

## Izvodi

Neka je realna f-ja  $f$  definirana na otvorenom intervalu  $(a, b)$  i neka je  $x_0 \in (a, b)$ .

Granična vrijednost  $\lim_{\Delta x \rightarrow 0} \frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x}$  - priznajući taj izvod

ako postoji nazivamo prvim izvodom f-je  $f$  u tački  $x_0$ , i označavamo:

$$f'(x_0) = \lim_{x \rightarrow x_0} \frac{f(x) - f(x_0)}{x - x_0}$$

lijevi i desni izvod:

$$f'_+(x_0) = \lim_{x \rightarrow x_0^+} \frac{f(x) - f(x_0)}{x - x_0} \quad \text{desni izvod f-je u tački } x_0$$

$$f'_-(x_0) = \lim_{x \rightarrow x_0^-} \frac{f(x) - f(x_0)}{x - x_0} \quad \text{lijevi izvod f-je u tački } x_0$$

F-ja  $f$  je diferencijabilna u tački  $x_0$  ako i samo ako:  $f'_+(x_0) = f'_-(x_0)$  (su lijevi i desni izvodi jednaki).

Diferencijabilnost povlači neprekidnost. Ako je f-ja diferencijabilna onda je i neprekidna.

Obrnuto ne važi. (6) +

① Po definiciji naći izvod f-je:

$$f(x) = \sqrt{1+2x}$$

Neka je  $x_0 \in \mathbb{R}$  proizvoljna tačka.

$$f'(x_0) = \lim_{\Delta x \rightarrow 0} \frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x} = \lim_{\Delta x \rightarrow 0} \frac{\sqrt{1+2(x_0 + \Delta x)} - \sqrt{1+2x_0}}{\Delta x}$$

$$= \lim_{\Delta x \rightarrow 0} \frac{\sqrt{1+2(x_0 + \Delta x)} - \sqrt{1+2x_0}}{\Delta x} \cdot \frac{\sqrt{1+2(x_0 + \Delta x)} + \sqrt{1+2x_0}}{\sqrt{1+2(x_0 + \Delta x)} + \sqrt{1+2x_0}} =$$

$$= \lim_{\Delta x \rightarrow 0} \frac{1 - 2(x_0 + \Delta x) - (1 - 2x_0)}{\Delta x (\sqrt{1 - 2(x_0 + \Delta x)} + \sqrt{1 - 2x_0})} = \lim_{\Delta x \rightarrow 0} \frac{2 \cdot \Delta x}{\Delta x (\sqrt{1 - 2(x_0 + \Delta x)} + \sqrt{1 - 2x_0})}$$

$$= \frac{2}{\sqrt{1 - 2x_0} + \sqrt{1 - 2x_0}} = \frac{1}{\sqrt{1 - 2x_0}} \quad ; \quad x_0 \neq -\frac{1}{2}$$

$$f'(x) = \frac{1}{\sqrt{1 - 2x}} \quad ; \quad x \neq -\frac{1}{2}$$

$$f(x) = x^2$$

$$f'(x) = 2x \quad ;$$

$$f'(-1) = 2 \cdot (-1) = -2$$

$$f(-1) = (-1)^2 = 1$$

$$f'(1) = 0$$

② Ispitati diferencijabilnost funkcije  $f(x) = \frac{1}{2} \cdot \sqrt{(x-2)^2}$  u tački  $x_0 = 2$

- Napomena: Funkcija  $f$  je diferencijabilna u tački  $x_0$  ako i samo ako je

$$f'_+(x_0) = f'_-(x_0)$$

- Napomena: Diferencijabilnost povlači neprekidnost (obezbeđeno ne važi!)  
Neprekidnost ne povlači diferencijabilnost (u opštem slučaju)

$$f'_+(2) = \lim_{\Delta x \rightarrow 0^+} \frac{f(2+\Delta x) - f(2)}{\Delta x} = \lim_{\Delta x \rightarrow 0^+} \frac{\frac{1}{2} \sqrt{(2+\Delta x - 2)^2} - 0}{\Delta x} =$$

$$= \lim_{\Delta x \rightarrow 0^+} \frac{\frac{1}{2} \sqrt{\Delta x^2}}{\Delta x} = \lim_{\Delta x \rightarrow 0^+} \frac{\frac{1}{2} |\Delta x|}{\Delta x} = \lim_{\Delta x \rightarrow 0^+} \frac{\frac{1}{2} \cdot \Delta x}{\Delta x} = \frac{1}{2}$$

$$f'_-(2) = \lim_{\Delta x \rightarrow 0^-} \frac{f(2+\Delta x) - f(2)}{\Delta x} = \dots = \lim_{\Delta x \rightarrow 0^-} \frac{\frac{1}{2} \cdot (-\Delta x)}{\Delta x} = -\frac{1}{2}$$

$f'_+(2) \neq f'_-(2) \Rightarrow$  funkcija nije diferencijabilna u tački  $x_0 = 2$

③ Odrediti parametre  $a, b$  tako da funkcija  $f(x) = \begin{cases} x^2, & x \leq 1 \\ ax+b, & x > 1 \end{cases}$  bude diferencijabilna na  $\mathbb{R}$ .

\* Na intervalima  $(-\infty, 1)$  i  $(1, +\infty)$  f-ja se podlapa sa polinomima  $x^2$  i  $ax+b$  pa je diferencijabilna jer su polinomi dif. fu-je

Ostaje da odredimo parametre  $a, b$  tako da f-ja bude diferencijabilna i u tački  $x_0 = 1$

Potreban ali ne i dovoljan uslov da f-ja  $f$  bude diferencijabilna u tački  $x_0 = 1$  je da ona u njoj bude neprekidna tj. da važi:

$$\lim_{x \rightarrow 1} f(x) = \lim_{x \rightarrow 1^+} f(x) = \lim_{x \rightarrow 1^-} f(x) = f(1) \quad *$$

$$\lim_{x \rightarrow 1^+} f(x) = \lim_{x \rightarrow 1^+} (ax+b) = a+b$$

$$\lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^-} x^2 = 1$$

$$f(1) = 1^2 = 1$$

$$(*) \Rightarrow a+b=1$$

\* Da bi funkcija  $f$  bila diferencijabilna u tački  $x_0=1$  potrebno je i dovoljno

da važi  $f'_+(1) = f'_-(1)$  (\*\*)

$$f'_+(1) = \lim_{\Delta x \rightarrow 0^+} \frac{f(1+\Delta x) - f(1)}{\Delta x} = \lim_{\Delta x \rightarrow 0^+} \frac{a \cdot (1+\Delta x) + b - 1}{\Delta x} =$$

$$\lim_{\Delta x \rightarrow 0^+} \frac{a + a \cdot \Delta x + b - 1}{\Delta x} = \lim_{\Delta x \rightarrow 0^+} \frac{a \cdot \Delta x}{\Delta x} = a$$

$$f'_-(1) = \lim_{\Delta x \rightarrow 0^-} \frac{f(1+\Delta x) - f(1)}{\Delta x} = \lim_{\Delta x \rightarrow 0^-} \frac{(1+\Delta x)^2 - 1}{\Delta x} = \lim_{\Delta x \rightarrow 0^-} \frac{1 + 2\Delta x + \Delta x^2 - 1}{\Delta x} =$$

$$= \lim_{\Delta x \rightarrow 0^-} \frac{\Delta x(2 + \Delta x)}{\Delta x} = 2$$

$$(**) \Rightarrow a=2$$

$$\begin{cases} a+b=1 \\ a=2 \end{cases} \Rightarrow \begin{cases} a=2 \\ b=-1 \end{cases}$$

### TABLICA OSNOVNIH IZVODA

1)  $(c)' = 0$

2)  $(x^n)' = n \cdot x^{n-1}$

3)  $(\sqrt{x})' = \frac{1}{2\sqrt{x}}$

4)  $(x)' = 1$

5)  $(e^x)' = e^x$

6)  $(a^x)' = a^x \cdot \ln a$

7)  $(\ln x)' = \frac{1}{x}$

8)  $(\log_a x)' = \frac{1}{x \cdot \ln a}$ ,  $a > 0$ ,  $a \neq 1$

9)  $(\sin x)' = \cos x$

10)  $(\cos x)' = -\sin x$

11)  $(\operatorname{tg} x)' = \frac{1}{\cos^2 x}$

12)  $(\operatorname{ctg} x)' = -\frac{1}{\sin^2 x}$

Ulogak od  $12^n$   
 srijedom od  $10^k$   
 135. kabinet desno od b. b)

$$13. (\arcsin x)' = \frac{1}{\sqrt{1-x^2}}, \quad |x| < 1$$

$$14. (\arccos x)' = \frac{-1}{\sqrt{1-x^2}}, \quad |x| < 1$$

$$15. (\operatorname{arctg} x)' = \frac{1}{1+x^2}$$

$$16. (\operatorname{arcctg} x)' = \frac{-1}{1+x^2}$$

Osobine :

$$1) (c \cdot f)' = c \cdot f', \quad c \in \mathbb{R}$$

$$2) f \pm g = f' \pm g'$$

$$3) (f \cdot g)' = f' \cdot g + f \cdot g'$$

$$4) \left(\frac{f}{g}\right)' = \frac{f' \cdot g - f \cdot g'}{g^2}, \quad g \neq 0$$

$$\checkmark \textcircled{1} f(x) = x^4$$

$$f'(x) = 4x^3$$

$$\checkmark \textcircled{2} f(x) = (5x^3 + 4x)^4$$

$$\left\{ \begin{array}{l} f = f(u), \quad u = g(x) \\ f' \cdot x = f' \cdot u \cdot u' \cdot x \end{array} \right.$$

$$f(x) = u^4, \quad u = 5x^3 + 4x$$

$$f'(x) = 4 \cdot u^3 \cdot u' \cdot x$$

$$f'(x) = 4(5x^3 + 4x)^3 \cdot (15x^2 + 4)$$

① Nadi prvi izvod f-je:

$$y = \sqrt[3]{1+x^4}$$

$$y = (1+x^4)^{\frac{1}{3}}$$

$$y'(x) = \frac{1}{3} (1+x^4)^{\frac{1}{3}-1} \cdot (1+x^4)'$$

$$y'(x) = \frac{1}{3} (1+x^4)^{-\frac{2}{3}} \cdot (0+4x^3) = \frac{4x^3}{3 \cdot (1+x^4)^{\frac{2}{3}}} = \frac{4x^3}{3 \cdot \sqrt[3]{(1+x^4)^2}}$$

$$(\sqrt{x})' = \frac{1}{2\sqrt{x}}$$

$$(x^n)' = n \cdot x^{n-1}$$

②  $y = \cos(2x^3 + 5x)$

$$(\cos x)' = -\sin x$$

$$y'(x) = -\sin(2x^3 + 5x) \cdot (2x^3 + 5x)'$$

$$= -\sin(2x^3 + 5x) \cdot (6x^2 + 5)$$

$$= -(6x^2 + 5) \cdot \sin(2x^3 + 5x)$$

③  $y = \sin^3 \ln x^5$

$$\sin^2 x = (\sin x)^2$$

$$y = (\sin(\ln x^5))^3$$

$$\neq \sin x^2$$

$$y'(x) = 3 \cdot (\sin(\ln x^5))^2 \cdot (\sin(\ln x^5))'$$

$$= 3 (\sin(\ln x^5))^2 \cdot \cos(\ln x^5) \cdot (\ln x^5)'$$

$$= 3 (\sin(\ln x^5))^2 \cdot \cos(\ln x^5) \cdot \frac{1}{x^5} \cdot (x^5)'$$

$$(\ln x)' = \frac{1}{x}$$

$$= 3 (\sin(\ln x^5))^2 \cdot \cos(\ln x^5) \cdot \frac{1}{x^5} \cdot 5x^4$$

$$① y = \operatorname{tg}(2 \cdot e^{7x})$$

$$y'(x) = \frac{1}{\cos^2(2 \cdot e^{7x})} \cdot (2 \cdot e^{7x})'$$

konstante uvoziemo da izvedemo

$$\operatorname{tg}' = \frac{1}{\cos^2 x}$$

$$y'(x) = \frac{1}{\cos^2(2 \cdot e^{7x})} \cdot 2 \cdot (e^{7x})'$$

$$y'(x) = \frac{1}{\cos^2(2 \cdot e^{7x})} \cdot 2 \cdot e^{7x} \cdot (7x)'$$

$$y'(x) = \frac{1}{\cos^2(2 \cdot e^{7x})} \cdot 2 \cdot e^{7x} \cdot 7$$

$$② y = (3^2 \cdot \operatorname{tg}^3 x)^{3^x}$$

$$a^x = a^x \cdot \ln a$$

$$y'(x) = 3^2 \cdot \operatorname{tg}^3 x \cdot \ln 3 \cdot (2 \cdot \operatorname{tg}^3 x)'$$

konst. izvedemo

$$\operatorname{tg}^3 x = (\operatorname{tg} x)^3$$

$$y'(x) = 3^2 \cdot \operatorname{tg}^3 x \cdot \ln 3 \cdot 2 \cdot (\operatorname{tg}^3 x)'$$

$$= 3^2 \cdot \operatorname{tg}^3 x \cdot \ln 3 \cdot 2 \cdot ((\operatorname{tg} x)^3)'$$

$$= 3^2 \cdot \operatorname{tg}^3 x \cdot \ln 3 \cdot 2 \cdot 3 \cdot (\operatorname{tg} x)^2 \cdot (\operatorname{tg} x)'$$

$$= 3^2 \cdot \operatorname{tg}^3 x \cdot \ln 3 \cdot 6 (\operatorname{tg} x)^2 \cdot \frac{1}{\cos^2 x}$$

$$③ y = e^{x \cdot \ln(x+2)}$$

uao proizvod:  $f' \cdot g + f \cdot g'$

$$y'(x) = e^{x \cdot \ln(x+2)} \cdot (x \cdot \ln(x+2))'$$

$$y'(x) = e^{x \cdot \ln(x+2)} \cdot \left( 1 \cdot \ln(x+2) + x \cdot \frac{1}{x+2} \right) \cdot (x+2)'$$

$$= e^{x \cdot \ln(x+2)} \cdot \left( \ln(x+2) + \frac{x}{x+2} \right)$$

$$\frac{1}{2} \cdot \frac{1}{e^{2x}} \cdot \frac{-2e^{-2x}}{e^{2x}-1} = \frac{1}{1-e^{2x}}$$

$$y = x^{a^d} + a^{x^d} + a^{a^x}$$

$$\begin{cases} (x^n)' = n x^{n-1} \\ a^x = a^x \cdot \ln a \end{cases}$$

$$= a^d \cdot x^{a^d-1} + a^{x^d} \cdot \ln a \cdot (x^d)' + a^{a^x} \cdot \ln a \cdot (a^x)'$$

$$= a^d \cdot x^{a^d-1} + a^{x^d} \cdot \ln a \cdot d x^{d-1} + a^{a^x} \cdot \ln a \cdot a^x \cdot \ln a$$

Logaritmesai izvodi

$$y = x^x / \ln$$

$$\ln y = \ln x^x$$

$$\ln y = x \ln x$$



→ logaritamski izvodi ←

①  $y = x^x$  / ln

$$\ln x^n = n \cdot \ln x$$

$$\ln y = \ln x^x$$

$$\ln y = x \cdot \ln x \quad /'$$

$$\frac{1}{y} \cdot y' = 1 \cdot \ln x + x \cdot \frac{1}{x}$$

$$\frac{1}{y} \cdot y' = \ln x + 1$$

$$y' = y \cdot (\ln x + 1)$$

$$y' = x^x \cdot (\ln x + 1)$$

②  $y = x^{\sin x}$ ,  $x > 0$

$$\ln y = \ln x^{\sin x}$$

$$\ln y = \sin x \cdot \ln x \quad /'$$

$$\frac{1}{y} \cdot y' = \cos x \cdot \ln x + \sin x \cdot \frac{1}{x}$$

$$y' = y \left( \cos x \cdot \ln x + \frac{1}{x} \cdot \sin x \right)$$

$$y' = x^{\sin x} \left( \cos x \cdot \ln x + \frac{1}{x} \sin x \right)$$

⑧

## Izvod parametarski zadate funkcije

$$\begin{cases} x = f(t) \\ y = g(t) \end{cases}$$

$$y'_x = \frac{g'_t}{f'_t}$$

● Pokazati da funkcija  $x = 2t + 3t^2$  zadovoljava jednačinu  $2y^{13} + y^{12} - y = 0$   
 $y = t^2 + 2t^3$

$$x = f(t), \quad f(t) = 2t + 3t^2$$

$$y = g(t), \quad g(t) = t^2 + 2t^3$$

$$y'_x = \frac{g'_t}{f'_t}$$

$$g'_t = 2t + 6t^2$$

$$f'_t = 2 + 6t$$

$$y'_x = \frac{2t + 6t^2}{2 + 6t} = \frac{t(2 + 6t)}{2 + 6t} = t$$

$$M(5, 3)$$

$$y'_{(5)} = 1$$

Za  $x=5$ ,  $y=3$  imamo da je  $t=1$

Izvod implicitno zadate funkcije

● Naći prvi izvod  $f$   $y = f(x)$  ako je ona zadata implic. sledećom jedn.

$$x^2 + y^2 = 4 \quad / \quad '$$

$$2x + 2y \cdot y' = 0$$

$$x + y \cdot y' = 0$$

$$y' = -\frac{x}{y}$$

⑤ Neka je  $y=f(x)$  funkcija koja je zadata implicitno jednačinom  $x \cdot y + y^3 = 1$ . Naći  $y'$  u 0.

$$x \cdot y + y^3 = 1 \quad | \cdot$$

$$1 \cdot y + x \cdot y' + 3y^2 \cdot y' = 0$$

$$y + (x + 3y^2) \cdot y' = 0$$

$$y' = -\frac{y}{x + 3y^2}, \quad y = f(x)$$

$$y'(0) = ?$$

$$x=0, \quad \begin{cases} y=f(x) \\ y=f(0) \end{cases}$$

$$x=y+y^3=1$$

$$x=0 \quad 0 \cdot y + y^3 = 1$$

$$y^3 = 1, \quad y = 1$$

$$x=0, \quad f(0) = 1$$

⑥ Naći  $y'$  ako je funkcija  $y=f(x)$  zadata implicitno jednačinom

$$3x^2 \cdot y^3 - 5xy^2 = y \quad | \cdot$$

$$6x \cdot y^3 + 3x^2 \cdot 3y^2 \cdot y' - 5 \cdot y^2 - 5x \cdot 2y \cdot y' = y'$$

$$6x \cdot y^3 - 5y^2 = y' - 9x^2 y^2 \cdot y' + 10xy \cdot y'$$

$$6xy^3 - 5y^2 = y' (1 - 9x^2 y^2 + 10xy) \cdot y'$$

$$y' = \frac{6xy^3 - 5y^2}{1 - 9x^2 y^2 + 10xy}$$

⑦ Funkcija  $y$  je definisana jednačinom  $\operatorname{arctg}\left(\frac{y}{x}\right) = \frac{1}{2} \ln x^2 + y^2$ . Dokazati da funkcija zadovoljava jednačinu  $y' \cdot (x-y) = x+y$

$$\operatorname{arctg}\left(\frac{y}{x}\right) = \frac{1}{2} \ln(x^2 + y^2) \quad |'$$

$$\frac{1}{1 + \left(\frac{y}{x}\right)^2} \cdot \left(\frac{y}{x}\right)' = \frac{1}{2} \frac{1}{x^2 + y^2} \cdot (x^2 + y^2)'$$

$$\frac{1}{x^2 + y^2} \left(\frac{y}{x}\right)' = \frac{1}{2} \left(\frac{1}{x^2 + y^2} \cdot (2x + 2y \cdot y')\right)$$

$$\frac{x^2}{x^2 + y^2} \cdot \left(\frac{y}{x}\right)' = \frac{x + y \cdot y'}{x^2 + y^2}$$

$$x^2 \left(\frac{y}{x}\right)' = x + y \cdot y'$$

$$x^2 \cdot \frac{y' \cdot x - y \cdot 1}{x^2} = x + y \cdot y'$$

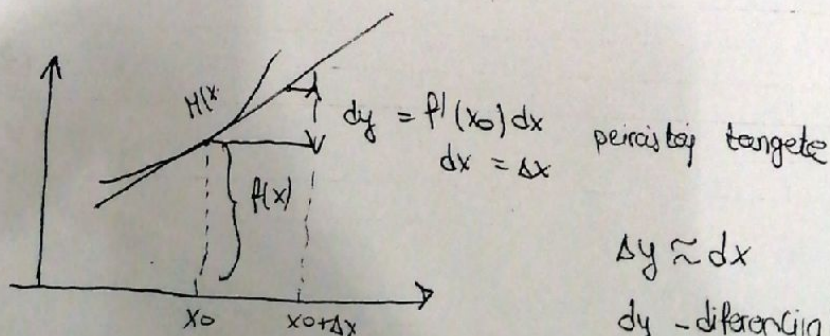
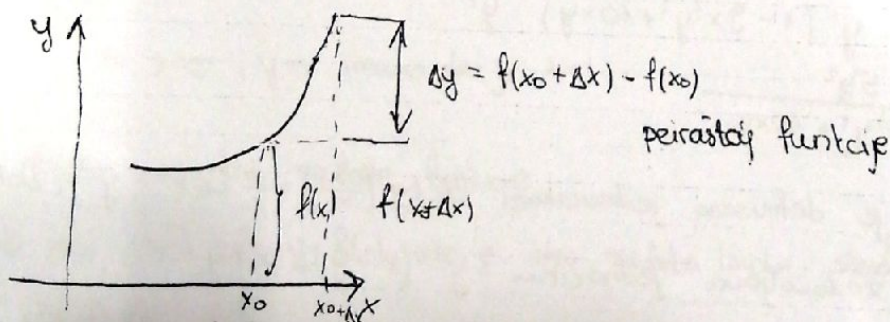
$$y' \cdot x - y = x + y \cdot y'$$

$$y' \cdot x - y \cdot y' = x + y$$

~~\_\_\_\_\_~~

$$(x - y) \cdot y' = x + y$$

Diferencijal i njegova primjena



$$\Delta y \approx dx$$

$dy$  - diferencijal funkcije  
 prava koja funkciju

Odeaditi diferen. sled f-ja

a)  $f(x) =$

$$f(x) = 2^x - 3^{-x} + \sqrt{x}$$

$$df = f'(x) \cdot dx$$

$$df = (2^x \cdot \ln 2 - 3^{-x} \ln 3 \cdot (-1) + \frac{1}{2\sqrt{x}}) dx$$

b)  $y = \ln(\cos \frac{1}{x}) - \frac{1}{x}$

$$y' dy = f'(x) dx$$

$$dy = (\frac{1}{\cos \frac{1}{x}} \cdot (-\sin \frac{1}{x}) \cdot (\frac{1}{x})' - (\frac{1}{x})') dx$$

$$dy = (-\operatorname{tg} \frac{1}{x} (-\frac{1}{x^2}) - (-\frac{1}{x^2})) dx$$

$$dy = \frac{1}{x^2} (\operatorname{tg} \frac{1}{x} + 1) \cdot dx$$

$$(\frac{2}{x^3})' = -6x^{-4} =$$

$$-\frac{6}{x^4}$$

$$(\frac{1}{x})' = (x^{-1})' = (-1) \cdot x^{-2} = -\frac{1}{x^2}$$

② Neka je  $f(x) = x^2$  približno izračunati  $f(5,123)$

Neka je  $x=0$ ,  $\Delta x = 0,123$

Težimo  $f(x_0 + \Delta x)$

$$\Delta y \approx dy$$

$$f(x_0 + \Delta x) - f(x_0) \approx f'(x_0) \cdot dx$$

$$f(x_0 + \Delta x) \approx f(x_0) + f'(x_0) dx, dx = \Delta x$$

$$f(5,123) \approx f(5) + f'(5) \cdot 0,123$$

$$f(5) = 5^2 = 25$$

$$f'(x) = 2x$$

$$f'(5) = 2 \cdot 5 = 10$$

$$5,123^2 \approx 25 + 10 \cdot 0,123 = 25 + 1,23 = 26,23$$

$$5,123^2 \approx 26,23$$

③ Približno izračunati  $\sqrt[3]{1,02}$

Kada  $f(x) = \sqrt[3]{x}$  približno treba izračunati  $f(1,02)$

Kada je  $x_0 = 1$   $\Delta x = 0,02$

Približno računamo

$$f(x_0 + \Delta x)$$

$$\Delta y \approx dy$$

$$f(x_0 + \Delta x) - f(x_0) = f'(x_0) \cdot dx$$

$$f(x_0 + \Delta x) \approx f'(x_0) + f'(x_0) \cdot dx$$

$$f(1,02) \approx f(1) + f'(1) \cdot$$

$$f(1)$$

$$f'(x) = (\sqrt[3]{x})' = \left(x^{\frac{1}{3}}\right)' = \frac{1}{3} x^{\frac{1}{3}-1} = \frac{1}{3} \cdot x^{-\frac{2}{3}} = \frac{1}{3} \cdot \frac{1}{x^{\frac{2}{3}}} =$$

$$\frac{1}{3} \cdot \frac{1}{\sqrt[3]{x^2}}$$

$$f'(1) = \frac{1}{3} \cdot \frac{1}{\sqrt[3]{1^2}} = \frac{1}{3}$$

$$f(1,02) \approx 1 + \frac{1}{3} \cdot 0,02$$

$$\sqrt[3]{1,02} \approx \frac{3,02}{3}$$

Napomena

$$\sqrt[3]{0,98}$$

$$f(x) = \sqrt[3]{x}$$

$$x = 1$$

$$\Delta x = -0,02$$

$$f(x_0 + \Delta x)$$

## Jzvodi, diferencijalni višeg reda

$$y'' = (y')'$$

$$y'' = (y'')' \quad y^{(n)} = (y^{(n-1)})'$$

1. Napiši  $y'$ ,  $y''$ ,  $y'''$  za  $y = x \cdot \ln x$

$$y = x \cdot \ln x$$

$$y' = 1 - \ln x + x \cdot \frac{1}{x}$$

$$y' = \ln x + 1$$

$$(y'') = (y')' = (\ln x + 1)' = \frac{1}{x}$$

$$y''' = (y'')' = \left(\frac{1}{x}\right)' = -\frac{1}{x^2}$$

$$y^{(4)} = (y''')' = \frac{2}{x^3}$$

2. Napiši  $n$ -ti izvod funkcije

a)  $y = x^n$

$$y' = n \cdot x^{n-1}$$

$$y'' = n \cdot (n-1) \cdot x^{n-2}$$

$$y''' = n \cdot (n-1) \cdot (n-2) \cdot x^{n-3}$$

$\vdots$

$$y^{(k)} = n \cdot (n-1) \cdot \dots \cdot (n-(k-1)) \cdot x^{n-k} \quad \text{za } k \leq n$$

$$y^{(n)} = n \cdot (n-1) \cdot \dots \cdot 2 \cdot 1 \cdot x^{n-n}$$

$$y^{(n)} = n \cdot (n-1) \cdot \dots \cdot 2 \cdot 1 = n!$$

$$y^{(n+1)} = 0$$

$$y^{(k)} = 0, \quad k > n$$

$$b) y = \frac{1}{1+x}$$

$$y' = ((1+x)^{-1})' = (-1)(1+x)^{-2} \cdot 1 = \frac{-1}{(1+x)^2}$$

$$y'' = (y')' = (-1(1+x)^{-2})' = 2(1+x)^{-3} \cdot 1 = \frac{2}{(1+x)^3}$$

$$y''' = (y'')' = (2(1+x)^{-3})' = -6(1+x)^{-4} \cdot 1 = \frac{-6}{(1+x)^4}$$

$$y^{(n)} = (-1)^n \cdot \frac{n!}{(1+x)^{n+1}}$$

3) Dajte su funkcije

a)  $f(x) = 6x^4 + 2x^3 - 3x^2 + 5x + 1$

b)  $f(x) = \frac{1}{1-x}$ ,  $x \neq 1$ . Očekivati  $f(0)$ ,  $f'(0)$ ,  $f''(0)$ ,  $f'''(0)$

a)  $f'(x) = 24x^3 + 6x^2 - 6x + 5$ ,  $f(0) = 1$   
 $f'(0) = 5$

$f''(x) = 72x^2 + 12x - 6$ ,  $f''(0) = -6$

$f'''(x) = 144x + 12$ ,  $f'''(0) = 12$

b)  $f(0) = \frac{1}{1-0} = 1$

$f'(x) = ((1-x)^{-1})' = -1 \cdot (1-x)^{-2} \cdot (-1) = \frac{1}{(1-x)^2}$ ,  $f'(0) = 1$

$f''(x) = ((1-x)^{-2})' = -2 \cdot (1-x)^{-3} \cdot (-1) = \frac{2}{(1-x)^3}$ ,  $f''(0) = 2$

$f'''(x) = (2 \cdot (1-x)^{-3})' = -6 \cdot (1-x)^{-4} \cdot (-1) = \frac{6}{(1-x)^4}$ ,  $f'''(0) = 6$