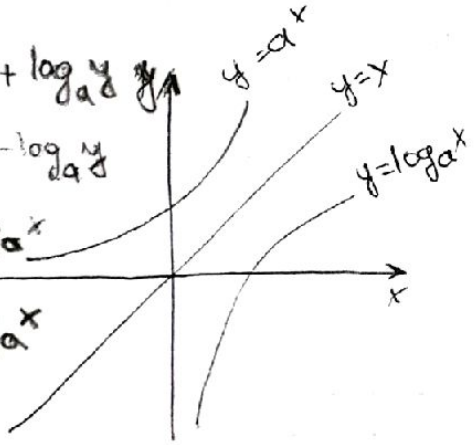


$\log_a(xy) = \log_a x + \log_a y$

$\log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y$

$\log_a x^n = n \log_a x$

$\log_{a^n} x = \frac{1}{n} \log_a x$

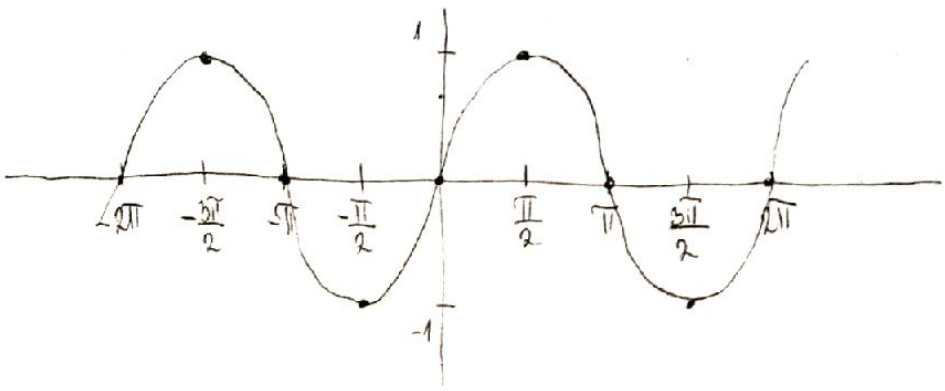


⑥ Trigonometrijske funkcije

1) $y = \sin x$

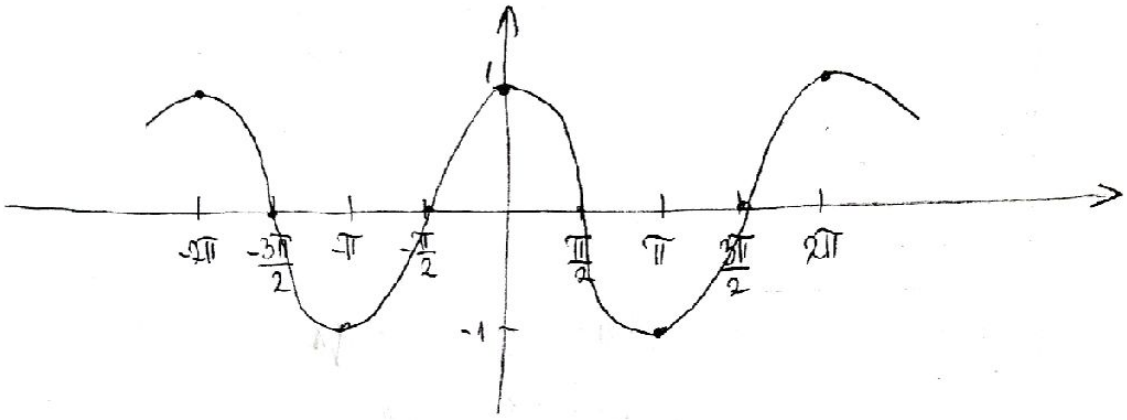
$\sin(-x) = -\sin x$

$\sin x$ - neparna f.

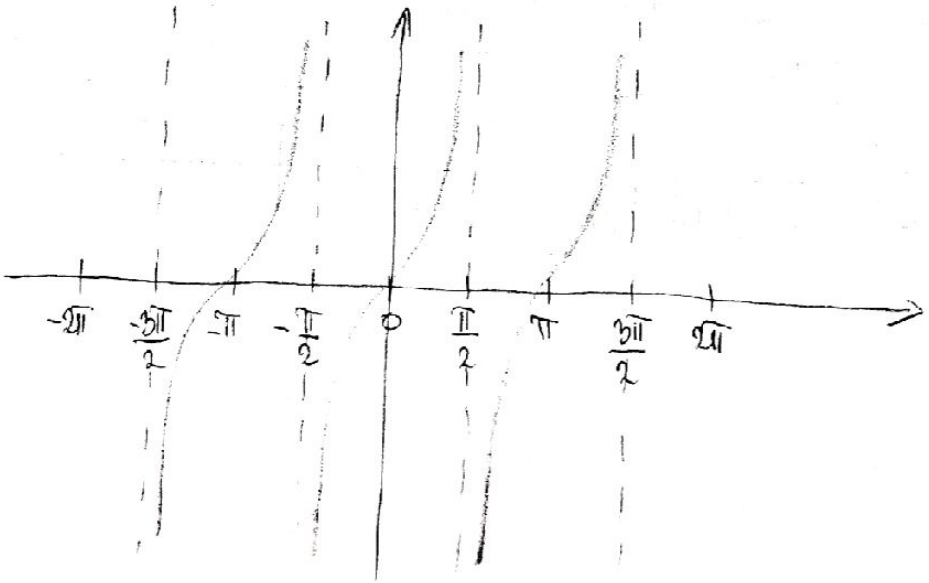


2) $y = \cos x$

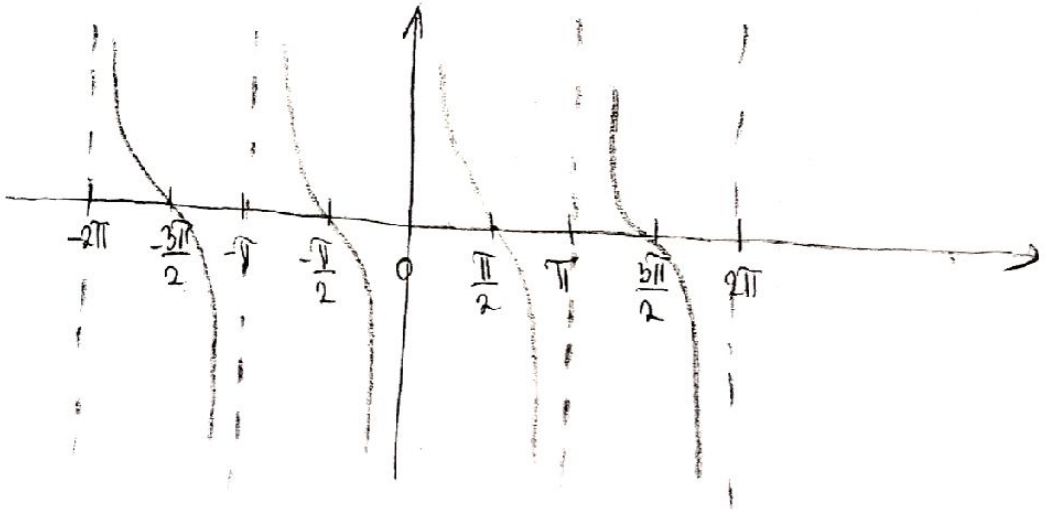
$\cos(x) = \cos x$ - para f.



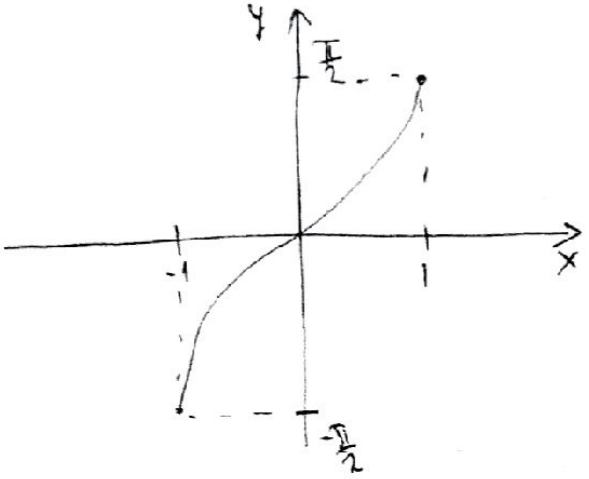
3) $y = tgx$ $\quad D: R \setminus \{k\frac{\pi}{2}\}\ (2k+1)\frac{\pi}{2}$



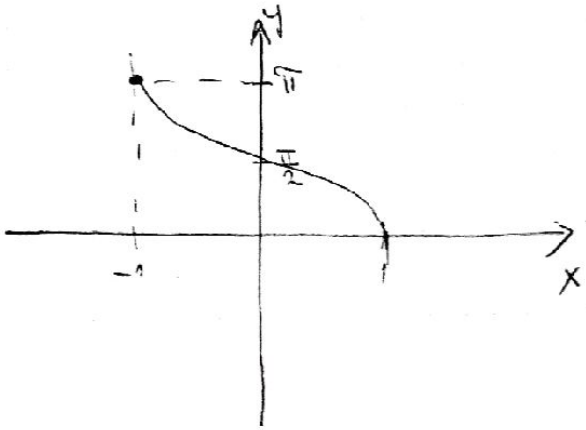
4) $y = ctgx$ $\quad ctg: R \setminus \{k\pi\} \to R$



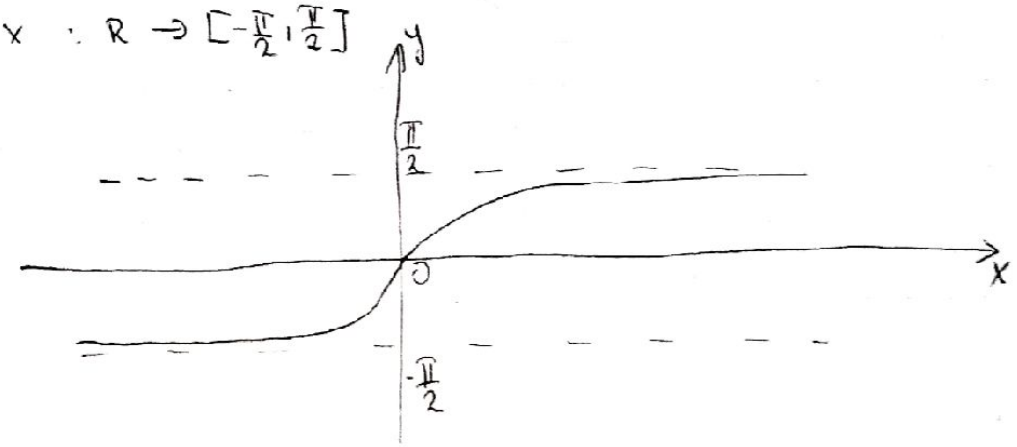
5) $y = \arcsin x$ $\quad \arcsin : [-1,1] \to [-\frac{\pi}{2}, \frac{\pi}{2}]$



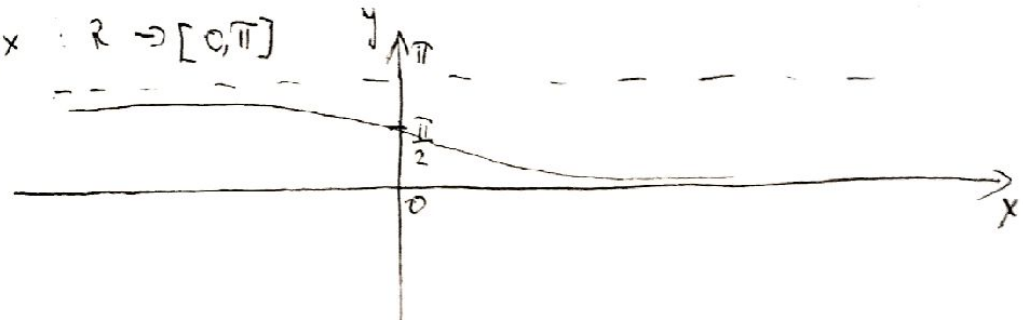
6) $y = \arccos$ $\quad \arccos : [-1,1] \to [0,\pi]$



7) $\operatorname{arctg} x : \mathbb{R} \to [-\frac{\pi}{2}, \frac{\pi}{2}]$



8) $\operatorname{arctg} x : \mathbb{R} \to [0,\pi]$



1.

(*) $f(x)=\sqrt{x}$

$Df=[0,+\infty)$

(*) $f(x)=-\frac{1}{x}$

$Df=(-\infty,0)\cup(0,+\infty)$

(*) $f(x)=\log a^x$, $a>0$, $a\neq 1$

$Df=(0,+\infty)$

(*) $f(x)=\sqrt{x^2-2}$

3.

$x^2-2\geq 0$

$x^2\geq 2$

$|x|\geq\sqrt{2}$

$Df=(-\infty,-\sqrt{2}]\cup[\sqrt{2},+\infty)$

(*) $f(x)=\frac{5}{2+\sin x}+\sqrt{x+3}$

1° $2+\sin x\neq 0$

(Uočimo da je $2+\sin x\neq 0$ $\forall x\in\mathbb{R}$ jer je $-1\leq\sin x\leq 1$ $\forall x\in\mathbb{R}$)

$D_1=\mathbb{R}$

2° $x+3\geq 0$

$x\geq -3$

~~$x\in(-\infty,-3)$~~ $x\in[-3,+\infty)$

$D_2=[-3,+\infty)$

$Df=D_1\cap D_2=[-3,+\infty)$

4.

(*) $f(x)=\log_2 2x^2+3x-2$

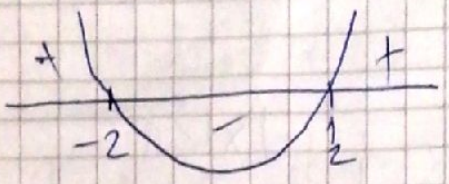
$2x^2+3x-2>0$

$2x^2+3x-2=0$

$x_{1/2}=\frac{-3\pm\sqrt{9+16}}{4}$

$x_1 = \frac{1}{2} \quad x_2 = -2$

$2x^2 + 3x - 2 = 2(x+2)\cdot(x-\frac{1}{2})$



$Df = (-\infty, -2) \cup (\frac{1}{2}, +\infty)$

⊛ $f(x) = \sqrt{\frac{1}{x} - 1} + \ln 2x + 1$

$1^\circ \quad \frac{1}{x} - 1 \geq 0 \wedge x \neq 0$

$\frac{1-x}{x} \geq 0 \wedge x \neq 0$

	$-\infty$	0	1	$+\infty$
$1-x$	+	+	0	−
x	−	+	+	+
	⊖	⊕	⊖	

$D_1 = (0, 1]$

$2^\circ \quad 2x > 0$

$x > 0$

$D_2 = (0, +\infty)$

$Df = D_1 \cap D_2 = (0, 1]$

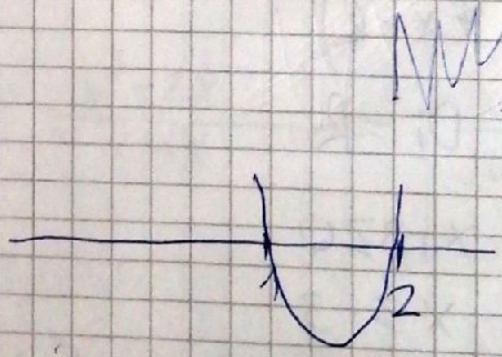
⊛ $f(x) = \frac{x+2}{\sqrt{x^2 - 3x + 2}}$

$x^2 - 3x + 2 > 0$

$x^2 - 3x + 2 = 0$

$x_{1/2} = \frac{3 \pm \sqrt{9-8}}{2}$

$x_1 = 2 \quad x_2 = 1$



$Df = (-\infty, 1) \cup (2, +\infty)$

⊛ $f(x) = \sqrt{\arcsin(\log_2 x)}$

$1^\circ \quad \arcsin(\log_2 x) \geq 0$

$0 \leq \log_2 x \leq 1$

$\log_2 1 \leq \log_2 x \leq \log_2 2$

$1 \leq x \leq 2$

$D_1 = [1, 2]$

2^0 $x > 0$ zbog $\log_2 x$ imamo uvjet

$D_2 = (0, +\infty)$

$D_f = D_1 \cap D_2 = [1, 2]$

Transformacije grafika

① Horizontalno istezanje i skupljanje grafika fje

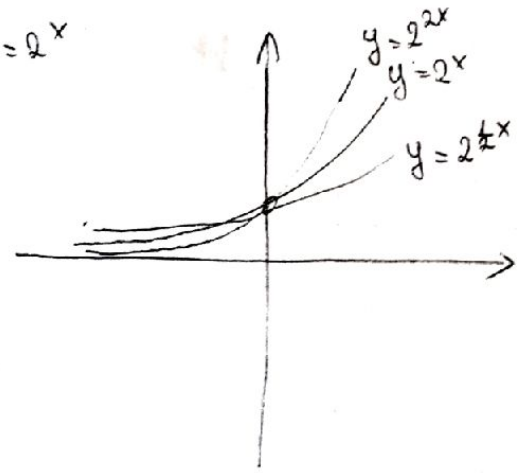
$y = f(x)$

$y = f(cx)$, $c > 0$

a) $c > 1$, horizontalno skupljanje

b) $0 < c < 1$, horizontalno istezanje

Primjer $y = 2^x$

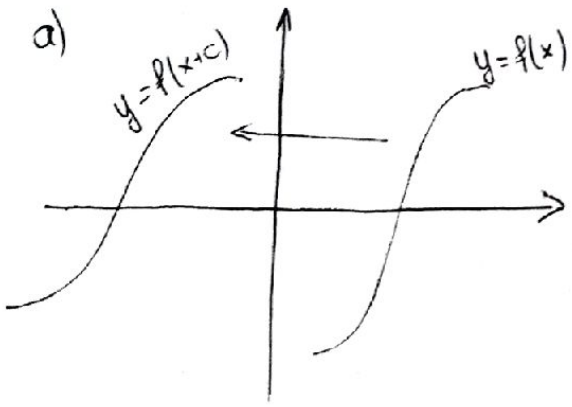


② Horizontalno pomjeranje grafika

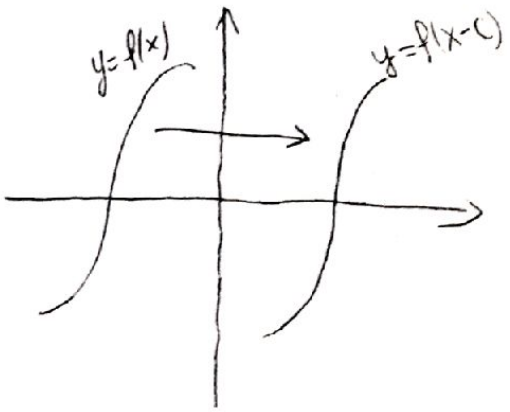
a) $y = f(x + c)$, $c > 0$, ulijevo

b) $y = f(x - c)$, $c > 0$, udesno

a)



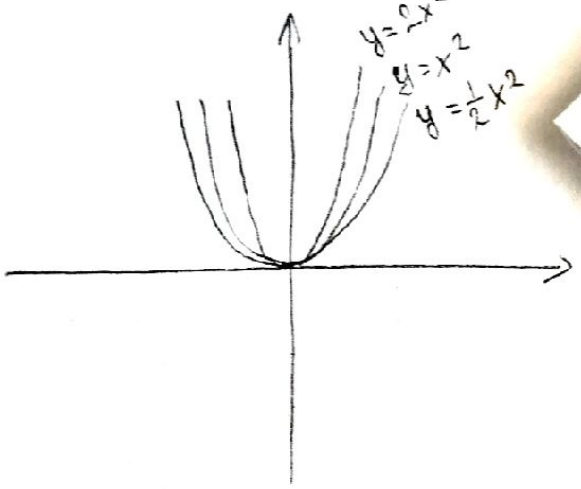
b)



③ Vertikalno istezanje i stiskanje

a) $y = c \cdot f(x)$, $c > 1$ istezanje

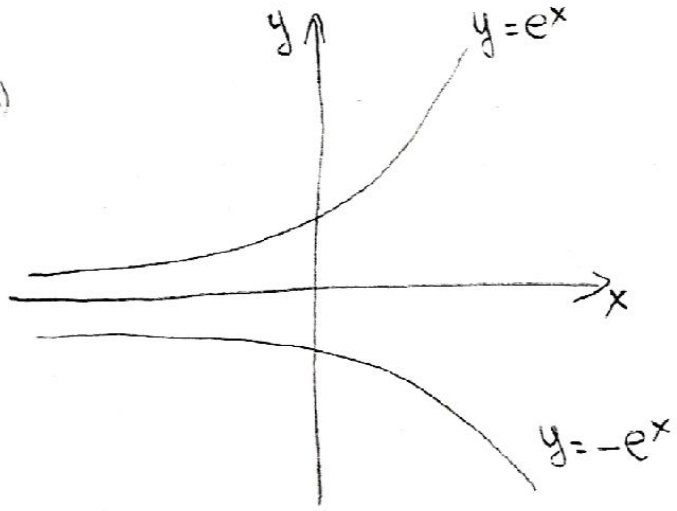
b) $y = c \cdot f(x)$, $0 < c < 1$ stiskanje



④ Odraz grafika oko Ox-osi

$y = f(x)$

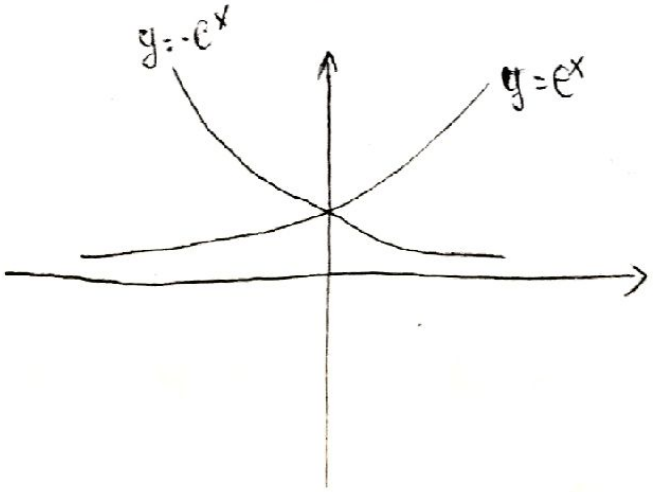
$y = -f(x)$



⑤ Odraz grafika oko Oy-osi

$y = f(x)$

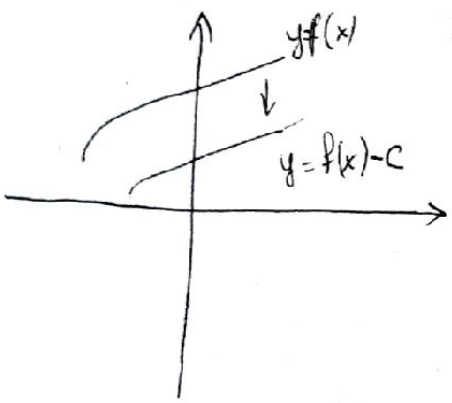
$y = f(-x)$

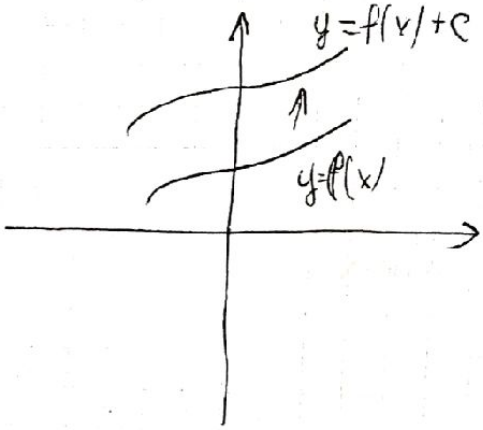


⑥ Vertikalno pomjeranje grafika

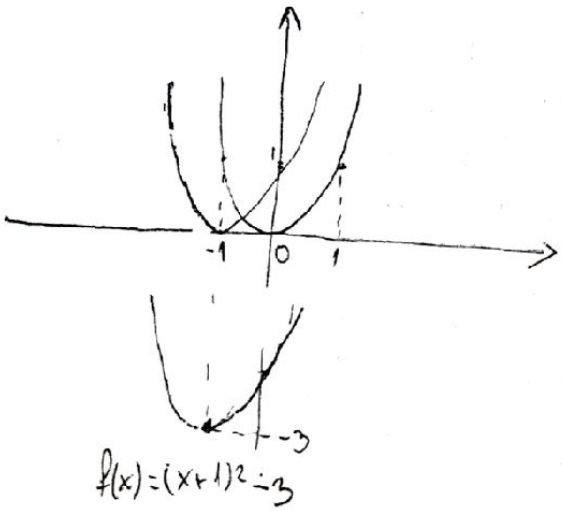
a) $y = f(x) + C$, podižemo grafik

b) $y = f(x) - C$, spuštamo grafik

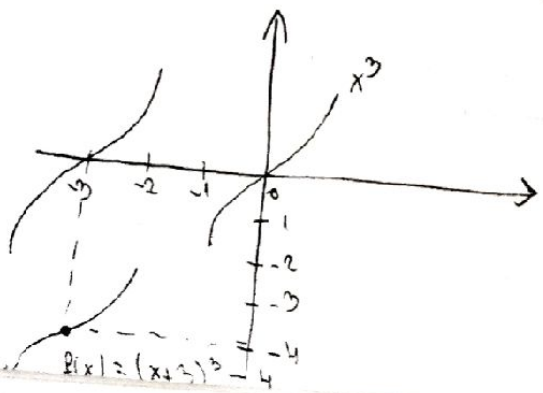




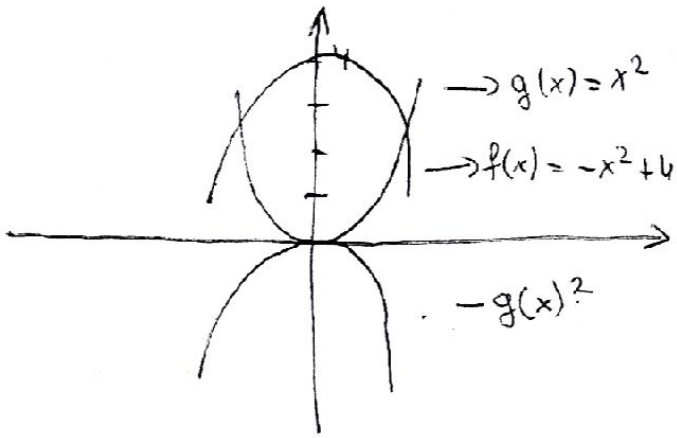
① Ispitati grafik funkcije $f(x) = x^2$ i nacrtati grafik f-je $f(x) = (x+1)^2 - 3$



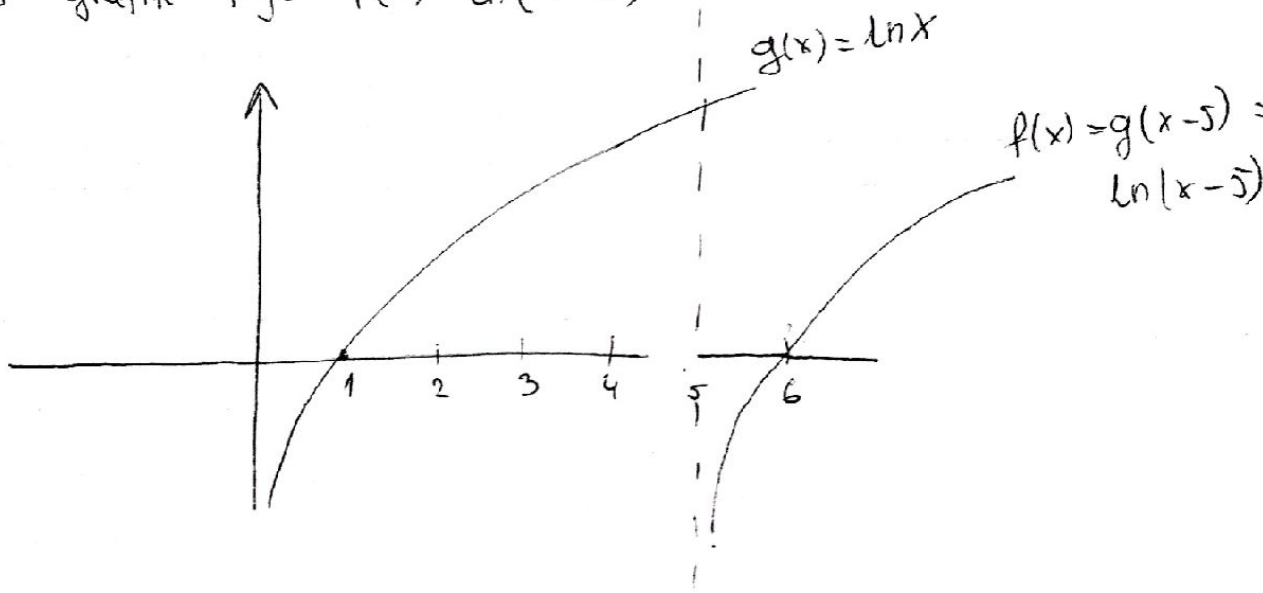
② Iskoristiti grafik f-je $f(x) = x^3$ i nacrtati grafik za funk. $f(x) = (x+3)^3 - 4$



③ Ispitivati grafik f-je $f(x) = -x^2 + 4$



④ Skicirati grafik f-je $f(x) = \ln(x-5)$



Vježba: $e^{-3x} + 2$

$e^{2-x} - 1$

$\frac{1}{2}x + 1$

$\left(\frac{1}{2}\right)^{3-x} - 2$