VJEŽBE XIII

Def: F-ja f ima $\lim\limits_{x\to x_0} f(x)$ ako $\exists A$ tako da $\forall \varepsilon>0, \exists \delta>0$ t.j. $\forall x: 0<|x-x_0|<\delta \Rightarrow |x-x_0| = \sqrt{(x_1-x_1^0)^2+(x_2-x_2^0)^2+\dots(x_m-x_m^0)^2}$ bude $|f(x)-A|<\varepsilon$.

① Naći $\lim\limits_{x\to 0}(\lim\limits_{y\to\infty} f(x,y))$ i $\lim\limits_{y\to\infty}(\lim\limits_{x\to 0} f(x,y))$, ako: $f(x,y)=\frac{1}{xy}\,tg\frac{xy}{1+xy}$

(uzastopne granične vrijednosti)

- $x\neq 0$ fiksiramo, $\lim\limits_{y\to+\infty}\frac{xy}{1+xy}=\lim\limits_{y\to\infty}\frac{x}{\frac{1}{y}+x}=1$, pa zbog neprekidnosti $tg\,x$, važi: $\lim\limits_{y\to+\infty}\frac{1}{xy}\,tg\frac{xy}{1+xy}=0\cdot tg1=0$ (Koši)

- fiksiramo y, važi: $\left[\lim\limits_{x\to 0}\frac{tg\,x}{x}=\lim\limits_{x\to 0}\frac{\sin x}{\cos x\cdot x}=1\right]$

$$\lim_{x\to 0} f(x,y)=\lim_{x\to 0}\frac{tg\frac{xy}{1+xy}}{\frac{xy}{1+xy}}\cdot(yx+1)^{-1}=\lim_{x\to 0}\frac{1}{xy+1}=1$$

$$\lim_{x\to 0}\left(\lim_{y\to\infty}\frac{1}{xy}\,tg\frac{xy}{1+xy}\right)=\lim_{x\to 0}0=0$$

$$\lim_{y\to 0}\left(\lim_{x\to 0}\frac{1}{xy}\,tg\frac{xy}{1+xy}\right)=\lim_{y\to 0}1=1$$

Def. granične vrijednosti (Hajne): F-ja ima $\lim\limits_{x\to x_0} f(x)$ ako $\exists A\in\mathbb{R}$ tako da za proizvoljno $\{x_n\}$, $x_n\in E\setminus\{x_0\}$, $E\subseteq\mathbb{R}^n$ koji konvergira ka x_0, odgovarajući $\{f(x_n)\}$ konvergira ka A.

Pokazati da je za f-ju $f(x,y)=\frac{x-y}{x+y}$, $\lim\limits_{x\to 0}(\lim\limits_{y\to 0} f(x,y))=1$, $\lim\limits_{y\to 0}(\lim\limits_{x\to 0} f(x,y))=-1$, a $\lim\limits_{\substack{x\to 0\\ y\to 0}} f(x,y)$ ne postoji.

?

$\lim\limits_{y\to 0}(\lim\limits_{x\to 0}\frac{x-y}{x+y})=\lim\limits_{y\to 0}\frac{-y}{y}=-1$; $\lim\limits_{x\to 0}(\lim\limits_{y\to 0}\frac{x-y}{x+y})=\lim\limits_{x\to 0}\frac{x}{x}=1$

Da li postoji $\lim\limits_{\substack{x\to 0\\ y\to 0}} f(x,y)=?$

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$\left\{\left(\frac{1}{n}, \frac{1}{n}\right)\right\}_{n\to\infty} \to (0,0)$ a $\left\{\left(\frac{2}{n}, \frac{1}{n}\right)\right\}_{n\to\infty} \to (0,0)$

$$\left.\begin{array}{l} f\left(\frac{1}{n}, \frac{1}{n}\right) = \frac{0}{\frac{2}{n}} = 0 \to 0,\ n\to\infty \\ f\left(\frac{2}{n}, \frac{1}{n}\right) = \frac{\frac{1}{n}}{\frac{3}{n}} = \frac{1}{3} \to \frac{1}{3},\ n\to\infty \end{array}\right\} \Rightarrow \nexists \lim_{\substack{x\to 0\\ y\to 0}} f(x,y)$$

② Ispitati neprekidnost funkcije: $f(x,y) = \begin{cases} \frac{e^{xy}-1}{2^{xy}-1} \cdot \frac{\sin\sqrt{x^2+y^2}}{\sqrt{x^2+y^2}} \\ 1, \ (x,y)=(0,0) \end{cases}$
u tački $(x,y)=(0,0)$

Za $(x,y) \neq (0,0)$ f-ja neprekidna kao kompozicija neprekidnih funkcija. Za $(x,y)=(0,0)$

$$\lim_{\substack{x\to 0\\ y\to 0}} \frac{e^{xy}-1}{2^{xy}-1} \cdot \frac{\sin\sqrt{x^2+y^2}}{\sqrt{x^2+y^2}} = \lim_{\substack{x\to 0\\ y\to 0}} \frac{\frac{e^{xy}-1}{xy}}{\frac{2^{xy}-1}{xy}} \cdot \frac{\sin\sqrt{x^2+y^2}}{\sqrt{x^2+y^2}}$$

$$\begin{array}{l} x\to 0 \\ y\to 0 \\ t\to 0 \end{array} \quad \lim_{t\to 0} \frac{e^t-1}{t} = 1, \ \lim_{t\to 0} \frac{a^t-1}{t} = \ln a; \ \lim_{t\to 0} \frac{\sin t}{t} = 1$$

$\lim\limits_{\substack{x\to 0\\ y\to 0}} \frac{1}{\ln 2} \cdot 1 = \frac{1}{\ln 2}$; $\lim\limits_{\substack{x\to 0\\ y\to 0}} f(x,y) = f(0,0)$ $\quad a \cdot \frac{1}{\ln 2} \neq 1 \Rightarrow$ nije neprek. u $(0,0)$

③ Naći graničnu vrijednost: $\lim\limits_{\substack{x\to \infty\\ y\to \infty}} \left(\frac{xy}{x^2+y^2}\right)^{x^2}$

$x^2+y^2 \geq 2xy \Rightarrow \frac{xy}{x^2+y^2} \leq \frac{xy}{2xy} = \frac{1}{2}$

$\left[(x-y)^2 \geq 0 \right.$!!!

$x^2-2xy+y^2 \geq 0$

$\left. x^2+y^2 \geq 2xy \right]$

$0 < \left(\frac{xy}{x^2+y^2}\right)^{x^2} \leq \left(\frac{1}{2}\right)^{x^2} \to 0,\ n\to+\infty$

$$\Rightarrow \lim_{\substack{x\to +\infty\\ y\to +\infty}} \left(\frac{xy}{x^2+y^2}\right)^{x^2} = 0$$

4) Odrediti parcijalne izvode funkcija:

a) $u = \operatorname{arctg} \frac{x}{y}$

$$\frac{\partial u}{\partial x} = \frac{1}{1+\left(\frac{x}{y}\right)^2} \cdot \left(\frac{x}{y}\right)'_x = \frac{1}{1+\left(\frac{x}{y}\right)^2} \cdot \frac{1}{y} = \frac{y^2}{y^2+x^2} \cdot \frac{1}{y}$$

$$\frac{\partial u}{\partial y} = \frac{1}{1+\left(\frac{x}{y}\right)^2} \cdot \left(\frac{x}{y}\right)'_y = \frac{1}{1+\left(\frac{x}{y}\right)^2} \cdot \left(\frac{-x}{y^2}\right) = \frac{y^2}{y^2+x^2} \cdot \left(\frac{-x}{y^2}\right) = \frac{-x}{y^2+x^2}$$

b) $u = x^2\sqrt{y} + 7y$

$$\frac{\partial u}{\partial y} = x^2 \cdot \frac{1}{2\sqrt{y}} + 7$$

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c) $u=\sin x^2\cdot\cos\sqrt{y}$

$$\frac{\partial u}{\partial x}=\cancel{\sin x^2\, 2x}\;\cos x^2\cdot 2x\cdot\cos\sqrt{y}$$

$$\frac{\partial u}{\partial y}=-\sin x^2\cdot\sin\sqrt{y}\cdot\frac{1}{2\sqrt{y}}$$

d) $n=x^x y^y$

$$\frac{\partial u}{\partial x}=y^y\cdot(\ln x+1)$$

$x^x=m\,/\ln$

$\ln x^x=\ln m$

$x\ln x=\ln m$

$\ln x+x\cdot\frac{1}{x}=\frac{1}{m}\cdot m'$

$m'=x^x(1+\ln x)$

$$\frac{\partial u}{\partial y}=x^x y^y(\ln y+1)$$

e) $u=x^{yz}+y^{xz}+z^{xy}$

$$\frac{\partial u}{\partial x}=yz\,x^{yz-1}+y^{xz}\cdot\ln y\cdot z+z^{xy}\ln z\cdot y$$

$$\frac{\partial u}{\partial y}=x^{yz}\cdot\ln x\cdot z+xz\,y^{xz-1}+z^{xy}\cdot\ln z\cdot x$$

$$\frac{\partial u}{\partial z}=x^{yz}\cdot\ln x\,y+y^{xz}\cdot\ln y\cdot x+xy\,z^{xy-1}$$

5.) Nađi druge parcijalne izvode $\frac{\partial^2 h}{\partial x^2}$; $\frac{\partial^2 h}{\partial x\partial y}$; $\frac{\partial^2 h}{\partial y^2}$ za fje:
→ jednom po x pa po y

$h(x,y)=\operatorname{arctg}\frac{y}{x}$

$$\frac{\partial^2 h}{\partial x^2}=\frac{\partial}{\partial x}\left(\frac{\partial h}{\partial x}\right)=\frac{\partial}{\partial x}\left(\frac{1}{1+\frac{y^2}{x^2}}\cdot\left(\frac{-y}{x^2}\right)\right)=\frac{\partial}{\partial x}\left(\frac{x^2}{x^2+y^2}\cdot\left(\frac{-y}{x^2}\right)\right)=$$

$$=-y\cdot\frac{\partial}{\partial x}\left(\frac{1}{x^2+y^2}\right)=-y\cdot\frac{-2x}{(x^2+y^2)^2}=\frac{2xy}{(x^2+y^2)^2}$$

$$\frac{\partial^2 h}{\partial x\partial y}=\frac{\partial}{\partial x}\left(\frac{\partial h}{\partial y}\right)=\frac{\partial}{\partial x}\left(\frac{1}{1+\left(\frac{y}{x}\right)^2}\cdot\frac{1}{x}\right)=\frac{\partial}{\partial x}\left(\frac{x^2}{x^2+y^2}\cdot\frac{1}{x}\right)=$$

$$=\frac{\partial}{\partial x}\left(\frac{x}{x^2+y^2}\right)=\frac{x^2+y^2-x(2x)}{(x^2+y^2)^2}=\frac{y^2-x^2}{(x^2+y^2)^2}$$

$$\frac{\partial^2 h}{\partial y^2}=\frac{1}{\partial y}\left(\frac{\partial h}{\partial y}\right)=\frac{\partial}{\partial y}\left(\frac{x}{x^2+y^2}\right)=\frac{-2xy}{(x^2+y^2)^2}$$

6) Nađi totalni diferencijal funkcije: $h(x,y)=\frac{xy}{x-y}$

$$dh=\frac{\partial h}{\partial x}\cdot dx+\frac{\partial h}{\partial y}\cdot dy$$

$$\frac{\partial h}{\partial x}=\frac{y(x-y)-xy}{(x-y)^2}=\frac{yx-y^2-xy}{(x-y)^2}=\frac{-y^2}{(x-y)^2}$$

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$$\frac{\partial f}{\partial y} = \frac{x(x-y)+xy}{(x-y)^2} = \frac{x^2 \cancel{-xy} \cancel{+xy}}{(x-y)^2} = \frac{x^2}{(x-y)^2}$$

$$df = \frac{-y^2}{(x-y)^2}dx + \frac{x^2}{(x-y)^2}dy$$

7) Približno izračunati $1{,}002 \cdot 2{,}003^2 \cdot 3{,}004^3$

$f(x,y,z) = (1+x)(2+y)^2(3+z)^3$

$f(x_0+\Delta x, y_0+\Delta y, z_0+\Delta z) - f(x_0,y_0,z_0) = df(x_0,y_0,z_0)$

$\Delta x = 0{,}002$; $\Delta x = 0{,}003$; $\Delta z = 0{,}004$

$x_0 = y_0 = z_0 = 0$

$f(0{,}002; 0{,}003; 0{,}004) - f(0,0,0) = df(0,0,0)$

$$df = \frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial y}dy + \frac{\partial f}{\partial z}dz$$

$$\frac{\partial f}{\partial x} = (2+y)^2(3+z)^3$$

$$\frac{\partial f}{\partial y} = (1+x)\,2(2+y)(3+z)^3$$

$$\frac{\partial f}{\partial z} = (1+x)(2+y)^2\,3(3+z)^2$$

$df(0,0,0) = ?$

$$\frac{\partial f}{\partial x}(0,0,0) = 2^2 \cdot 3^3 = 4 \cdot 27 = 108$$

$$\frac{\partial f}{\partial y}(0,0,0) = 108 \; ; \; \frac{\partial f}{\partial z}(0,0,0) = 108$$

$df(0,0,0) = 108 \cdot 0{,}002 + 108 \cdot 0{,}003 + 108 \cdot 0{,}004 = 0{,}972$

$f(0{,}002; 0{,}003; 0{,}004) = f(0,0,0) + df(0,0,0) = 108 + 0{,}972 = 108{,}972$

$\Delta x = dx$

$f = xy^2z^3$

$f(1{,}002; 2{,}003; 3{,}004) =$

$= f(1,2,3) + df(1,2,3)$

$$df = \frac{\partial f}{\partial x}(1,2,3)\Delta x + \frac{\partial f}{\partial y}(1,2,3)\Delta y + \frac{\partial f}{\partial z}(1,2,3)\Delta z$$

$\Delta x = 0{,}002$

$\Delta y = 0{,}003$

$\Delta z = 0{,}004$

– Diferencijal višeg reda –

$$u_{xx} = \frac{\partial^2 u}{\partial x^2} \; ; \; u_{xy} = \frac{\partial^2 u}{\partial x \partial y} \; ; \; u_{yy} = \frac{\partial^2 u}{\partial y^2}$$

$$d^2 u = \frac{\partial^2 u}{\partial x^2} \cdot dx^2 + 2 \cdot \frac{\partial^2 u}{\partial x \partial y} dx dy + \frac{\partial^2 u}{\partial y^2} \cdot dy^2$$

$$d^n u = \left(\frac{\partial u}{\partial x} \cdot dx + \frac{\partial u}{\partial y} \cdot dy\right)^n \; ; n \in N$$

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1) Za $u = \ln(x^2+y^2)$ dokazati $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$, odrediti du, d^2u.

$$\frac{\partial u}{\partial x} = \frac{1}{x^2+y^2}\cdot 2x \quad , \quad \frac{\partial u}{\partial y} = \frac{1}{x^2+y^2}\cdot 2y$$

$$\frac{\partial^2 u}{\partial x^2} = \frac{2(x^2+y^2) - 2x\cdot 2x}{(x^2+y^2)} = \frac{2y^2-2x^2}{(x^2+y^2)^2}$$

$$\frac{\partial^2 u}{\partial x \partial y} = \frac{-2x2y}{(x^2+y^2)^2}$$

$$\frac{\partial^2 u}{\partial y^2} = \frac{2(x^2+y^2) - 2y\cdot 2y}{(x^2+y^2)^2} = \frac{2x^2-2y^2}{(x^2+y^2)^2}$$

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \frac{2y^2-2x^2+2x^2-2y^2}{(x^2+y^2)^2} = 0$$

$$du = \frac{2x}{x^2+y^2}dx + \frac{2y}{x^2+y^2}dy$$

$$d^2u = \frac{2y^2-2x^2}{(x^2+y^2)^2}dx^2 - 2\frac{4yx}{(x^2+y^2)^2}dxdy + \frac{2x^2-2y^2}{(x^2+y^2)^2}dy^2$$

-Izvod implicitno zadate funkcije-

$$F = F(x,y) \; ; \; y = y(x)$$

$$\frac{\partial F}{\partial x} + \frac{\partial F}{\partial y}\cdot\frac{\partial y}{\partial x} = 0 \rightarrow \frac{\partial y}{\partial x} = + \frac{-\frac{\partial F}{\partial x}}{\frac{\partial F}{\partial y}}$$

$$F = F(x,y,z) = 0 \; ; \; z = z(x,y)$$

$$\frac{\partial z}{\partial x} = ? \qquad \frac{\partial F}{\partial x} + \frac{\partial F}{\partial z}\cdot\frac{\partial z}{\partial x} = 0 \rightarrow \frac{\partial z}{\partial x} = -\frac{\frac{\partial F}{\partial x}}{\frac{\partial F}{\partial z}}$$

$$\frac{\partial F}{\partial y} + \frac{\partial F}{\partial z}\cdot\frac{\partial z}{\partial y} = 0 \rightarrow \frac{\partial z}{\partial y} = \frac{-\frac{\partial F}{\partial y}}{\frac{\partial F}{\partial z}}$$

Def: Diferencijabilnost funkcije 2 ili 3 promjenljive

$f: A \rightarrow R$, $A \subseteq R^2$ $\; u = f(x,y)$ dif. u (x,y) ; $x = x(s,t)$; $y = y(s,t) \Rightarrow$

$\Rightarrow u = f(x,y) = f(x(s,t), y(s,t))$ u (s,t) ako važi :

$$\frac{\partial u}{\partial s} = \frac{\partial u}{\partial x}\cdot\frac{\partial x}{\partial s} + \frac{\partial u}{\partial y}\cdot\frac{\partial y}{\partial s}$$

$$\frac{\partial u}{\partial t} = \frac{\partial u}{\partial x}\cdot\frac{\partial x}{\partial t} + \frac{\partial u}{\partial y}\cdot\frac{\partial y}{\partial t}$$

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1) Odrediti $\frac{\partial u}{\partial t}$ ako je: a) $u=e^{x-2y}$, $x=\sin t$, $y=t^3$

$$\frac{\partial u}{\partial t}=\frac{\partial u}{\partial x}\cdot\frac{\partial x}{\partial t}+\frac{\partial u}{\partial y}\cdot\frac{\partial y}{\partial t}$$

$$\frac{\partial u}{\partial x}=e^{x-2y};\ \frac{\partial x}{\partial t}=\cos t;\ \frac{\partial u}{\partial y}=-2e^{x-2y},\ \frac{\partial y}{\partial t}=3t^2$$

$$\frac{\partial u}{\partial t}=e^{x-2y}\cdot\cos t+(-2e^{x-2y})\cdot 3t^2=e^{\sin t-2t^3}(\cos t-6t^2)$$

b) $\frac{\partial u}{\partial s};\ \frac{\partial u}{\partial z}=?$ $\quad u=x^2\ln y$; $x=\frac{s}{t}$; $y=3s-2t$ $\quad$ ⌜$z=t$⌟

$$\frac{\partial u}{\partial s}=\frac{\partial u}{\partial x}\cdot\frac{\partial x}{\partial s}+\frac{\partial u}{\partial y}\cdot\frac{\partial y}{\partial s}=2x\ln y\cdot\frac{1}{t}+x^2\cdot\frac{1}{y}\cdot 3=$$
$$=2\frac{s}{t}\ln(3s-2t)\frac{1}{t}+\frac{3s^2}{t^2(s3-2t)}$$

$$\frac{\partial u}{\partial z}=\frac{\partial u}{\partial x}\cdot\frac{\partial x}{\partial z}+\frac{\partial u}{\partial y}\cdot\frac{\partial y}{\partial z}=2x\ln y\cdot\frac{-s}{t^2}-2x^2\frac{1}{y}=$$
$$=2\frac{s}{t}\ln(3s-2t)\cdot\frac{-s}{t^2}=-2\cdot\frac{s^2}{t^2}\cdot\frac{1}{3s-2t}$$

Za $u=u(x,y,z)\Rightarrow \underbrace{\frac{\partial u}{\partial x},\ \frac{\partial u}{\partial y},\ \frac{\partial u}{\partial z}}_{\text{I reda}}$; $\frac{\partial^2 u}{\partial x^2},\ \frac{\partial^2 u}{\partial y^2},\ \frac{\partial^2 u}{\partial z^2},\ \frac{\partial^2 u}{\partial x\partial y},\ \frac{\partial^2 u}{\partial x\partial z},\ \frac{\partial^2 u}{\partial y\partial z}$ → II reda

$$d^2u=\frac{\partial^2 u}{\partial x^2}dx^2+2\frac{\partial^2 u}{\partial x\partial y}dxdy+\frac{\partial^2 u}{\partial y^2}dy^2+2\frac{\partial^2 u}{\partial x\partial z}dxdz+\frac{\partial^2 u}{\partial y\partial z}dydz+\frac{\partial^2 u}{\partial z^2}dz^2$$

1) Dokazati da za funkciju $u=\frac{1}{\sqrt{x^2+y^2+z^2}}$ važi: $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}=0$

2) Za funkciju $u=e^{xyz}$ odrediti $\frac{\partial^3 u}{\partial x\partial y\partial z}$

$$\frac{\partial u}{\partial x}=e^{xyz}\cdot yz;\ \frac{\partial u}{\partial y\partial x}=\frac{\partial}{\partial y}\left(\frac{\partial u}{\partial x}\right)=\frac{\partial}{\partial y}(yz\cdot e^{xyz})=$$
$$=z(e^{xyz}+y\cdot e^{xyz}\cdot xz)=e^{xyz}(z+xyz^2)$$

$$\frac{\partial}{\partial t}\left(\frac{\partial^2 u}{\partial y\partial x}\right)=\frac{\partial}{\partial t}(e^{xyz}(z+xyz^2))=e^{xyz}\cdot xy(z+xyz^2)+e^{xyz}(1+2xyz)$$

3) Izračunati izvod implicitno zadate funkcije:

a) $x+y=e^{x-y}$ $\qquad$ b) $x^2+y^2+z^2=0$

a) $/\frac{\partial}{\partial x}$ $\quad 1+\frac{\partial y}{\partial x}=e^{x-y}\left(1-\frac{\partial y}{\partial x}\right)$ $\qquad$ ili →

$$\frac{\partial y}{\partial x}(1+e^{x-y})=e^{x-y}-1$$

$$\frac{\partial y}{\partial x}=\frac{e^{x-y}-1}{1+e^{x-y}}$$

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$F(x,y) = x + y - e^{x-y} = 0$

$\frac{\partial F}{\partial x} = 1 - e^{x-y}$

$\frac{\partial F}{\partial y} = 1 + e^{x-y}$

$\frac{\partial y}{\partial x} = -\frac{1-e^{x-y}}{1+e^{x-y}}$

b) $x^2 + y^2 + z^2 = 0$ $\quad$ [z - zavisna, njen izvod z']

$\frac{\partial}{\partial x} = 2x + 2z \cdot \frac{\partial z}{\partial x} = 0 \qquad \frac{\partial}{\partial y} = 2y + 2z \cdot \frac{\partial z}{\partial y} = 0$

$\frac{\partial z}{\partial x} = \frac{-2x}{2z} = \frac{-x}{z} \qquad \frac{\partial z}{\partial y} = -\frac{y}{z}$

4) Ispitati da li funkcija z definisana sa $F(z-\sqrt{x}, \sqrt{x}-\sqrt{y}) = 0$ zadovoljava jednačinu: $\sqrt{x} \cdot \frac{\partial z}{\partial x} + \sqrt{y} \cdot \frac{\partial z}{\partial y} = \frac{1}{2}$

$u = z - \sqrt{x} \qquad v = \sqrt{x} - \sqrt{y}$

$\frac{\partial u}{\partial x} = -\frac{1}{2\sqrt{x}}, \quad \frac{\partial u}{\partial z} = 1, \quad \frac{\partial v}{\partial x} = \frac{1}{2\sqrt{x}}, \quad \frac{\partial v}{\partial y} = \frac{-1}{2\sqrt{y}}, \quad \frac{\partial u}{\partial y} = 0 \quad \frac{\partial v}{\partial z} = 0$

$\frac{\partial z}{\partial x} = \frac{-\frac{\partial F}{\partial x}}{\frac{\partial F}{\partial z}}; \quad \frac{\partial z}{\partial y} = -\frac{\frac{\partial F}{\partial y}}{\frac{\partial F}{\partial z}}$

$\frac{\partial F}{\partial x} = \frac{\partial F}{\partial u} \cdot \frac{\partial u}{\partial x} + \frac{\partial F}{\partial v} \cdot \frac{\partial v}{\partial x} = \frac{\partial F}{\partial u}\left(\frac{-1}{2\sqrt{x}}\right) + \frac{\partial F}{\partial v} \cdot \frac{1}{2\sqrt{x}} = \frac{\frac{\partial F}{\partial v} - \frac{\partial F}{\partial u}}{2\sqrt{x}}$

$\frac{\partial F}{\partial y} = \frac{\partial F}{\partial u} \cdot \frac{\partial u}{\partial y} + \frac{\partial F}{\partial v} \cdot \frac{\partial v}{\partial y} = \frac{\partial F}{\partial u} \cdot 0 - \frac{\partial F}{\partial v} \cdot \frac{1}{2\sqrt{y}} = \frac{-\frac{\partial F}{\partial v}}{2\sqrt{y}}$

$\frac{\partial F}{\partial z} = \frac{\partial F}{\partial u} \cdot \frac{\partial u}{\partial z} + \frac{\partial F}{\partial v} \cdot \frac{\partial v}{\partial z} = \frac{\partial F}{\partial u} + 0 = \frac{\partial F}{\partial u}$

$\frac{\partial t}{\partial x} = -\frac{\frac{\partial F}{\partial x}}{\frac{\partial F}{\partial z}} = -\frac{-\frac{\partial F}{\partial u} + \frac{\partial F}{\partial v}}{2\sqrt{x} \cdot \frac{\partial F}{\partial u}}$

$\frac{\partial z}{\partial y} = -\frac{\frac{\partial F}{\partial y}}{\frac{\partial F}{\partial z}} = -\frac{-\frac{\partial F}{\partial v}}{2\sqrt{y} \cdot \frac{\partial F}{\partial u}}$

$\sqrt{x} \cdot \frac{\partial z}{\partial x} + \sqrt{y} \cdot \frac{\partial z}{\partial y} = \sqrt{x} \cdot \frac{\frac{\partial F}{\partial u} - \frac{\partial F}{\partial v}}{2\sqrt{x} \cdot \frac{\partial F}{\partial u}} + \sqrt{y} \cdot \frac{\frac{\partial F}{\partial v}}{2\sqrt{y}\,\frac{\partial F}{\partial u}} =$

$\frac{\ldots}{2\frac{\partial F}{\partial u}} = \frac{1}{2}$

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