$$\int \frac{\frac{t^2}{1+t^2}}{\frac{1}{(1+t^2)^3}} \cdot \frac{dt}{1+t^2} = \int t^2 \cdot (1+t^2)\,dt = \int t^2 dt + \int t^4 dt = \frac{t^3}{3} + \frac{t^5}{5} + C =$$

$$= \frac{tg^3 x}{3} + \frac{tg^5 x}{5} + C$$

→ Određeni integrali ←

① $$\int_{-1}^{2} (x^3+x-1)\,dx = \int_{-1}^{2} x^3\,dx + \int_{-1}^{2} x\,dx - \int_{-1}^{2} dx = \frac{x^4}{4}\Big|_{-1}^{2} + \frac{x^2}{2}\Big|_{-1}^{2} - x\Big|_{-1}^{2} =$$

$$= \frac{1}{4}\left(2^4-(-1)^4\right) + \frac{1}{2}\left(2^2-(-1)^2\right) - \left(2-(-1)\right) = \frac{15}{4} + \frac{3}{2} - 3 = \frac{9}{4}$$

② $$\int_0^3 \frac{dx}{x+1} = \left[\begin{array}{l} x+1=t \\ dx=dt \end{array} \quad \begin{array}{c|c|c} x & 0 & 3 \\ \hline t & 1 & 4 \end{array}\right] = \int_1^4 \frac{dt}{t} = \ln|t|\Big|_1^4 =$$

$$= \ln|4| - \underbrace{\ln|1|}_{0} = \ln 4$$

③ $$\int_0^{\ln 2} x \cdot e^x\,dx = \left[\begin{array}{l} u=x \Rightarrow du=dx \\ dv=e^x dx \Rightarrow v=\int e^x dx = e^x \end{array}\right] = x \cdot e^x\Big|_0^{\ln 2} - \int_0^{\ln 2} e^x\,dx =$$

$$\ln 2 \cdot e^{\ln 2} - 0 \cdot e^0 - e^x\Big|_0^{\ln 2} = 2 \cdot \ln 2 - (e^{\ln 2} - e^0) = 2 \cdot \ln 2 - (2-1) = 2 \cdot \ln 2 - 1$$

④ Izračunati površinu figure ograničene parabolom $y=x^2+2x+2$ i pravama $y=0$, $x=2$ i $x=-3$

$y = x^2+2x+2$

$a=1 \quad b=2, \quad c=2$

$D = b^2-4ac$

$D = 4-4 \cdot 2 = -4 < 0$

Nema presjeka sa Ox-osom

$a=1>0 \quad \cup$

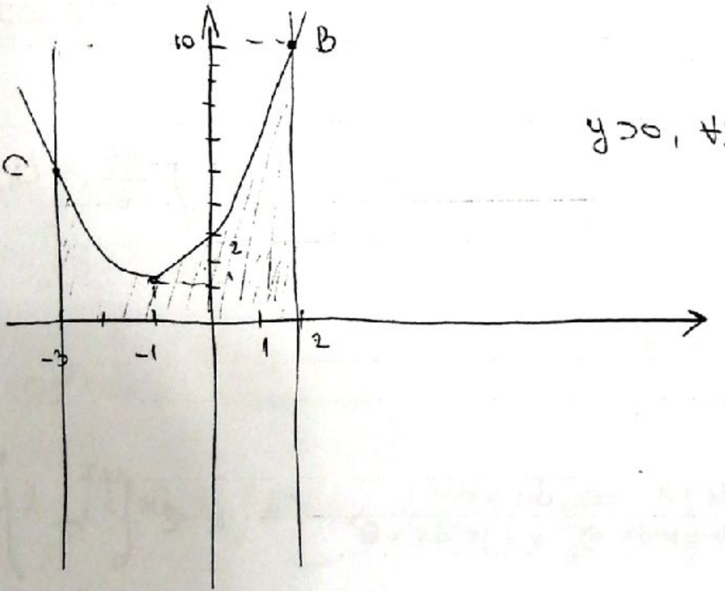
Presjek sa Oy-osom $(x=0)$

$y=2 \quad A(0,2)$

$T\left(-\frac{b}{2a}, -\frac{D}{4a}\right) \quad T\left(-\frac{2}{2}, -\frac{-4}{4}\right) \quad T(-1,1)$

$$\begin{cases} y=x^2+2x+2 \\ x=2 \end{cases} \quad \begin{array}{l} y=4+4+2=10 \\ B(2,10) \end{array}$$

$$\begin{cases} y=x^2+2x+2 \\ x=-3 \end{cases} \quad \begin{array}{l} y=9-6+2=5 \\ C(-3,5) \end{array}$$



$y>0, \forall x$

$$P=\int_{-3}^{2}(x^2+2x+2)\,dx=\frac{x^3}{3}\Big|_{-3}^{2}+2\cdot\frac{x^2}{2}\Big|_{-3}^{2}+2x\Big|_{-3}^{2}=$$

$$=\frac{1}{3}\left(8-(-27)\right)+4-9+2\left(2-(-3)\right)$$

(5.) Izračunati površinu figure ograničene krivom x^2-1, osom Ox i

pravama $x=0$ i $x=2$

$\uparrow$ y-osa

$y = x^2-1$

$x^2-1=0$

$x = \pm 1 \qquad N(1,0) \quad M(-1,0)$

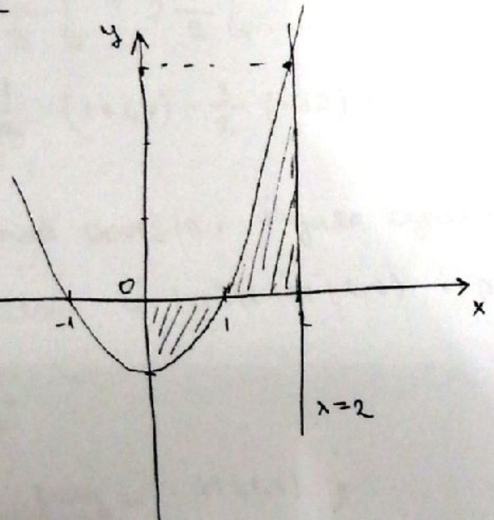
Presjek Oy-osom $(x=0)$

$y=-1 \qquad A(0,-1)$

$b=0$ pa je tjeme presjek sa Oy-osom

$T(0,-1)$

$$\begin{cases} y = x^2-1 \\ x=2 \end{cases} \Rightarrow y=3 \qquad B(2,3)$$



f-ja mijenja znak na intervalu $(0,2)$

$$P = \int_0^2 |x^2-1|\,dx = \int_0^1 |x^2-1|\,dx + \int_1^2 |x^2-1|\,dx =$$

$$= \int_0^1 -(x^2-1)\,dx + \int_1^2 (x^2-1)\,dx =$$

$$= -\int_0^1 x^2dx + \int_0^1 dx + \int_1^2 x^2dx - \int_1^2 dx =$$

$$= -\frac{x^3}{3}\Big|_0^1 + x\Big|_0^1 + \frac{x^3}{3}\Big|_1^2 - x\Big|_1^2 = -\frac{1}{3}(1^3-0^3)+1-0+\frac{1}{3}(2^3-1^3)-(2-1)=$$

$$= -\frac{1}{3}+1+\frac{7}{3}-1 = \frac{6}{3} = 2$$

(6) Izračunati površinu ograničenu krivom $y=x^2-3x$ i pravom $y=0$ $\leftarrow$ x-osa

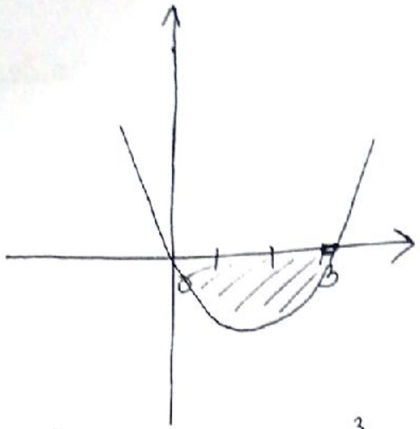
$y = x^2 - 3x$

$x^2 - 3x = 0$

$x(x-3) = 0$

$x = 0 \lor x = 3$

presjek i sa Oy osom → $N(0,0)$ $M(3,0)$



$$P = -\int_0^3 (x^2 - 3x)\,dx = -\int_0^3 x^2 dx + 3\int_0^3 x\,dx =$$

$$= -\frac{x^3}{3}\Big|_0^3 + 3\cdot\frac{x^2}{2}\Big|_0^3 = -\frac{1}{3}(3^3 - 0^3) + \frac{3}{2}(3^2 - 0^2) = -9 + \frac{27}{2} = \frac{9}{2}$$

⑦ Izračunati površinu figure ograničene parabolom $y = x^2 + 4x$ i pravom $y = x + 4$

$y = x^2 + 4x$

$x^2 + 4x = 0$

$x(x+4) = 0$

$x = 0 \lor x = -4$

$N(0,0)$ $M(-4,0)$

presjek i sa Oy-osom

$T\left(-\frac{b}{2a}, \frac{-D}{4a}\right)$

$T\left(-\frac{4}{2}, -\frac{16}{4}\right)$

$T(-2,-4)$

$a = 1 > 0$ $\cup$

$$\begin{cases} y = x^2 + 4x \\ y = x + 4 \end{cases}$$

$x^2 + 4x = x + 4$

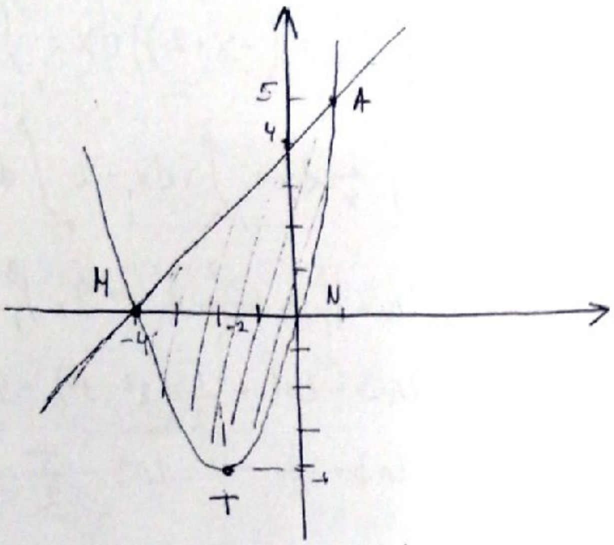
$x^2 + 3x - 4 = 0$

$x_{12} = \frac{-3 \pm \sqrt{9 - 4(-4)}}{2}$

$x_1 = 1$ $x_2 = -4$

$x = 1 \Rightarrow y = 1+4 = 5 \quad A(1,5)$

$x = -4 \Rightarrow y = -4+4 = 0 \quad M(-4,0)$



$$P = \int_{-4}^{1} \left(x+4-(x^2+4x)\right)dx = \int_{-4}^{1}(-x^2-3x+4)\,dx = \int_{-4}^{1} x^2 dx - 3\int_{-4}^{1} x\,dx + 4\int_{-4}^{1} dx =$$

$$= -\frac{x^3}{3}\Big|_{-4}^{1} - 3\frac{x^2}{2}\Big|_{-4}^{1} + 4x\Big|_{-4}^{1} = -\frac{1}{3}\left(1^3-(-4)^3\right) - \frac{3}{2}\left(1^2-(-4)^2 + 4(1-(-4))\right) =$$

$$= -\frac{1}{3}\cdot(1+64) - \frac{3}{2}(-15) + 4\cdot 5 = -\frac{65}{3} + \frac{45}{2} + 20 = \frac{-130+1}{6}$$

(8) Izračunati površinu figure ograničene krivom $y = \frac{1}{x}$, njenom tangentom u tački $M(1,1)$ i pravom $x=3$

$y = \frac{1}{x}$

J-na tang u $M(1,1)$ je

$y - 1 = y'(1)\cdot(x-1)$

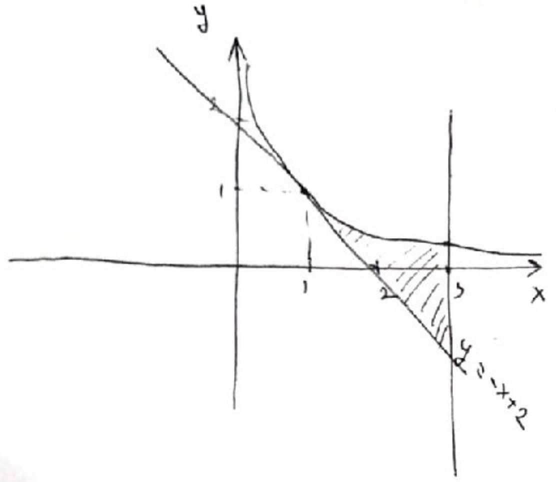
$y'(x) = -\frac{1}{x^2}$

$y'(1) = -\frac{1}{1^2} = -1$

$t: y-1 = -1(x-1)$

$t: y = -x+2$

$$\begin{cases} y=\frac{1}{x} \\ x=3 \end{cases} \Rightarrow y=\frac{1}{3} \qquad A\left(3,\frac{1}{3}\right)$$



$$P=\int_1^3\left(\frac{1}{x}-(-x+2)\right)dx=\int_1^3\left(\frac{1}{x}+x-2\right)dx=$$

$$=\int_1^3\frac{1}{x}\,dx+\int_1^3 x\,dx-2\int_1^3 dx=$$

$$=\ln x\Big|_1^3+\frac{x^2}{2}\Big|_1^2-2x\Big|_1^3=$$

$$=\ln 3-\ln 1+\frac{1}{2}(2^2-1^2)-2(3-1)=$$

$$=\ln 3+\frac{3}{2}-4=\ln 3-\frac{5}{2}$$

(9) Izračunati dužinu luka krive $y=\frac{1}{4}x^2-\frac{1}{2}\ln x$ na intervalu $[1,e]$.

$$l=\int_a^b\sqrt{1+y'^2}\,dx$$

$$l=\int_1^e\sqrt{1+y'^2}\,dx=$$

$$y'=\frac{1}{4}\cdot 2x-\frac{1}{2}\cdot\frac{1}{x}=\frac{x}{2}-\frac{1}{2\cdot x}=\frac{x^2-1}{2x}$$

$$l=\int_1^e\sqrt{1+\left(\frac{x^2-1}{2x}\right)^2}\,dx=\int_1^e\sqrt{\frac{4x^2+x^4-2x^2+1}{4x^2}}\,dx=\int_1^e\sqrt{\frac{x^4+2x^2+1}{4x^2}}\,dx=$$

$$=\int_1^e\sqrt{\frac{(x^2+1)^2}{(2x)^2}}\,dx=\int_1^e\frac{x^2+1}{2x}\,dx=\frac{1}{2}\int_1^e x\,dx+\frac{1}{2}\int_1^e\frac{1}{x}\,dx=$$

$$=\frac{1}{2}\cdot\frac{x^2}{2}\Big|_1^e+\frac{1}{2}\ln|x|\Big|_1^e=\frac{1}{4}(e^2-1)+\frac{1}{2}(\underbrace{\ln e}_{1}-\underbrace{\ln 1}_{0})=$$

$$=\frac{1}{4}(e^2-1)+\frac{1}{2}\cdot 1=\frac{1}{4}e^2+\frac{1}{4}=\frac{e^2+1}{4}$$

(10) Naći zapreminu koja nastaje rotacijom figure ograničene krivom $y=-x^2+2x$ i pravom $y=0$ oko Ox-ose

$y=-x^2+2x$

$-x^2+2x=0$

$x(-x+2)=0$

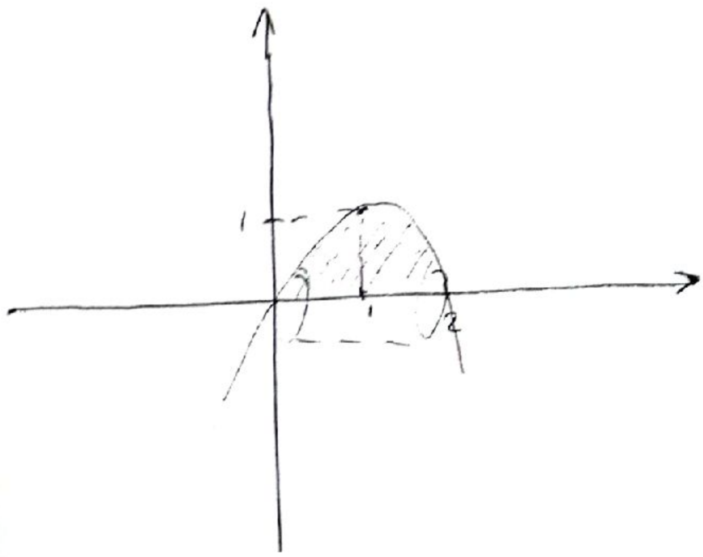
$x=0 \vee x=2$

$N(0,0)$ $M(2,0)$

presjek sa Oy-osom

$T\left(-\frac{2}{-2}, -\frac{4}{-4}\right)$ $T(1,1)$

$a=-1<0$ $\cap$



$$V=\pi\int_a^b f^2(x)dx$$

$$V=\pi\int_0^2(-x^2+2x)^2dx=\pi\int_0^2(x^4-4x^3+4x^2)dx=\pi\int_0^2 x^4dx-4\pi\int_0^3 x^3dx+4\pi\int_0^3 x^2dx=\pi\cdot\frac{x^5}{5}\Big|_0^2-4\pi\frac{x^4}{4}\Big|_0^2+4\pi\cdot\frac{x^3}{3}\Big|_0^2=$$

$$=\frac{\pi}{5}\cdot(2^5-0)-\pi\cdot(2^4-0^4)+\frac{4}{3}\pi(2^3-0^3)=\frac{32\pi}{5}-16\pi+\frac{32\pi}{3}=$$