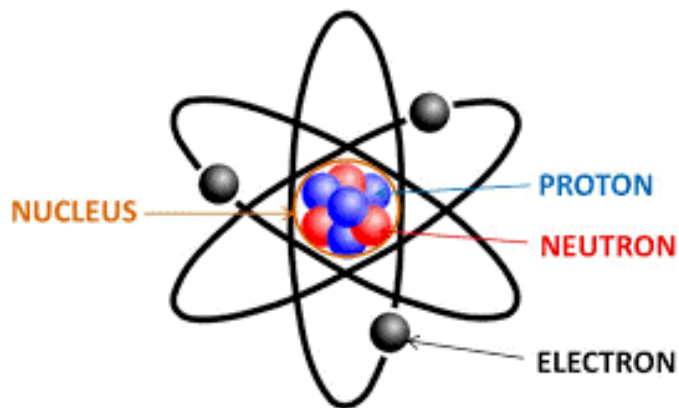


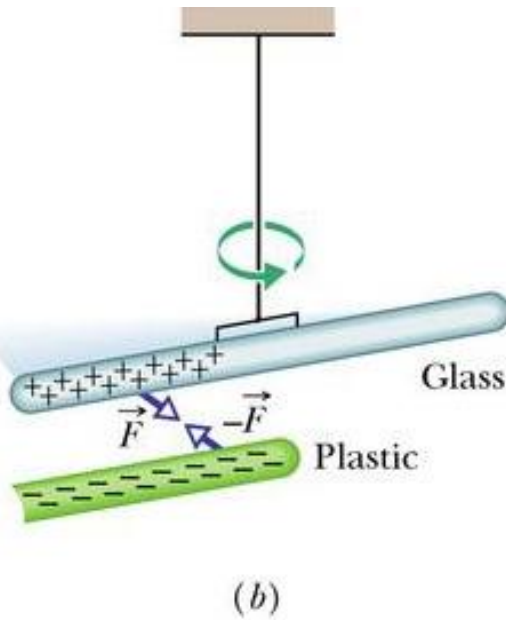
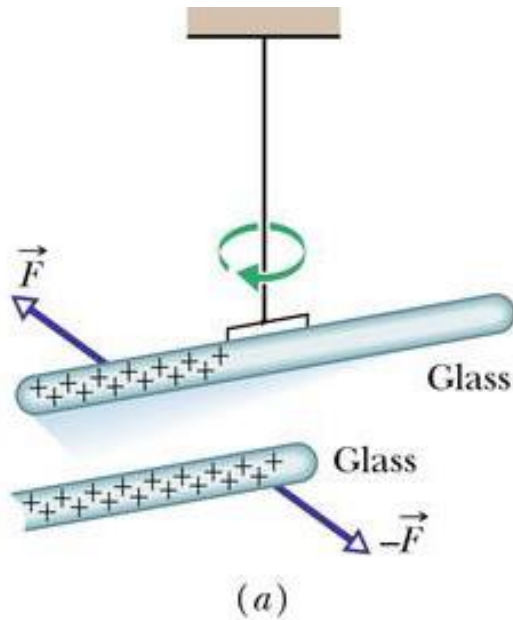
Elektro-statika



Naelekt		
Čestica	Simbol	risanje
Elektron	e ili e^-	$-e$
Proton	p	$+e$
Neutron	n	0

$$|e^+| = |e^-| = e = 1.6 \cdot 10^{-19} \text{ C}$$

Naelektrisanje materije



Zakon održanja količine naelektrisanja

$$q = n_1 \cdot e^+ - n_2 \cdot e^-, n_1, n_2 = \pm 1, \pm 2, \pm 3, \dots,$$

$$\sum q = \text{const.}$$

Jedinica za naelektrisanje je Kulon (C).

Kulonov zakon



(a) Odbijanje



(b) Odbijanje

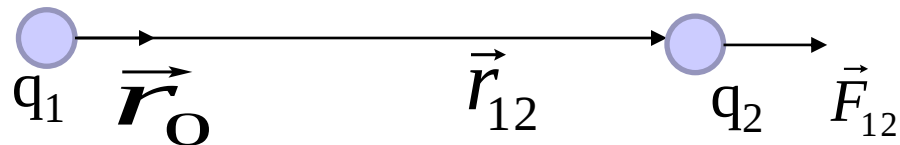


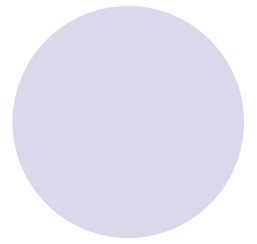
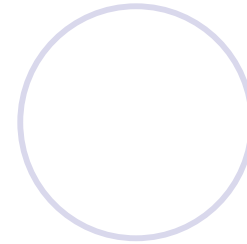
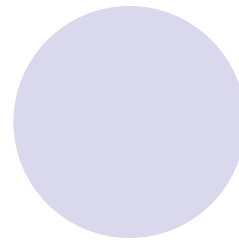
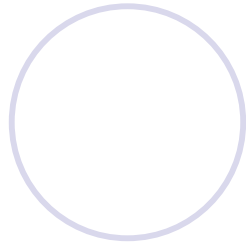
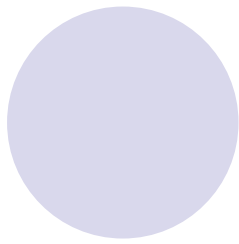
(c) Privlačenje

$$F = k \frac{q_1 q_2}{r^2}$$

$$\vec{F}_{12} = k \frac{q_1 q_2}{r_{12}^3} \vec{r}_{12}$$

$$\vec{F}_{12} = k \frac{q_1 q_2}{r^2} \vec{r}_0$$





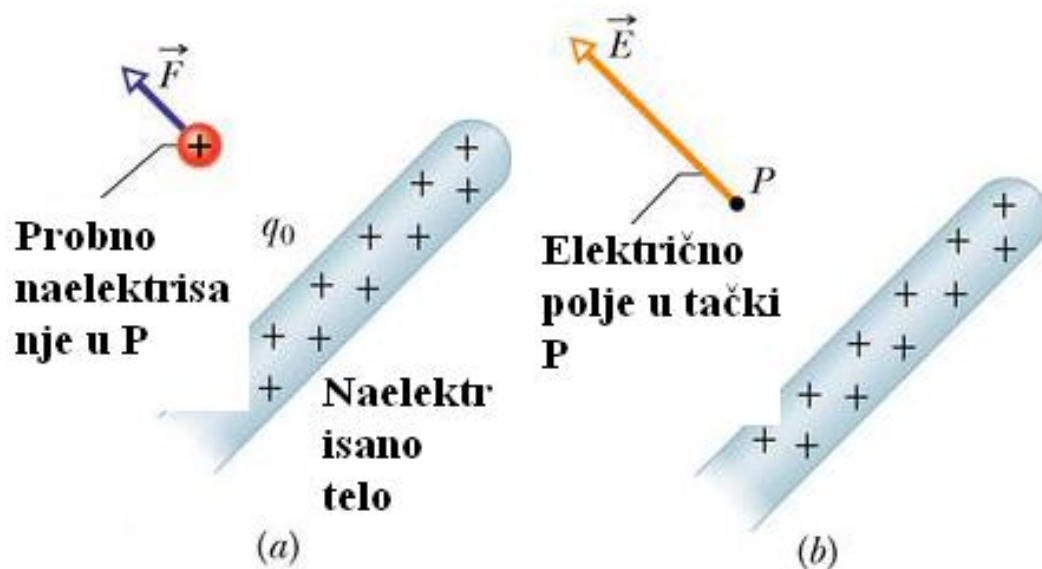
$$F = k \frac{q_1 q_2}{r^2} \Rightarrow k = \frac{F r^2}{q_1 q_2}$$

$$k = \frac{1}{4\pi\epsilon_0} = 8.99 \cdot 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2$$

$$\epsilon_0 = 8.85 \cdot 10^{-12} \text{ C}^2 / \text{N} \cdot \text{m}^2$$

$$F = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2}$$

Električno polje



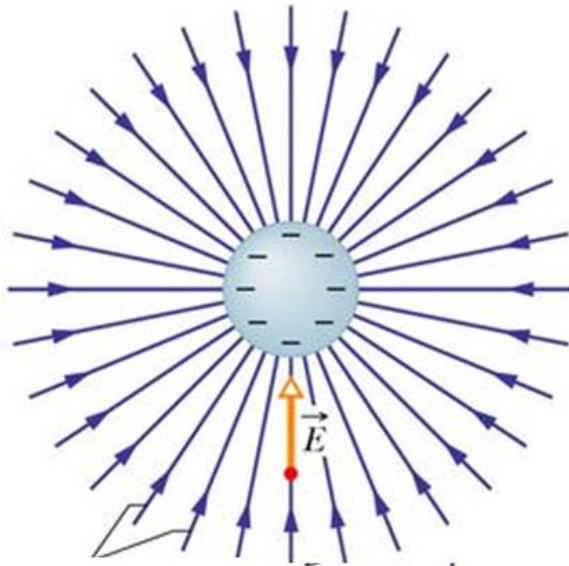
$$\vec{F} = q_0 \vec{E}$$

$$\vec{E} = \frac{\vec{F}}{q_0}$$

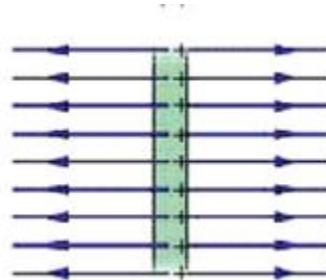
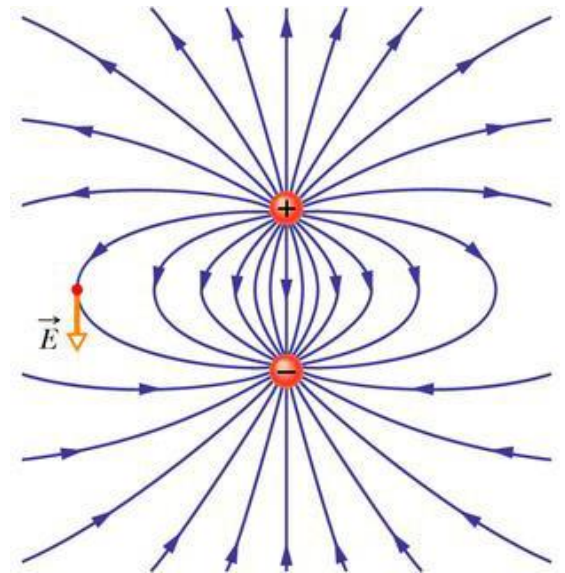
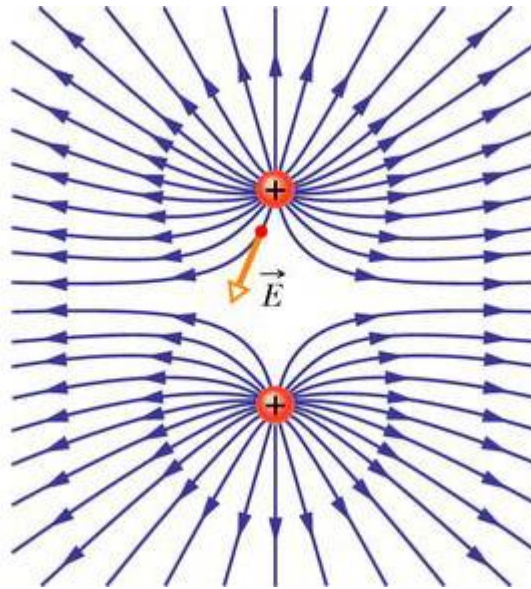
$$|\vec{E}| = \frac{|\vec{F}|}{q_0}$$

Jedinica za jačinu električnog polja je N/C.

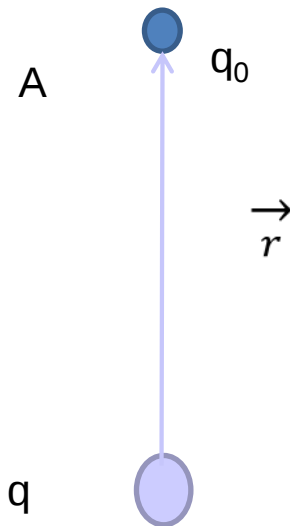
Linije sila električnog polja



Linije sila električnog polja



Polje tačkastog naelektrisanja

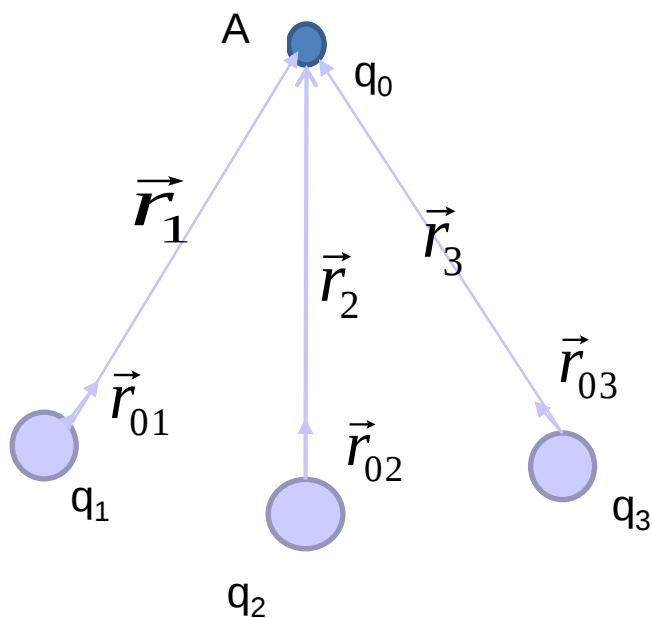


$$\vec{F} = \frac{1}{4\pi\epsilon_0} \frac{qq_0}{r^2} \vec{r}_0$$

$$\vec{E} = \frac{\vec{F}}{q_0} = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \vec{r}_0$$

$$E = \frac{F}{q_0} = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2}$$

Polje sistema tačkastih naelektrisanja



$$\vec{F}_1 = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_0}{r_1^2} \vec{r}_{01}$$

$$\vec{F}_2 = \frac{1}{4\pi\epsilon_0} \frac{q_2 q_0}{r_2^2} \vec{r}_{02}$$

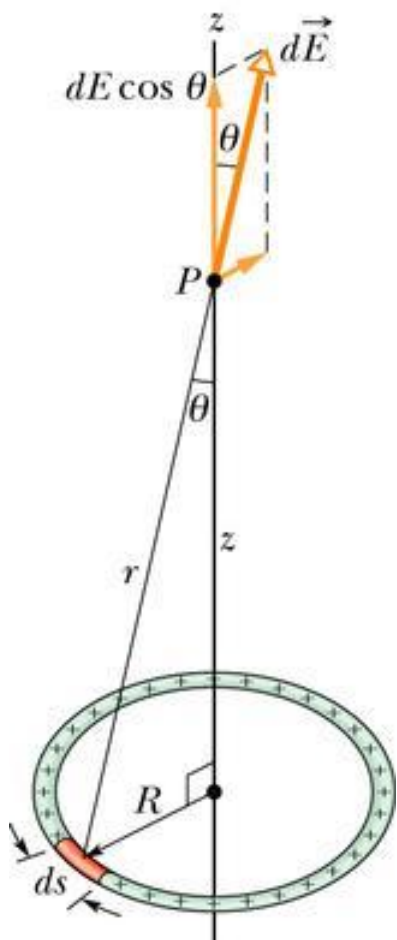
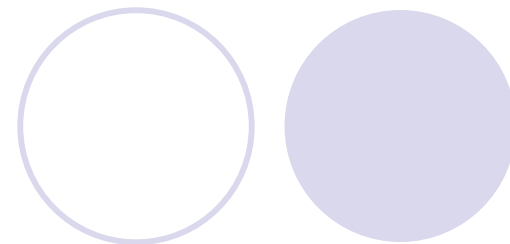
$$\vec{F}_3 = \frac{1}{4\pi\epsilon_0} \frac{q_3 q_0}{r_3^2} \vec{r}_{03}$$

$$\vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 = \frac{q_0}{4\pi\epsilon_0} \left(\frac{q_1}{r_1^2} \vec{r}_{01} + \frac{q_2}{r_2^2} \vec{r}_{02} + \frac{q_3}{r_3^2} \vec{r}_{03} \right)$$

$$\vec{E} = \frac{\vec{F}}{q_0} = \frac{1}{4\pi\epsilon_0} \left(\frac{q_1}{r_1^2} \vec{r}_{01} + \frac{q_2}{r_2^2} \vec{r}_{02} + \frac{q_3}{r_3^2} \vec{r}_{03} \right) = \frac{1}{4\pi\epsilon_0} \sum_{i=1}^3 \frac{q_i}{r_i^2} \vec{r}_{0i}$$

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \sum_{i=1}^n \frac{q_i}{r_i^2} \vec{r}_{0i} = \vec{E}_1 + \vec{E}_2 + \dots + \vec{E}_n = \sum_{i=1}^n \vec{E}_i$$

Polje na osi ravnomerno naelektrisanog prstena

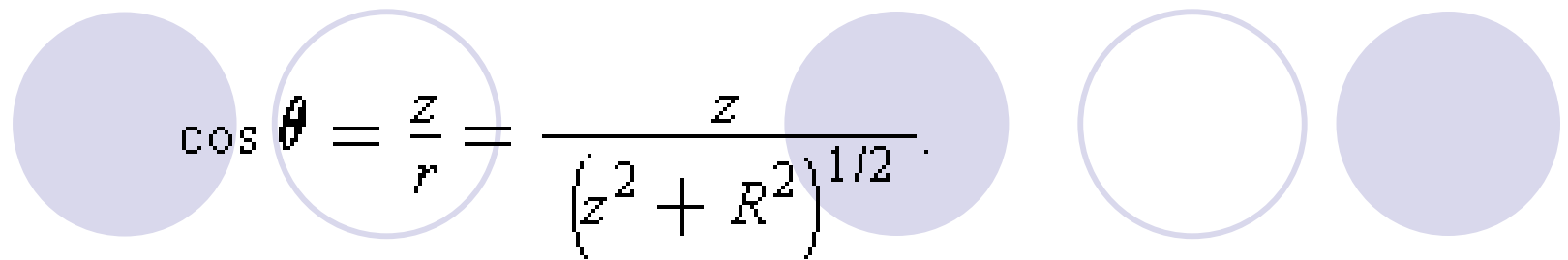


$$dq = \lambda ds.$$

λ je naelektrisanje po jedinici dužine

$$dE = \frac{1}{4\pi\epsilon_0} \frac{dq}{r^2} = \frac{1}{4\pi\epsilon_0} \frac{\lambda ds}{r^2}.$$

$$dE = \frac{1}{4\pi\epsilon_0} \frac{\lambda ds}{(z^2 + R^2)}.$$

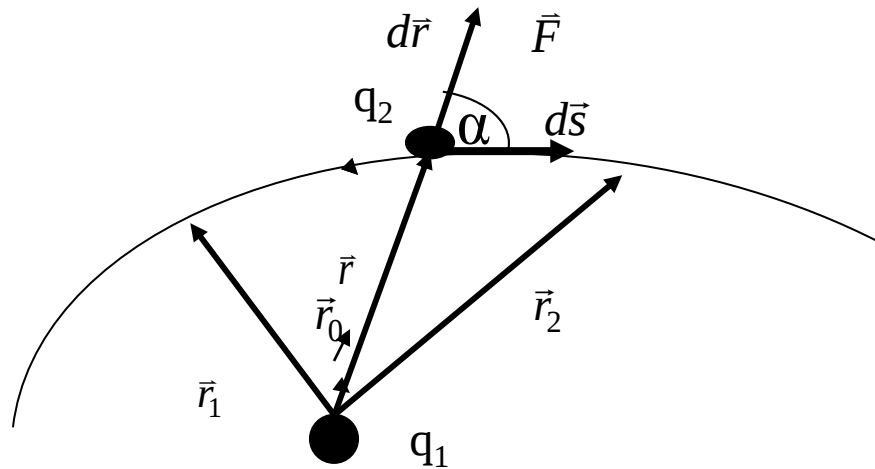


$$dE \cos \theta = \frac{z\lambda}{4\pi\epsilon_0 (z^2 + R^2)^{3/2}} ds.$$

$$E = \int dE \cos \theta = \frac{z\lambda}{4\pi\epsilon_0 (z^2 + R^2)^{3/2}} \int_0^{2\pi R} ds = \frac{z\lambda (2\pi R)}{4\pi\epsilon_0 (z^2 + R^2)^{3/2}}.$$

$$E = \frac{qz}{4\pi\epsilon_0 (z^2 + R^2)^{3/2}} \quad (\text{naelektrisani prsten}).$$

Električna potencijalna energija



$$\vec{F} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2} \vec{r}_0 = F(r) \vec{r}_0$$

$$A_{12} = \int_1^2 F(r) \cdot \vec{r}_0 \cdot d\vec{s} = \int_1^2 F(r) \cdot dr =$$

$$= \frac{1}{4\pi\epsilon_0} q_1 q_2 \int_1^2 \frac{dr}{r^2} = \frac{1}{4\pi\epsilon_0} q_1 q_2 \left(\frac{1}{r_1} - \frac{1}{r_2} \right)$$

$$E_p = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r} + \text{const.}$$

Kad je $r = \infty$, $E_p = 0$.

$$A_{12} = E_{p1} - E_{p2}$$

$$E_p = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r}$$

Električni potencijal V

Električna potencijalna energija po jedinici naelektrisanja u datoj tački polja ne zavisi od probnog naelektrisanja i predstavlja karakteristiku polja i zove se električni potencijal.

$$V = \frac{E_p}{q_2}$$

$$V = \frac{1}{4\pi\epsilon_0} \frac{q}{r}$$

$$E_p = qV$$

$$V = \frac{A_\infty}{q} = \frac{1}{q} \int_C^\infty \vec{F} d\vec{r} = -\frac{1}{q} \int_\infty^C \vec{F} d\vec{r}$$

$$A_{12} = q(V_1 - V_2)$$

$$V = -\int_\infty^C \vec{E} d\vec{r}$$

Potencijal date tačke polja brojno je jednak radu koji izvrše sile polja na premeštanju jediničnog probnog naelektrisanja iz date tačke u beskonačnost.

Jedinica za električni potencijal je Volt ($V=J/C$).

Električni napon

$$V_A = -\frac{1}{q} \int_{\infty}^A \vec{F} d\vec{r} = -\int_{\infty}^A \vec{E} d\vec{r} = -\int_{\infty}^A E dr \cos \Theta$$

$$V_B - V_A = -\int_{\infty}^B \vec{E} d\vec{r} + \int_{\infty}^A \vec{E} d\vec{r} = -\int_A^B \vec{E} d\vec{r} = -\int_A^B E dr \cos \Theta$$

Razlika potencijala između dve tačke polja ili napon je brojno jednak radu koji treba izvršiti da se jedinično naelektrisanje prenese iz tačke A u tačku B.

Električni potencijal i električno polje

$$dV = -\vec{E}d\vec{r} = -E \cos \Theta dr, \text{ kako je } E \cos \Theta = E_x$$

$$E_x = -\frac{\partial V}{\partial x}, \quad E_y = -\frac{\partial V}{\partial y}, \quad E_z = -\frac{\partial V}{\partial z}$$

$$\vec{E} = E_x \vec{i} + E_y \vec{j} + E_z \vec{k} = -\left(\frac{\partial V}{\partial x} \vec{i} + \frac{\partial V}{\partial y} \vec{j} + \frac{\partial V}{\partial z} \vec{k} \right)$$

$$\nabla = \vec{i} \frac{\partial}{\partial x} + \vec{j} \frac{\partial}{\partial y} + \vec{k} \frac{\partial}{\partial z}, \text{ diferencijalni operator nabra,}$$

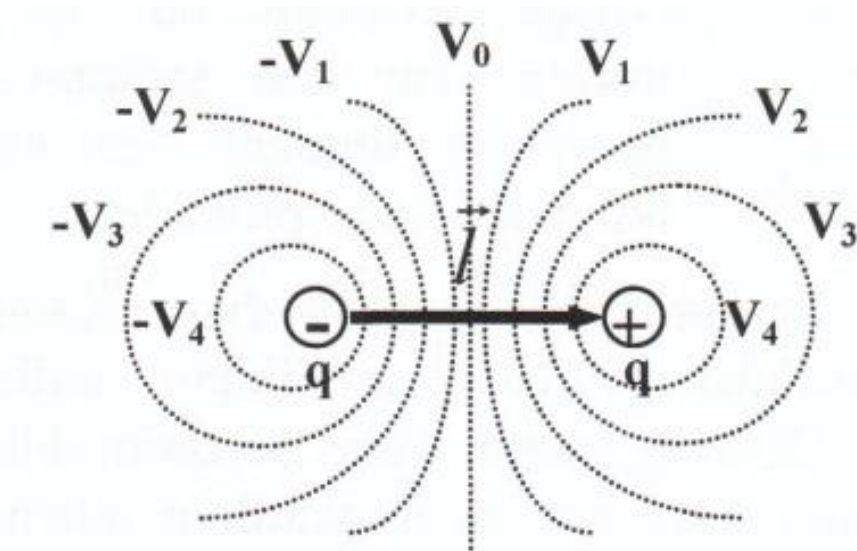
$$\vec{E} = -\nabla V(\vec{r}) = -\text{grad } V(\vec{r})$$

Ekvipotencijalne površi

$$\vec{E} = -\text{grad}V(\vec{r}) \Rightarrow E_x = -\frac{\partial V}{\partial x}, E_y = -\frac{\partial V}{\partial y}, E_z = -\frac{\partial V}{\partial z}$$

$$E = -\frac{\Delta V}{\Delta r}$$

$$\Delta r = -\frac{1}{E} \Delta V$$



Polazeći od činjenice da je električno polje dato gradijentom potencijala uvodimo nov način prikazivanja polja pomoću *ekvipotencijalnih površina*.

Ekvipotencijalna površina u prostoru je skup svih tačaka u kojima je potencijal jednake vrednosti.

Kapacitet izolovanog provodnika

$$V = \frac{1}{4\pi\epsilon_0} \frac{q}{r} \Rightarrow q = 4\pi\epsilon_0 r V$$

$$q = CV$$

$$C = \frac{q}{V} = 4\pi\epsilon_0 r$$

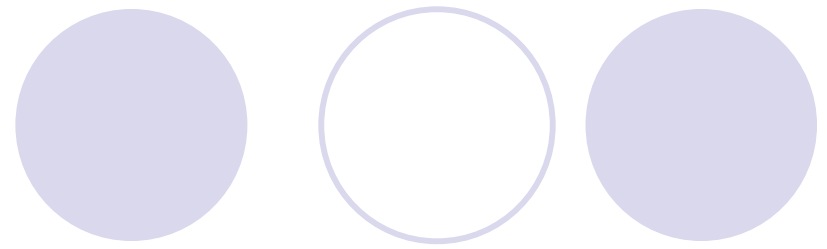
C je kapacitet, konstanta koja ne zavisi od q ili V .

$$1 \text{ farad} = 1 \text{ F} = 1 \text{ coulomb per volt} = 1 \text{ C} / \text{V}.$$

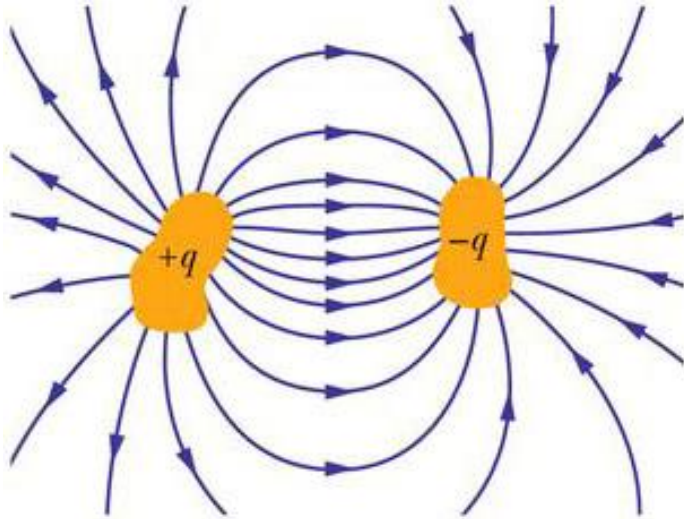
$$1 \mu\text{F} = 10^{-6} \text{ F}$$

$$1 \text{ pF} = 10^{-12} \text{ F}$$

Kondenzatori



Električni kondenzator je sistem dva provodnika postavljenih jedan pored drugog i naelektrisana jednakim količinama naelektrisanja suprotnog znaka.



$$C = \frac{q}{V}$$

Kapacitet od jednog farada ima kondenzator kome se razlika potencijala promeni za 1V, kada se naelektrisanje promeni za 1C.

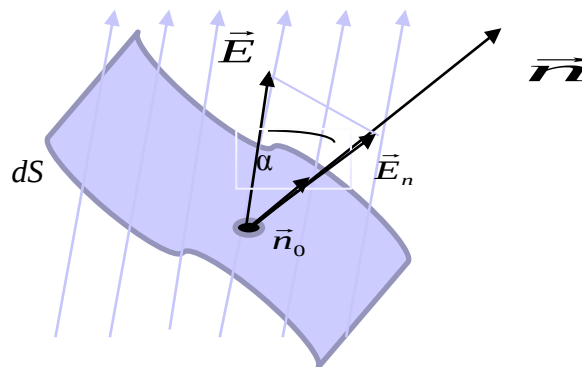
Električni fluks. Gausova teorema

$$d\Phi = \vec{E}d\vec{S} = EdS \cos \alpha = E_n dS, \text{ gde je } d\vec{S} = dS\vec{n}_0$$

$$\Phi = \oint_S \vec{E}d\vec{S}$$

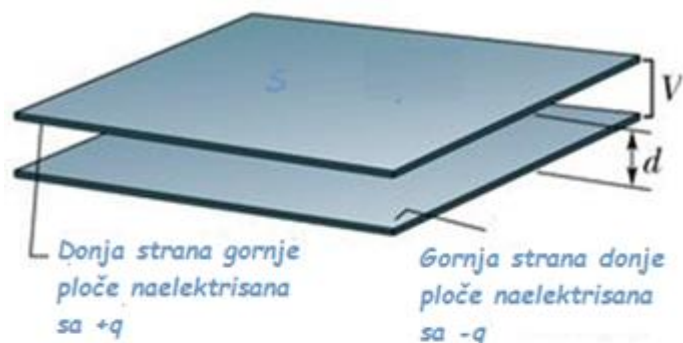
$$\Phi = \oint_S \vec{E}d\vec{S} = \frac{1}{\epsilon_0} \sum_{i=1}^n q_i$$

$$\epsilon_0 \oint_S \vec{E}d\vec{S} = \sum_{i=1}^n q_i$$

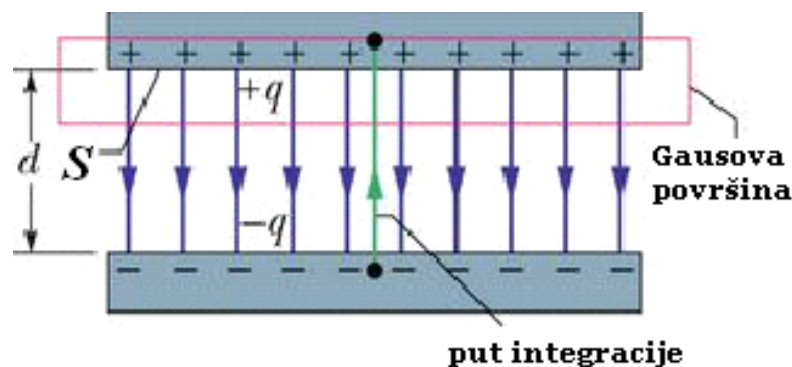
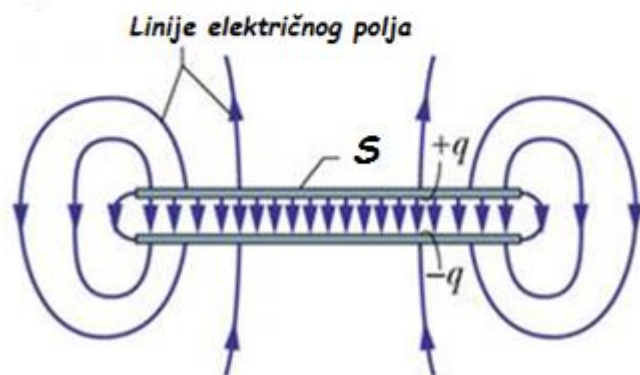


Gausova teorema glasi: Fluks vektora električnog polja kroz proizvoljnu zatvorenu površinu S jednak je algebarskom zbiru naelektrisanja unutar te površine, podeljenim sa ϵ_0 .

Pločasti kondenzator



(a)



$$ES = \frac{q}{\epsilon_0} \Rightarrow E = \frac{q}{\epsilon_0 S}$$

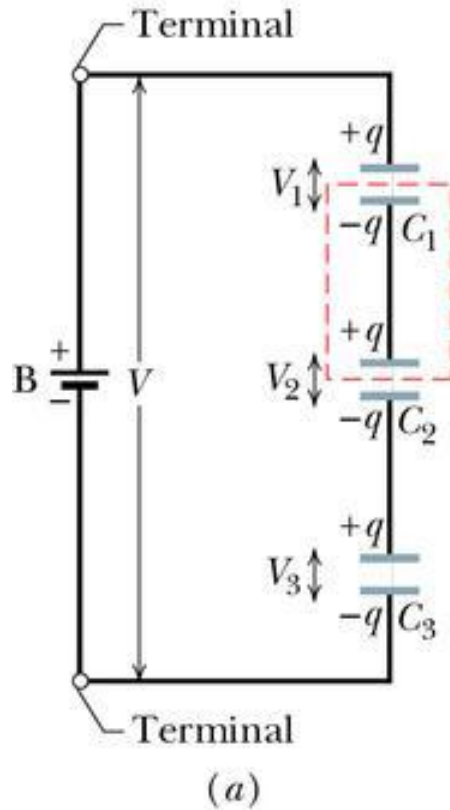
$$V = \int_{+}^{-} E dl = E \int_0^d dl = Ed = \frac{q}{\epsilon_0 S} d$$

$$C = \frac{q}{V} = \frac{q}{\frac{q}{\epsilon_0 S} d} = \frac{\epsilon_0 S}{d}$$

Vezivanje kondenzatora

- Paralelna veza
- Redna veza
- Kada imamo više kondenzatora u strujnom kolu možemo ih zameniti jednim ekvivalentnim kondenzatorom, koji ima ukupni kapacitet svih kondenzatora.

Redna veza kondenzatora



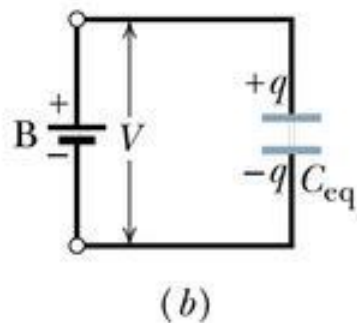
$$V_1 = \frac{q}{C_1}, V_2 = \frac{q}{C_2}, V_3 = \frac{q}{C_3}$$

$$V = V_1 + V_2 + V_3 = q \left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \right)$$

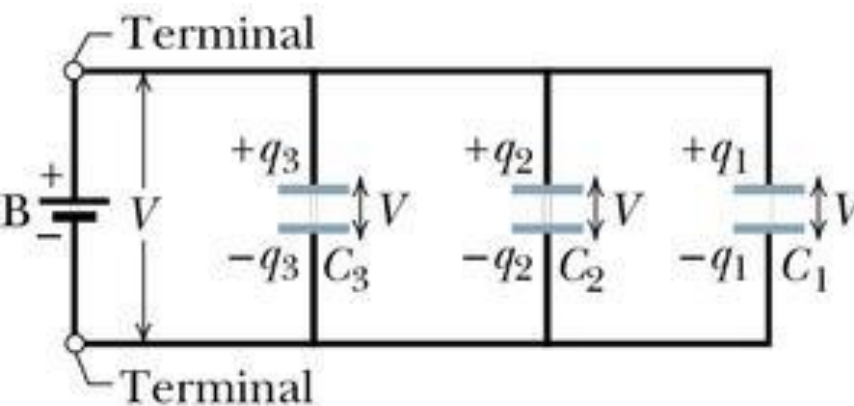
$$C_{eq} = \frac{q}{V} = \frac{1}{1/C_1 + 1/C_2 + 1/C_3},$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}.$$

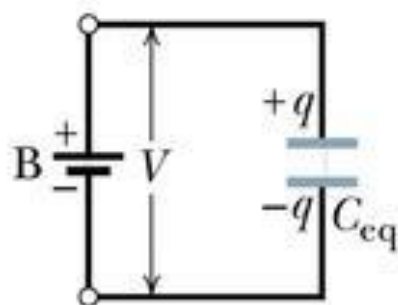
$$\frac{1}{C_{eq}} = \sum_{j=1}^n \frac{1}{C_j}$$



Paralelna veza kondenzatora



(a)



(b)

$$q_1 = C_1 V, \quad q_2 = C_2 V, \quad q_3 = C_3 V$$

$$q = q_1 + q_2 + q_3 = (C_1 + C_2 + C_3)V.$$

$$C_{eq} = \frac{q}{V} = C_1 + C_2 + C_3.$$

$$C_{eq} = \sum_{j=1}^n C_j$$

Energija napolnjenog kondenzatora. Energija električnog polja

Proces punjenja kondenzatora je prenos naelektrisanja sa obloge nižeg na oblogu višeg potencijala. Pri tome se vrši rad nasuprot dejstvu elektrostatičkih sila.

$$dA = Vdq = \frac{q}{C} dq$$

$$A = \int_0^Q Vdq = \frac{1}{C} \int_0^Q qdq = \frac{Q^2}{2C}$$

$$E_p = \frac{Q^2}{2C} = \frac{QV}{2} = \frac{CV^2}{2}$$

$$W = \frac{CV^2}{2} = \frac{1}{2} \epsilon_0 \frac{S}{d} V^2 = \frac{1}{2} \epsilon_0 \frac{V^2}{d^2} Sd$$

$$W = \frac{1}{2} \epsilon_0 E^2 Sd = \frac{1}{2} \epsilon_0 E^2 V_{\text{zapremina}}$$

$$w = \frac{W}{V_z} = \frac{1}{2} \epsilon_0 E^2$$