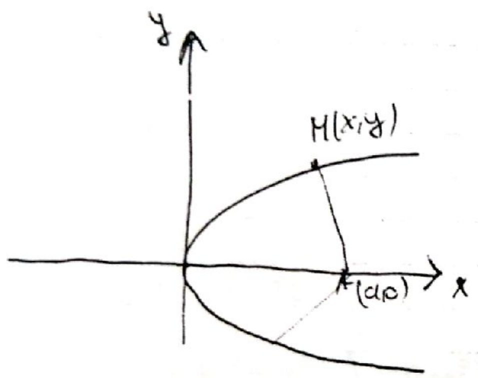
Eksteremne vrijednosti

1. Na paraboli $y^2 = 2px$ naći tačku najbližu tački A ~~se~~ $(a, 0)$



$d(M, A) = \sqrt{(x-a)^2 + y^2}$

$f(x) = \sqrt{(x-a)^2 + y^2} = \sqrt{(x-a)^2 + 2px}$

Tražimo minimum fu-je f.

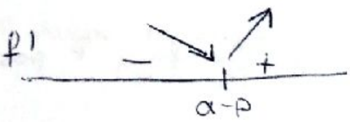
$$f'(x) = \frac{1}{2\sqrt{(x-a)^2 + \cancel{y^2}\, 2px}} \cdot \left(2\cdot(x-a) + 2p\right)$$

$$f'(x) = \frac{x-a+p}{\sqrt{(x-a)^2+2px}}$$

$$f'(x) = 0 \iff x-a+p=0$$
$$x = a-p$$

$$f'(x) > 0 \iff x-a+p>0$$
$$\iff x > a-p \quad \nearrow$$

$$f(x) < 0 \iff x-a+p<0$$
$$\iff x < a-p \quad \searrow$$

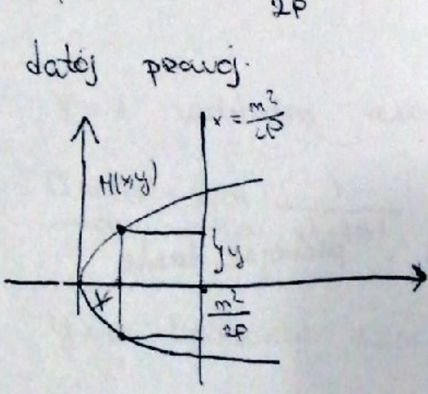


U tački $x=a-p$ funkcija f postiže minimum.

$$y^2 = 2px$$
$$y^2 = 2p(a-p)$$
$$y = \pm\sqrt{2p(a-p)}$$

$$M_1\left(a-p,\ \sqrt{2p(a-p)}\right)$$
$$M_2\left(a-p,\ -\sqrt{2p(a-p)}\right)$$

2. Naći pravougaonik maksimalne površine ograničen parabolom $y^2=2px$ i pravom $x=\frac{m^2}{2p}$ ako jedna stranica tog pravougaonika leži na datoj pravoj.



$$P = a\cdot b$$
$$b = 2y$$
$$a = \frac{m^2}{2p} - x$$

$$P = \left(\frac{m^2}{2p} - x\right)\cdot 2y$$
$$P(y) = \left(\frac{m^2}{2p} - \frac{y^2}{2p}\right)\cdot 2y$$
$$P(y) = \frac{1}{p}\left(m^2y - y^3\right)$$

$$P'(y) = \frac{1}{p}(m^2 - 3y^2)$$

$$P'(y) = 0 \Leftrightarrow m^2 - 3y^2 = 0$$
$$\Leftrightarrow y^2 = \frac{m^2}{3}$$
$$\Leftrightarrow y = \pm \frac{m}{\sqrt{3}}$$

Uzećemo $y = \frac{m}{\sqrt{3}}$ (jer M pripada I kvadrantu)

Tražim dr. izvod

$$P''(y) = \frac{1}{p}(-6y)$$

$P''\left(\frac{m}{\sqrt{3}}\right) = \frac{1}{p}\left(-6 \cdot \frac{m}{\sqrt{3}}\right) < 0 \Leftrightarrow$ u tački $y = \frac{m}{\sqrt{3}}$ f-ja P postiže maksimum.

$$P_{max} = P\left(\frac{m}{\sqrt{3}}\right) = \frac{1}{p}\left(m^2 \cdot \frac{m}{\sqrt{3}} - \frac{m^3}{3\sqrt{3}}\right) = \frac{1}{p} m^3 \frac{2}{3\sqrt{3}} = \frac{2\sqrt{3}\, m^2}{9p}$$

$$x = \frac{y^2}{2p}$$

$$x = \frac{\frac{m^2}{3}}{2p} = \frac{m^2}{6p} \qquad M\left(\frac{m^2}{6p}, \frac{m\sqrt{3}}{3}\right)$$

Grafik funkcije

1. Oblast def.
2. Parnost
3. Nule funkcije i znak (i presjek sa Oy-osom)
4. ponašanje na krajevima oblasti definisanosti
5. Prvi izvod i tok funkcije, ekstremne vrijednosti
6. Drugi izvod, intervali konveksnosti i konkavnosti i prevojne tačke

Grafik funkcije

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Funkcije

(1.) $y = \dfrac{x}{(x-1)^2}$

razlomak imenioc $> 0 \rightarrow$ različit od 0
paran (potkorijena veličina veća od 0
ln od nekog izraz, izraz > 0

1. $D_f: (x-1)^2 \neq 0$

$x - 1 \neq 0$

$x \neq 1$

$D_f: (-\infty, 1) \cup (1, +\infty)$

2. $f(-x) = \dfrac{-x}{(-x-1)^2} = \dfrac{-x}{(x+1)^2} \begin{array}{l} \neq f(x) \\ \neq f(-x) \end{array}$

ni parna ni neparna

3. $y = 0 \iff \dfrac{x}{(x-1)^2} = 0$

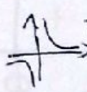
$\iff x = 0 \qquad (0,0)$

$\dfrac{x}{(x-1)^2}$

$y > 0 \iff x > 0 \qquad x \in (0, +\infty)$

$y < 0 \iff x < 0 \qquad x \in (-\infty, 0)$

4. $\lim\limits_{x \to 1+} f(x) = \lim\limits_{x \to 1+} \dfrac{x}{(x-1)^2} = +\infty$



$\frac{1}{x}$

$\lim\limits_{x \to 1-} f(x) = \lim\limits_{x \to 1-} \dfrac{x}{(x-1)^2} = +\infty$

$x = 1$ vertikalna asimptota

$\lim\limits_{x \to +\infty} f(x) = \lim\limits_{x \to +\infty} \dfrac{x}{x^2 - 2x + 1} = \lim\limits_{x \to +\infty} \dfrac{x}{x^2\left(1 - \frac{2}{x} + \frac{1}{x^2}\right)} = \lim\limits_{x \to +\infty} \dfrac{1}{x} \cdot \dfrac{1}{1 - \frac{2}{x} + \frac{1}{x^2}} = 0$

$y = 0$ horizontal. asim. $(x \to +\infty)$

$\lim\limits_{x \to -\infty} f(x) = 0 \qquad y = 0 \quad H.A. \quad (x \to -\infty)$

Napomena: $y=0$ jednačina x-ose

Nema kose asimp. (jer ima H)

5. $y = \dfrac{x}{(x-1)^2}$

$$y' = \frac{1\cdot(x-1)^2 - x\cdot 2(x-1)\cdot 1}{(x-1)^4}$$

$$y' = \frac{(x-1)[(x-1) - 2x]}{(x-1)^4}$$

$$y' = \frac{-1 - x}{(x-1)^3}$$

$y' = -\dfrac{(1+x)}{(x-1)^3}$, $y' = 0 \iff x = -1$

y' nije def. u $x=1$, ali $x = 1 \notin D_f$.

	$-\infty$	-1	1	$+\infty$
$1+x$	$-$	0	$+$	$+$
$(x-1)^3$	$-$		$-$ 0	$+$
"$-$"	$-$		$-$	$-$
y'	$-$		$+$ nedef.	$-$
y	$\searrow$		$\nearrow$ nedef.	$\searrow$

U $x=-1$ f postiže minimum $\quad y_{min} = f(-1) = \dfrac{-1}{(-1-1)^2} = \dfrac{-1}{4}$

$M\left(-1, -\frac{1}{4}\right)$

6. $y' = -\dfrac{1+x}{(x-1)^3}$

$$y'' = -\frac{1(x-1)^3-(1+x)\cdot 3(x-1)^2\cdot 1}{(x-1)^6}$$

$$y'' = -\frac{(x-1)^2(x-1-3(1+x))}{\cancel{x-1}\,(x-1)^6}$$

$$y'' = -\frac{-2x-4}{(x-1)^4}$$

$$y'' = 2\cdot\frac{x+2}{(x-1)^4}$$

$y''=0 \iff x=-2$

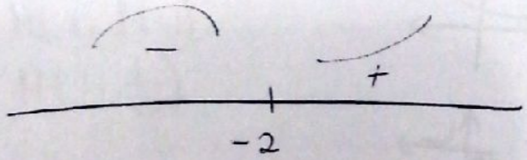
y'' nije def u $x=1$, $x=1 \notin Df$.

$y''>0 \iff x+2>0$

$\iff x>-2$

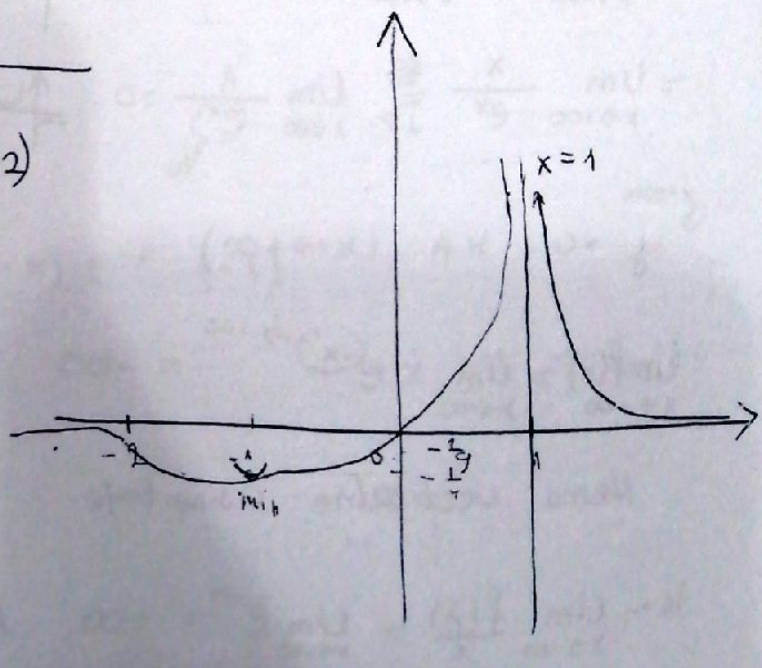
$y''<0 \iff x+2<0$

$\iff x<-2$



Prevojna tačka $P(-2, f(-2))$

$$f(-2) = \frac{-2}{(-2-1)^2} = -\frac{2}{9}$$



① $y = x \cdot e^{-x}$

1. $Df = \mathbb{R}$

2. $y(-x) = -x \cdot e^{-(-x)} = -x \cdot e^{x} \neq y(x)$
$\neq -y(x)$

y nije parna

y nije neparna

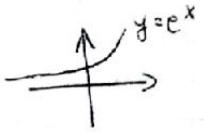
3. $y = 0 \iff x \cdot \dfrac{1}{\underbrace{e^x}_{>0}} = 0$

$\iff x = 0$

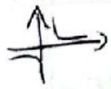
$y > 0 \iff x > 0$

$y < 0 \iff x < 0$

4. $\lim\limits_{x \to +\infty} f(x) = \lim\limits_{x \to +\infty} x \cdot e^{-x} \overset{\infty \cdot 0}{=}$



$= \lim\limits_{x \to +\infty} \dfrac{x}{e^x} \overset{\frac{\infty}{\infty}}{\underset{L.P.}{=}} \lim\limits_{x \to +\infty} \dfrac{1}{\underbrace{e^x}_{\infty}} = 0$



x-osa

$y = 0$ H.A. $(x \to +\infty)$

$\lim\limits_{x \to -\infty} f(x) = \lim\limits_{x \to -\infty} x \cdot e^{\overset{\to +\infty}{-x}} = -\infty$

Nema vertikalne asimptote

$k = \lim\limits_{x \to -\infty} \dfrac{f(x)}{x} = \lim\limits_{x \to -\infty} e^{-x} = +\infty$ Nema kose asimptote za $x \to -\infty$

Ako nema hor. tražimo kosu!

5. $y = x \cdot e^{-x}$

$y = e^{-x} + x \cdot e^{-x} \cdot (-1)$

$y' = (1 - x) \cdot e^{-x}$ ili

$y' = \dfrac{1-x}{e^{x}}$

$y' = 0 \iff 1 - x = 0$

$\iff x = 1$

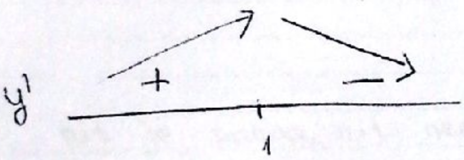
y' je definisano za $\forall x \in R$

$y' > 0 \iff 1 - x > 0$

$\iff x < 1$

$y' < 0 \iff 1 - x < 0$

$\iff x > 1$



U tački $x = 1$ f-ja postiže maksimum. To je $f_{max} = y_{max} = f(1) =$

$1 \cdot e^{-1} = \dfrac{1}{e}$

$M(1, f(1))$

$M\left(1, \dfrac{1}{e}\right)$

6. $y' = (1 - x) \cdot e^{-x}$

$y'' = (y')' = (-1) \cdot e^{-x} + (1 - x) \cdot e^{-x} \cdot (-1)$

$y'' = (-1 - 1 \cdot (1 - x)) \cdot e^{-x}$

$y'' = (x - 2) \cdot e^{-x}$

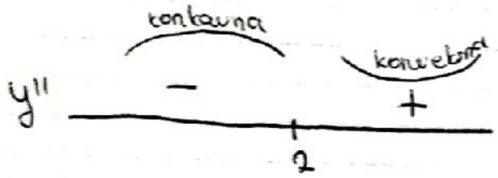
$y'' = \dfrac{x-2}{e^{x}}$

$y'' = 0 \iff x - 2 = 0$
$\iff x = 2$

y'' je definisano za $\forall x \in R$

$y'' > 0 \iff x - 2 > 0$
$\iff x > 2$

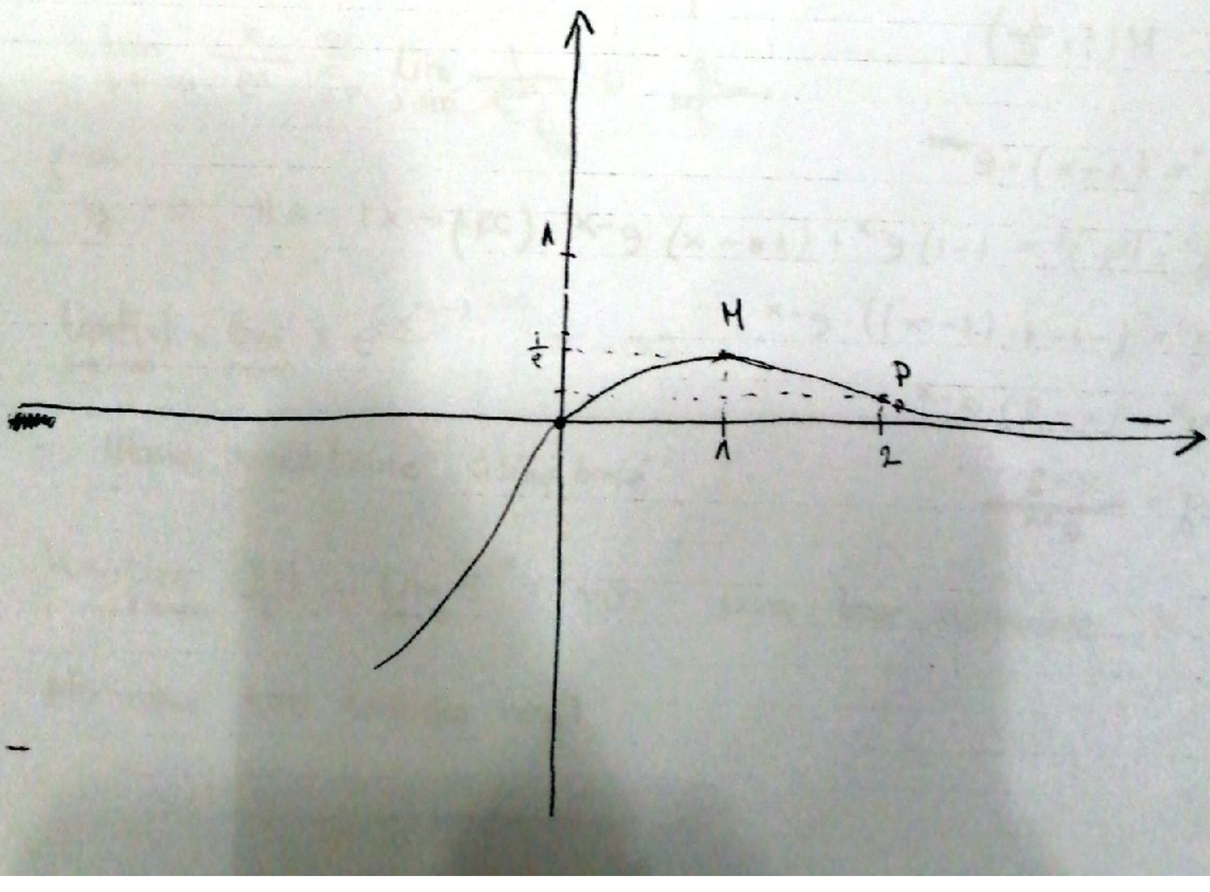
$y'' < 0 \iff x - 2 < 0$
$x < 2$



$P(2, f(2))$

$f(2) = 2 \cdot e^{-2} = \frac{2}{e^2}$

$P\left(2, \frac{2}{e^2}\right)$



(2)

$$y = \frac{6x - x^2 - 9}{x - 2}$$

$$y = \frac{-x^2 + 6x - 9}{x - 2}$$

$$y = \frac{-(x-3)^2}{x-2} = \frac{(x-3)^2}{2-x}$$

1. $D_f: \ x - 2 \neq 0$

$$x \neq 2$$

$D_f \ (-\infty, 2) \cup (2, +\infty)$

2. $$y(-x) = \frac{-(-x-3)^2}{-x-2} = \frac{(x+3)^2}{x+2} \neq y(x)$$
$$\neq -y(x)$$

nit je parna, nit neparna

3. $y = 0 \iff -\dfrac{(x-3)^2}{x-2} = 0$

$\iff x = 3 \qquad N(3,0)$

$y > 0 \iff 2 - x > 0$
$\iff x < 2$

$y < 0 \iff 2 - x < 0$
$x > 2$

4. $$\lim_{x \to 2+} f(x) = \lim_{x \to 2+} \frac{(x-3)^2}{\underset{\downarrow 0^-}{2-x}} = -\infty$$

$x = 2 \quad$ V. A.

$$\lim_{x\to 2-} f(x) = \lim_{x\to 2-} \frac{(x-3)^2}{2-x} = +\infty$$

$$\lim_{x\to +\infty} f(x) = \lim_{x\to +\infty} \frac{(x-3)^2}{2-x} = \lim_{x\to +\infty} \frac{x^2\left(1-\frac{6}{x}+\frac{9}{x^2}\right)}{x\left(\frac{2}{x}-1\right)} = \lim_{x\to +\infty} \frac{x\cdot\left(1-\frac{6}{x}+\frac{9}{x^2}\right)}{\frac{2}{x}-1} = -\infty$$

nema konačnog br. nema H. A.

$$\lim_{x\to -\infty} f(x) = \lim_{x\to -\infty} \frac{x\cdot\left(1-\frac{6}{x}+\frac{9}{x^2}\right)}{\frac{2}{x}-1} = +\infty$$

Nema H. A.

$$k = \lim_{x\to \pm\infty} \frac{f(x)}{x} = \lim_{x\to \pm\infty} \frac{(x-3)^2}{x\cdot(2-x)} = \lim_{x\to \pm\infty} \frac{x^2\left(1-\frac{6}{x}+\frac{9}{x^2}\right)}{x^2\left(\frac{2}{x}-1\right)} = \frac{1}{-1} = -1$$

$$n = \lim_{x\to +\infty} \left(f(x)-kx\right)$$

$$n = \lim_{x\to \pm\infty} \left(\frac{(x-3)^2}{2-x} - (-1)\cdot x\right) = \lim_{x\to \pm\infty} \frac{x^2-6x+9+2x-x^2}{2-x} =$$

$$= \lim_{x\to \pm\infty} \frac{9-4x}{2-x} = \lim_{x\to \pm\infty} \frac{x\left(\frac{9}{x}-4\right)}{x\left(\frac{2}{x}-1\right)} = 4$$

$y = kx+n$

$y = -x+4$ K. A.

5. $y = \dfrac{(x-3)^2}{2-x}$

$$y' = \frac{2\cdot(x-3)(2-x)-(x-3)^2(-1)}{(2-x)^2}$$

$$y' = \frac{(x-3)\left[2\cdot(2-x)+(x-3)\right]}{(2-x)^2}$$

$$y' = \frac{(x-3)\cdot(1-x)}{(2-x)^2}$$

$$y'=0 \iff (x-3)(1-x)=0$$
$$\iff x=3 \lor x=1$$

y' nije definisano za $x=2$ ali $x=2 \notin D_f$.

	$-\infty$		1		3		$+\infty$
$x-3$		$-$		$-$	0	$+$	
$1-x$		$+$	0	$-$		$-$	
y'		$-$	0	$+$	0	$-$	
y		$\searrow$		$\nearrow$		$\searrow$	

U $x=1$ f-postiže minimum

$$y_{min} = f(1) = \frac{(1-3)^2}{2-1} = 4$$

$M_1(1,4)$

U $x=3$ f postiže maksimum

$$y_{max} = f(3) = \frac{(3-3)^2}{2-3} = 0$$

$M_2(3,0)$

6. $y' = \dfrac{-x^2+4x-3}{(2-x)^2}$

$$y'' = \frac{(-2x+4)(2-x)^2 - (-x^2+4x-3)\cdot 2(2-x)\cdot(-1)}{(2-x)^4}$$

$$y'' = \frac{2(2-x)\left[(2-x)^2 + (-x^2+4x-3)\right]}{(2-x)^4}$$

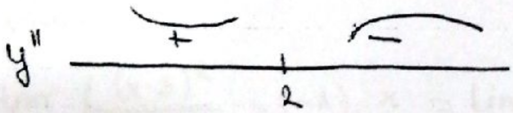
$$y'' = 2\cdot\frac{4-4x+x^2-x^2+4x-3}{(2-x)^3} = \frac{2}{(2-x)^3}$$

$y'' \neq 0 \quad , \forall x \in Df$

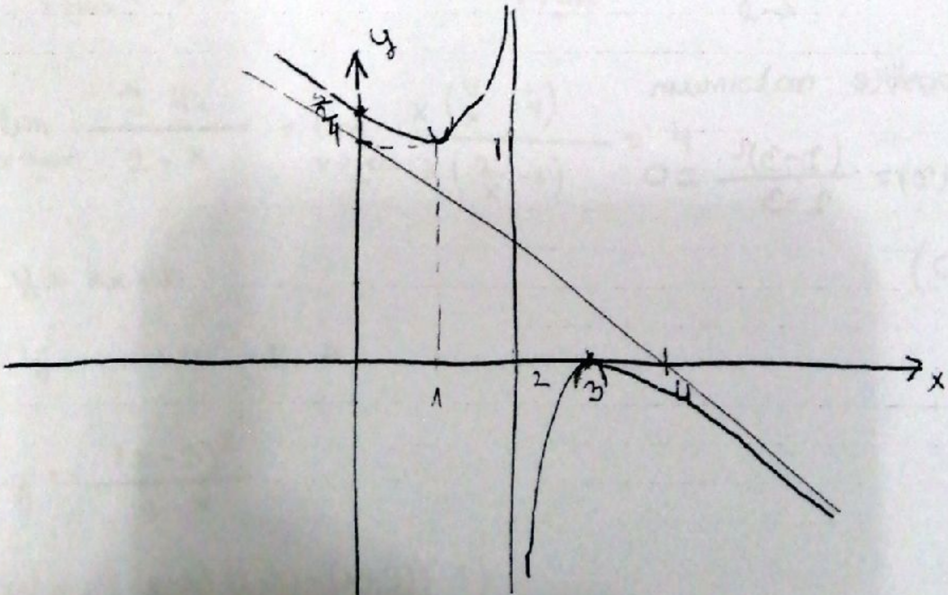
y'' nije definisano za $x=2$, ali $x=2 \notin Df$

$y'' > 0 \Leftrightarrow 2-x > 0$
$\Leftrightarrow x < 2$

$y'' < 0 \Leftrightarrow 2-x < 0$
$x > 2$



Nemamo prevojnu tačku.



$\cap Oy\,(x=0)$
$y(0) = \dfrac{(3-0)^2}{\cdot} = \dfrac{9}{\cdot}$

4.

	$(-\infty, 1)$	1	$(1, 2)$	2	$(2, 3)$	3	$(3, +\infty)$
y	$+$		$+$	nedef.	$-$	0	$-$
y'	$-$	0	$+$	nedef.	$+$	0	$-$
y''	$+$		$+$	nedef.	$-$		$-$

5. $y = \dfrac{\ln x + 1}{x}$

1. Df : $x \neq 0 \ \wedge \ x > 0$

   Df : $(0, +\infty)$

2. nit parna nit neparna
   $\Downarrow$ x-osu $\quad$ $\Downarrow$ koordinantni početak

3. $y = 0 \iff \ln x + 1 = 0$
   $\iff \ln x = -1$
   $\iff x = e^{-1}$
   $x = \frac{1}{e}$

$y > 0 \iff \dfrac{\ln x + 1}{\underset{>0 \ \forall x \in Df}{x}} > 0$

$y > 0 \iff \ln x + 1 > 0$
$\iff \ln x > -1$
$\iff \ln x > \ln e^{-1}$
$\iff x > e^{-1}$
$\iff x \in \left(\frac{1}{e}, +\infty\right)$

$y < 0 \iff \ln x + 1 < 0$
$\iff \ln x < -1$
$\iff \ln x < \ln e^{-1}$
$\iff x < \frac{1}{e}$
$x \in \left(0, \frac{1}{e}\right)$

4. $\lim\limits_{x\to 0+} f(x) = \lim\limits_{x\to 0+} \frac{\ln x+1}{x} = \lim\limits_{x\to 0+} \big(\underbrace{\ln x+1}_{-\infty}\big)\cdot\big(\underbrace{\tfrac{1}{x}}_{+\infty}\big) = -\infty$

$x=0$ vertikalna asimptota

$\lim\limits_{x\to +\infty} f(x) = \lim\limits_{x\to +\infty} \frac{\ln x+1}{x} \overset{\frac{\infty}{\infty}}{\underset{L'H}{=}} \lim\limits_{x\to +\infty} \frac{\frac{1}{x}}{1} = \lim\limits_{x\to +\infty} \frac{1}{x} = 0$

$x\to\infty)$ $y=0$ H. A. $x\to+\infty$

Nema kose

5. $y = \frac{\ln x+1}{x}$

$y' = \frac{\frac{1}{x}\cdot x - (\ln x+1)\cdot 1}{x^2}$

$y' = \frac{1-\ln x-1}{x^2}$

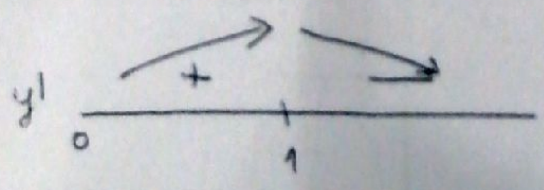
$y' = -\frac{\ln x}{x^2}$

$y'=0 \iff \ln x=0$
$\iff x=1$

y' nije definirana za $x=0$ ali $x=0 \notin Df$

$y'>0 \iff \ln x<0$
$\iff x<1$
$x\in(0,1)$

$y'<0 \iff \ln x>0$
$\iff x>1$



U točki $x=1$ f postiže maksimum

$y_{max} = f(1) = \frac{\ln 1+1}{1} = \frac{0+1}{1} = 1$

$M(1,1)$

⑥ $y'' = \left(-\frac{\ln x}{x^2}\right)'$

$$y'' = -\frac{\frac{1}{x}\cdot x^2 - \ln x \cdot 2x}{x^4} = -\frac{x - 2x\cdot \ln x}{x^4}$$

$$y'' = -\frac{1-2\ln x}{x^3} = \frac{2\ln x - 1}{x^3} \quad x^3 > 0, \forall x \in D(f)$$

$\boxed{y'' = 0} \Leftrightarrow 2\ln x - 1 = 0$

$\Leftrightarrow \ln x = \frac{1}{2}$

$\Leftrightarrow x = e^{\frac{1}{2}}$

$\boxed{\Leftrightarrow x = \sqrt{e}}$

$\boxed{y'' > 0} \Leftrightarrow 2\ln x - 1 > 0$

$\Leftrightarrow \ln x > \frac{1}{2}$

$\Leftrightarrow \ln x > \ln e^{\frac{1}{2}}$

$\Leftrightarrow x > e^{\frac{1}{2}}$

$\Leftrightarrow x > \sqrt{e}$

$\boxed{y'' < 0} \Leftrightarrow 2\ln x - 1 < 0$

$\Leftrightarrow \ln x < \frac{1}{2}$

$\Leftrightarrow x < \sqrt{e}$

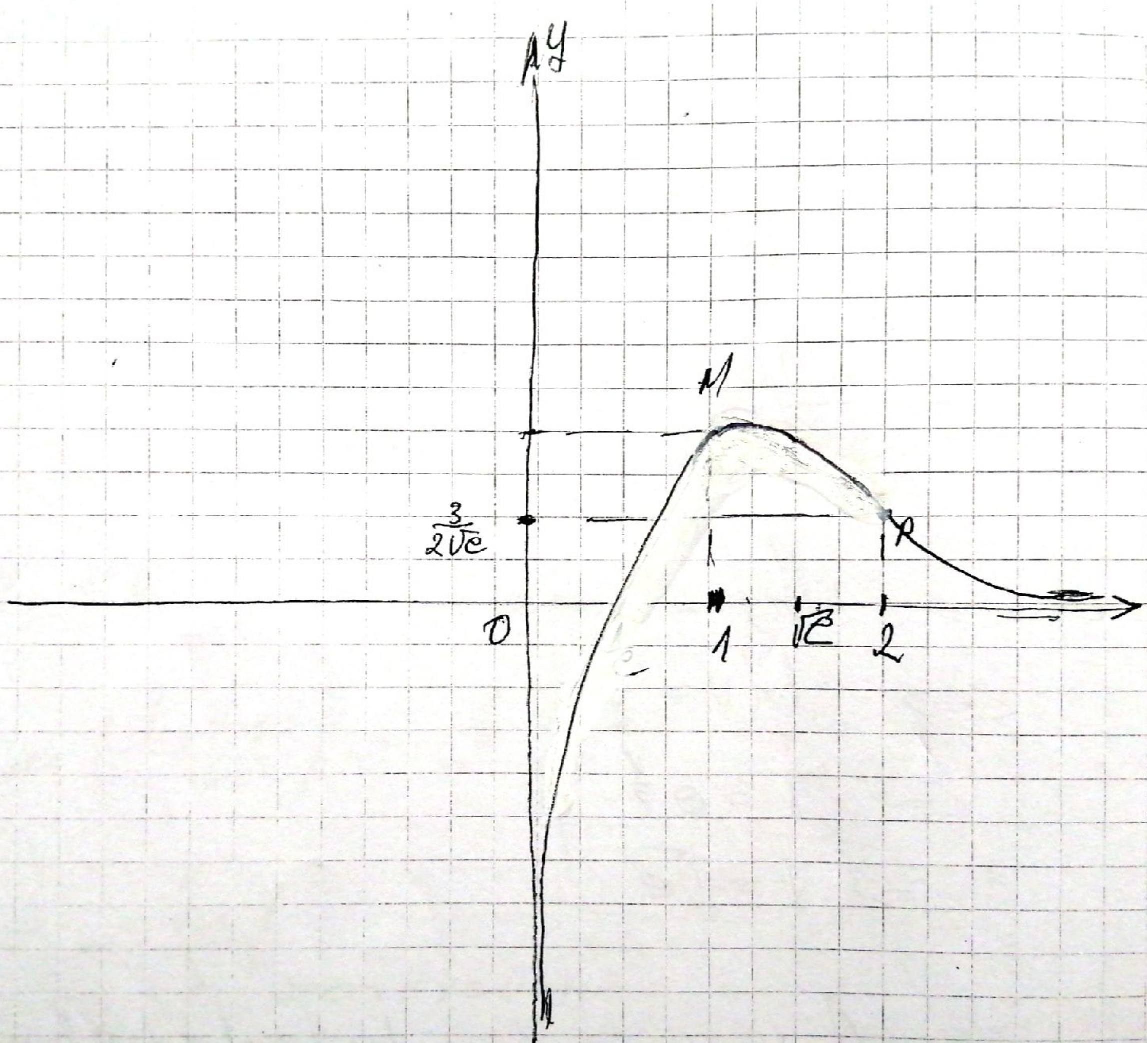
$\Leftrightarrow x \in (0, \sqrt{e})$

$-$ $\quad \sqrt{e} \quad$ $+$

prevojna tačka $P(\sqrt{e}, f(\sqrt{e}))$

$$f(\sqrt{e}) = \frac{\ln e^{\frac{1}{2}} + 1}{e^{\frac{1}{2}}} = \frac{\frac{1}{2}+1}{\sqrt{e}} = \frac{3}{2\sqrt{e}}, \quad P\left(\sqrt{e}, \frac{3}{2\sqrt{e}}\right)$$

7



(4) $y = \frac{x}{x^2-1}$ $\qquad$ $y = \frac{x}{(x-1)(x+1)}$

$x \neq 1 \quad x \neq -1$

(1°) $D(y): x^2 - 1 \neq 0$

$x^2 \neq 1$

$x \neq \pm 1$

$D(y): (-\infty, -1) \cup (-1, 1) \cup (1, +\infty)$

(2°) $y(-x) = \frac{-x}{(-x)^2-1} = \frac{-x}{x^2-1} = -\frac{x}{x^2-1} = -y(x)$

f-ja je neparna

Grafik f-je je simetričan na koordinatni početak

(3°) $y = 0 \Leftrightarrow x = 0$

$y = \frac{x}{(x-1)(x+1)}$

	$-\infty$	-1		0		1		$+\infty$
x	$-$		$-$	0	$+$		$+$	
$x-1$	$-$		$-$		$-$	0	$+$	
$x+1$	$-$	0	$+$		$+$		$+$	$+$
y (znak f-je)	$-$	nedef	$+$		$-$	ned.	$+$	

$\rightarrow$ 4.

$$\lim_{x\to 1+} f(x) = \lim_{x\to 1+} \frac{x}{\underbrace{(x-1)}_{0+}(x+1)} = +\infty$$

$x = 1$ vertikalna A.

$$\lim_{x\to 1-} f(x) = \lim_{x\to 1-} \frac{x}{\underbrace{(x-1)}_{0-}(x+1)} = -\infty$$

$$\lim_{x\to -1+} f(x) = \lim_{x\to -1+} \frac{x}{(x-1)\underbrace{(x+1)}_{0+}} = +\infty$$

$$\lim_{x\to -1-} f(x) = \lim_{x\to -1-} \frac{x}{(x-1)\underbrace{(x+1)}_{0-}} = -\infty$$

$\frac{-1}{-1(0)+} \rightarrow -1$

$x = -1$ V.A.

$$\lim_{x\to \pm\infty} f(x) = \lim_{x\to \pm\infty} \frac{x}{x^2-1} = \lim_{x\to\pm\infty} \frac{x}{x^2(1-\frac{1}{x^2})} = \lim_{x\to\pm\infty} \underbrace{\frac{1}{x}}_{0} \cdot \frac{1}{1-\underbrace{\frac{1}{x^2}}_{0}} = 0$$

$y = 0$ H.A., $x \to +\infty$

takođe $\lim_{x\to -\infty} f(x) = 0$, $y = 0$ H.A, $x \to -\infty$

Nema kose asimptote!

$\rightarrow$ 5. $y = \frac{x}{x^2-1}$

$$y' = \frac{x^2 - 1 - x\cdot 2x}{(x^2-1)^2}$$

$$y' = \frac{-x^2-1}{(x^2-1)^2}$$

$$y' = -\frac{x^2+1}{(x-1)^2(x+1)^2}$$

$y' = 0 \Leftrightarrow x^2 + 1 = 0$ nemoguće

y' nije def. u $x=1$ i $x=-1$, $x=1 \notin D_f$
$x=-1 \notin D_f$

Vočimo $y' = -\dfrac{\overbrace{(x^2+1)}^{>0}}{\underset{>0}{(x-1)^2}\underset{>0}{(x+1)^2}}$ $\forall x \in D_f$

$y' < 0$, $\forall x \in D_f$

y stalno opada i ne postiže ekstremum.

→ 6. $y' = -\dfrac{x^2+1}{(x^2-1)^2}$

$$y'' = -\frac{2x(x^2-1)-(x^2+1)\cdot 2(x^2-1)\cdot 2x}{(x^2-1)^4}$$

$$y'' = -2\,\frac{x\cancel{(x^2-1)}\cdot\left[x^2-1-2(x^2+1)\right]}{(x^2-1)^{\cancel{4}\,3}}$$

$$y'' = -2\cdot\frac{x\cdot(x^2-1-2x^2-2)}{(x^2-1)^3} = -2\cdot\frac{x\cdot(-x^2-3)}{(x^2-1)^3}$$

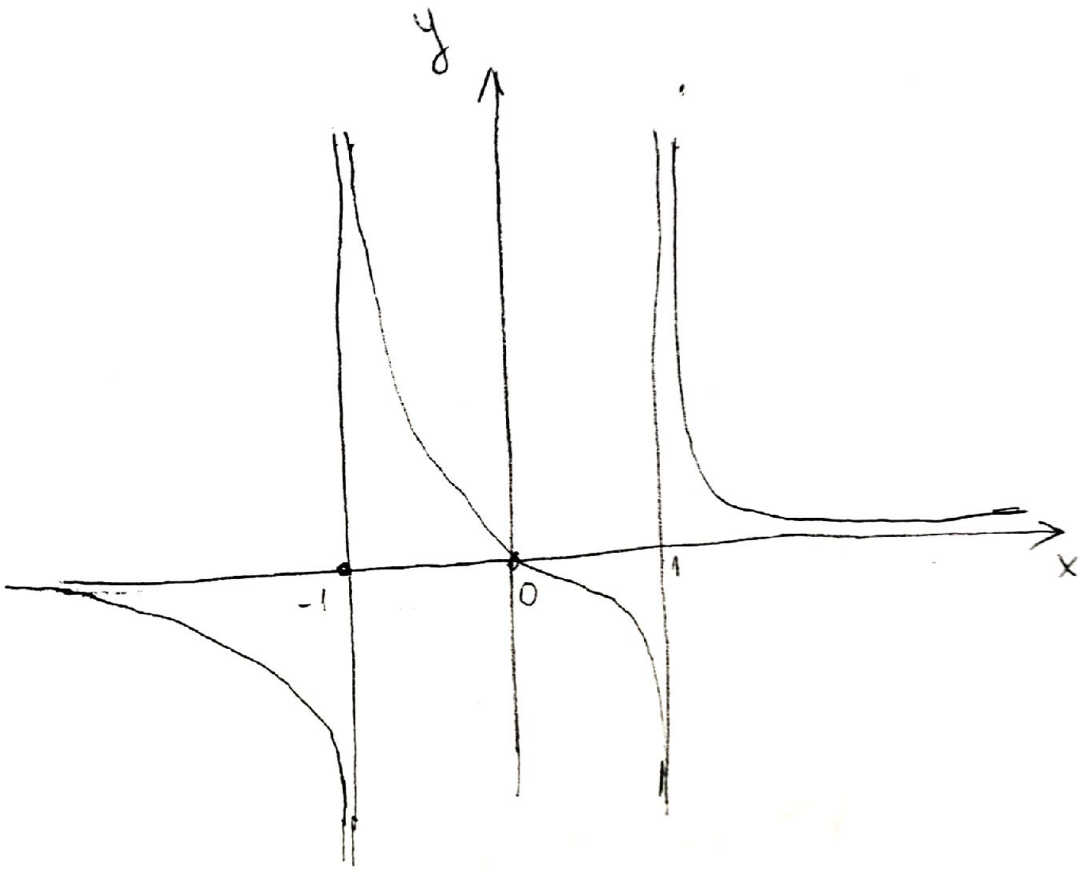
$$y'' = 2\cdot\frac{x(x^2+3)}{(x-1)^3\cdot(x+1)^3}$$

$y''=0 \iff x=0$

y'' nije definisano u $x=1$ i $x=-1$ ali $x=1 \notin D_f$
$x=-1 \notin D_f$

	$-\infty$	-1		0		1		$-\infty$
x	−		−	0	+		+	+
$x-1$	−		−		−	0	+	+
$x+1$	−	0	+		+		+	+
y''	−	nedef.	+	0	−	nedef.	+	
y		nedef.		prevoj		nedef.		

⇓
nemamo prevoj



③ $y = e^{\frac{1}{x}}$

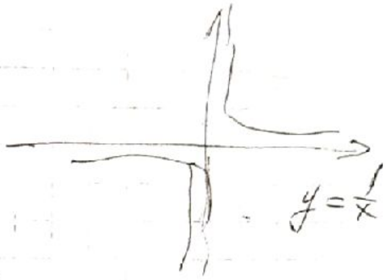
nikad nije negativna e

① $D(y): \ x \neq 0$

$D(y): \ (-\infty, 0) \cup (0, +\infty)$

② $y(-x) = e^{\frac{1}{-x}}$

ni parna ni neparna



3° $y > 0, \ \forall x \in D(y)$

4° $\lim\limits_{x \to 0^+} f(x) = \lim\limits_{x \to 0^+} e^{\overset{\to +\infty}{\frac{1}{x}}} = e^{+\infty} = +\infty$

$\lim\limits_{x \to 0^-} f(x) = \lim\limits_{x \to 0^-} e^{\overset{\to -\infty}{\frac{1}{x}}} = e^{-\infty} = 0$

$x = 0$, V.A. - vertikalna asimptota

$\lim\limits_{x \to +\infty} f(x) = \lim\limits_{x \to +\infty} e^{\overset{\to 0}{\frac{1}{x}}} = e^0 = 1 \qquad \left(\frac{1}{+\infty} = 0\right)$

$\lim\limits_{x \to -\infty} f(x) = \lim\limits_{x \to -\infty} e^{\overset{\to 0}{\frac{1}{x}}} = e^0 = 1$

$y = 1$ je horizontalna HA.

Nema kose asimptote!

5° $y = e^{\frac{1}{x}}$

$y' = e^{\frac{1}{x}} \cdot \left(\frac{1}{x}\right)' = e^{\frac{1}{x}} \cdot \left(-\frac{1}{x^2}\right) = -\overset{>0, \forall x \in D(y)}{e^{\frac{1}{x}}} \cdot \underset{>0, \forall x \in D(y)}{\frac{1}{x^2}} \neq 0$

$y' < 0 \ (\forall x \in D(y))$ <u>Stalno je f-ja opadajuća</u>)

$y'' = \left(-e^{\frac{1}{x}} \cdot \frac{1}{x^2}\right)' \Rightarrow$ izvod proizvoda

$$y'' = -\left(e^{\frac{1}{x}} \cdot \left(\frac{1}{x}\right)' \cdot \frac{1}{x^2} + e^{\frac{1}{x}} \cdot \left(\frac{1}{x^2}\right)'\right)$$

$$= -\left(e^{\frac{1}{x}} \cdot \left(-\frac{1}{x^2}\right) \cdot \frac{1}{x^2} + e^{\frac{1}{x}} \cdot \left(-\frac{2}{x^3}\right)\right)$$

$$= e^{\frac{1}{x}} \cdot \left(\frac{1}{x^4} + \frac{2}{x^3}\right) = \underbrace{e^{\frac{1}{x}}}_{>0,\ \forall x \in D(y)} \cdot \frac{1+2x}{\underbrace{x^4}_{>0,\ \forall x \in D(y)}}$$

$$y'' = 0 \Leftrightarrow 1+2x = 0$$
$$\Leftrightarrow \boxed{x = -\frac{1}{2}}$$

y'' nije def. u $x=0$, ali $x = 0 \notin D(y)$

$y'' > 0 \Leftrightarrow 1+2x > 0$
$2x > -1$
$x > -\frac{1}{2}$

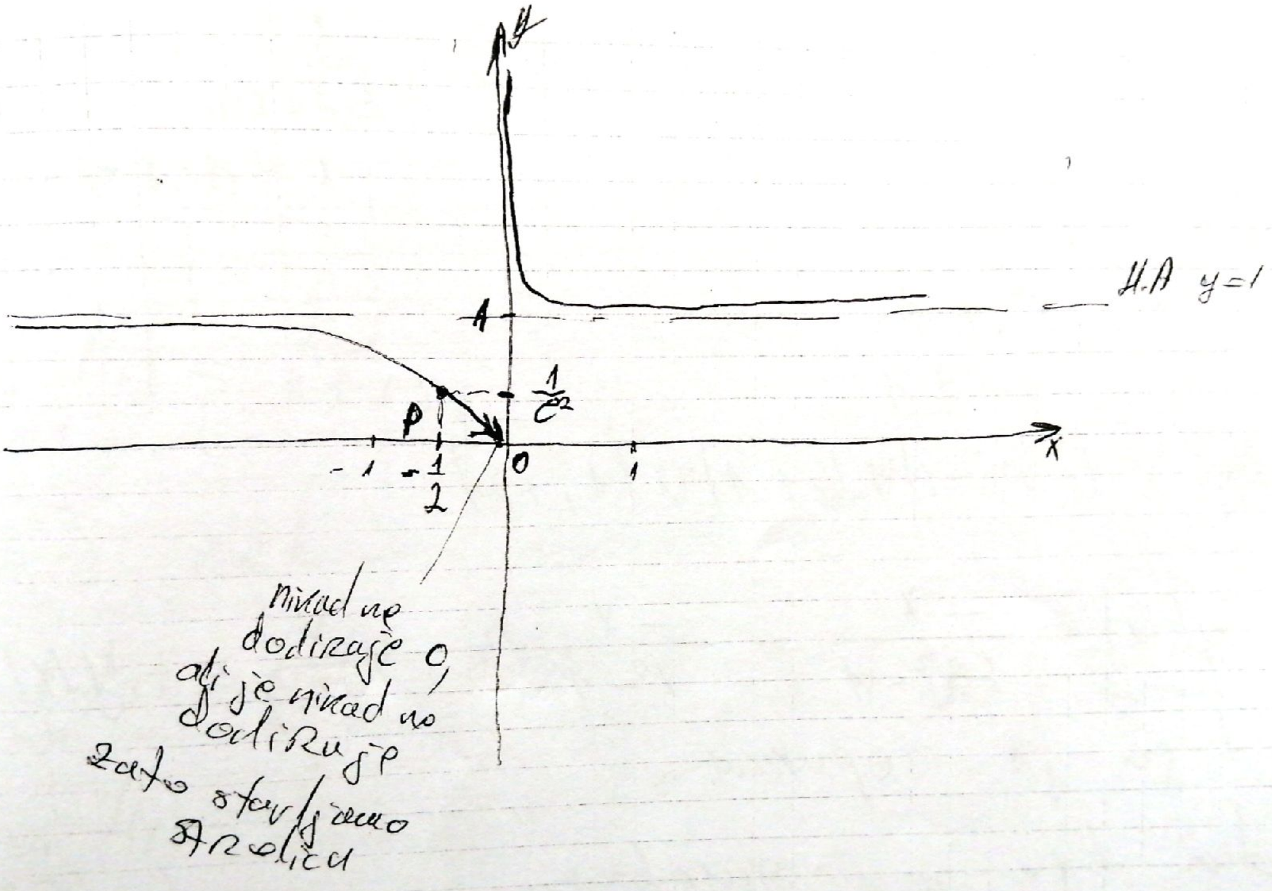
$y'' < 0 \Leftrightarrow 1+2x < 0$
$2x < -1$
$x < -\frac{1}{2}$

y'': $-$ na $\left(-\infty, -\frac{1}{2}\right)$, $+$ na $\left(-\frac{1}{2}, \infty\right)$; $-\frac{1}{2}$

prevojna tačka je $P\left(-\frac{1}{2}, f\left(-\frac{1}{2}\right)\right)$

$$f\left(-\frac{1}{2}\right) = e^{\frac{1}{-\frac{1}{2}}} = e^{-2} = \frac{1}{e^2}.$$

$$P\left(-\frac{1}{2}, \frac{1}{e^2}\right)$$


y
x
1
$\frac{1}{e^2}$
P
-1
$-\frac{1}{2}$
0
1
H.A y=1
nikad ne
dodiruje 0,
ali je nikad ne
dodiruje P
zato stavljamo
strelicu