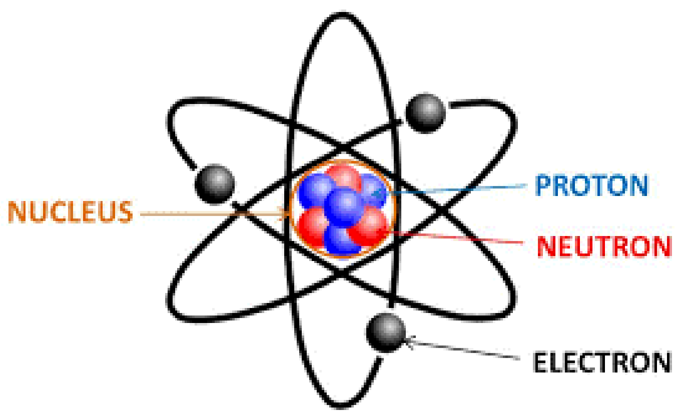
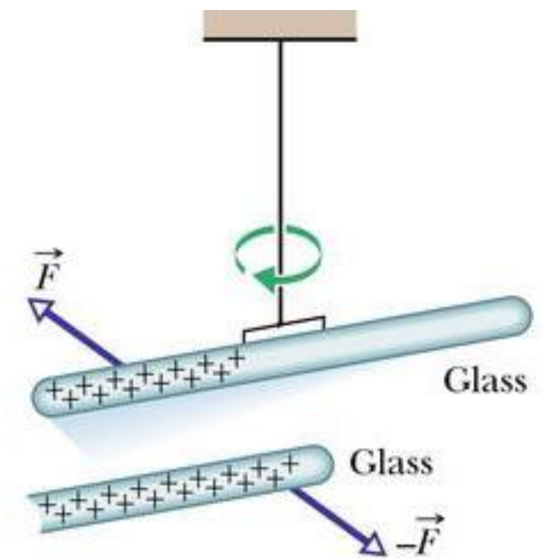
Elektro-statika



Čestica	Simbol	Naelektrisanje
Elektron	e ili e^-	$-e$
Proton	p	$+e$
Neutron	n	0

$$\left|e^+\right| = \left|e^-\right| = e = 1.6 \cdot 10^{-19}\,\mathrm{C}$$

Naelektrisanje materije



(*a*) (*b*)

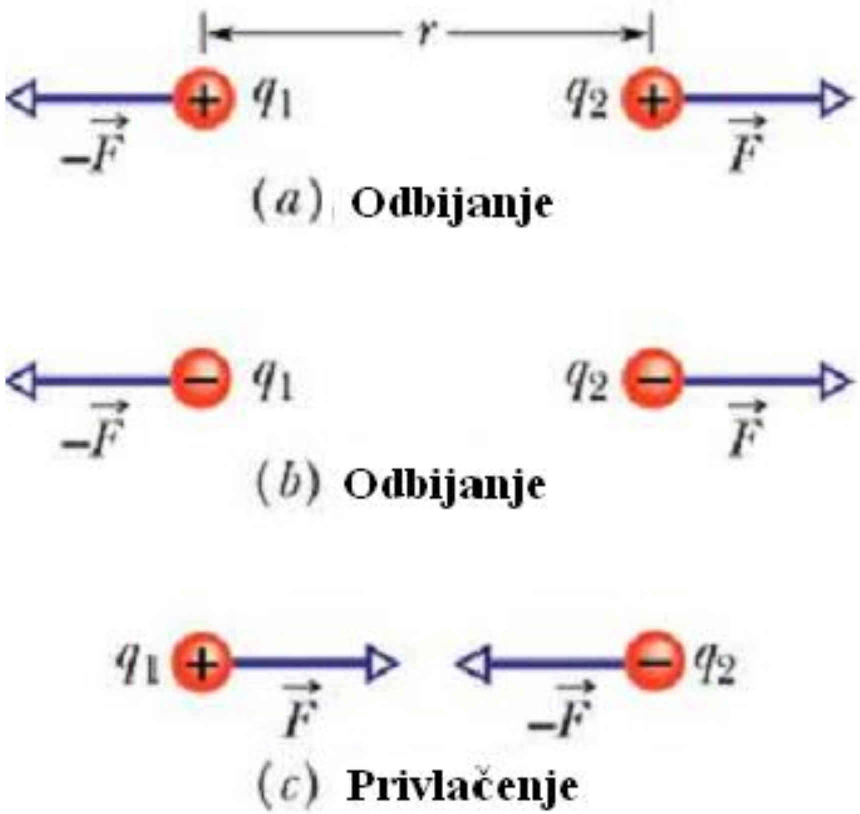
Zakon održanja količine naelektrisanja

$$q = n_1 \cdot e^+ - n_2 \cdot e^-, n_1, n_2 = \pm 1, \pm 2, \pm 3, \cdots,$$

$$\sum q = const.$$

Jedinica za naelektrisanje je Kulon (C).

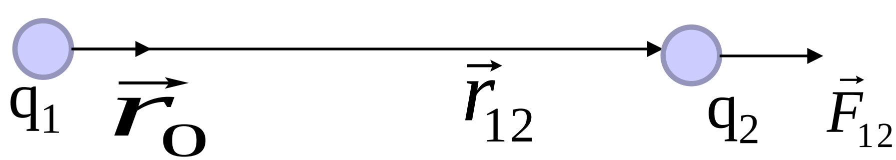
Kulonov zakon



$$F = k\frac{q_1 q_2}{r^2}$$

$$\vec{F}_{12} = k\frac{q_1 q_2}{r^3}\vec{r}_{12}$$

$$\vec{F}_{12} = k\frac{q_1 q_2}{r^2}\vec{r}_0$$



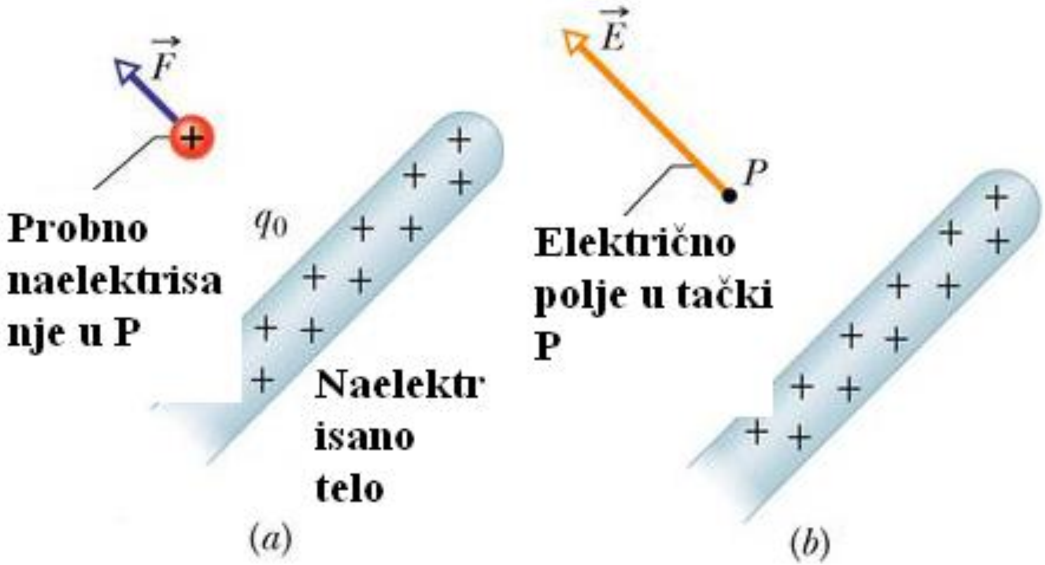
$$F = k\frac{q_1 q_2}{r^2} \Rightarrow k = \frac{Fr^2}{q_1 q_2}$$

$$k = \frac{1}{4\pi\varepsilon_0} = 8.99 \cdot 10^9\ \mathrm{N \cdot m^2 / C^2}$$

$$\varepsilon_0 = 8.85 \cdot 10^{-12}\ \mathrm{C^2 / N \cdot m^2}$$

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}$$

Električno polje



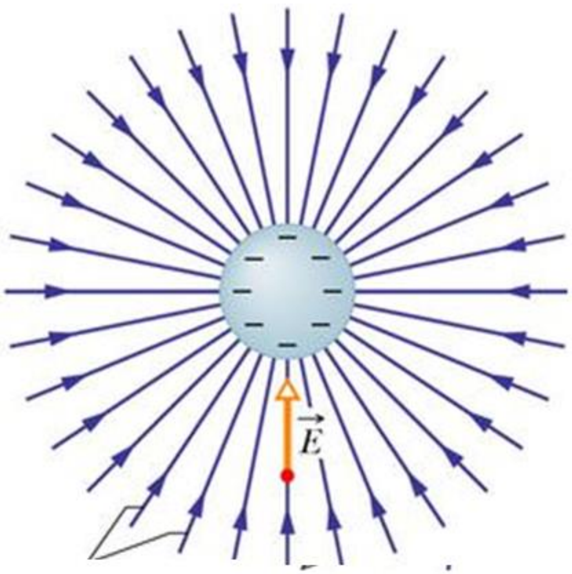
$$\vec{F} = q_0\vec{E}$$

$$\vec{E} = \frac{\vec{F}}{q_0}$$

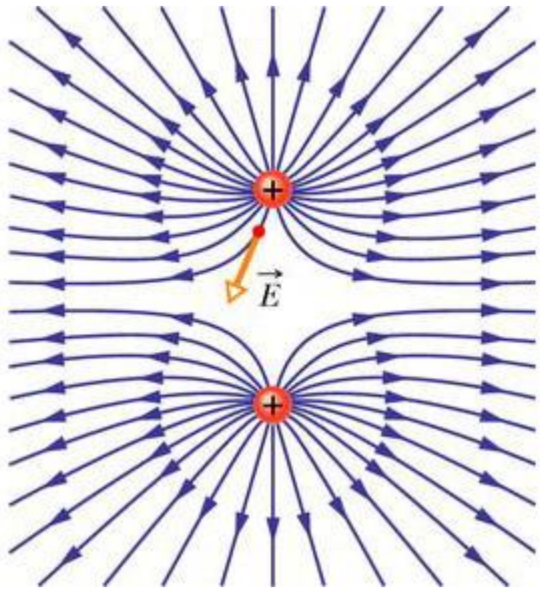
$$\left|\vec{E}\right| = \frac{\left|\vec{F}\right|}{q_0}$$

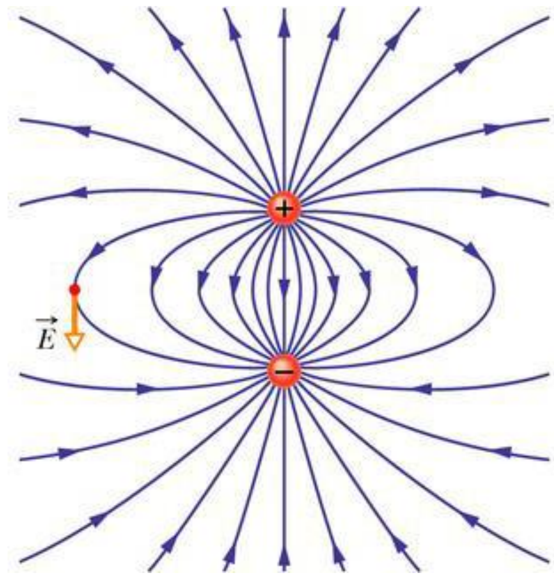
Jedinica za jačinu električnog polja je N/C.

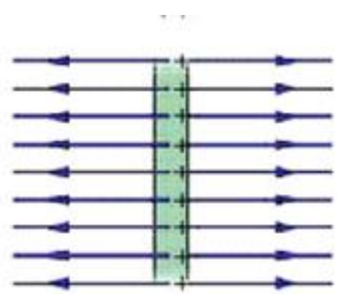
Linije sila električnog polja



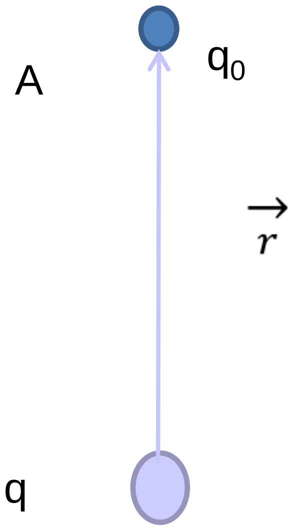
Linije sila električnog polja







Polje tačkastog naelektrisanja

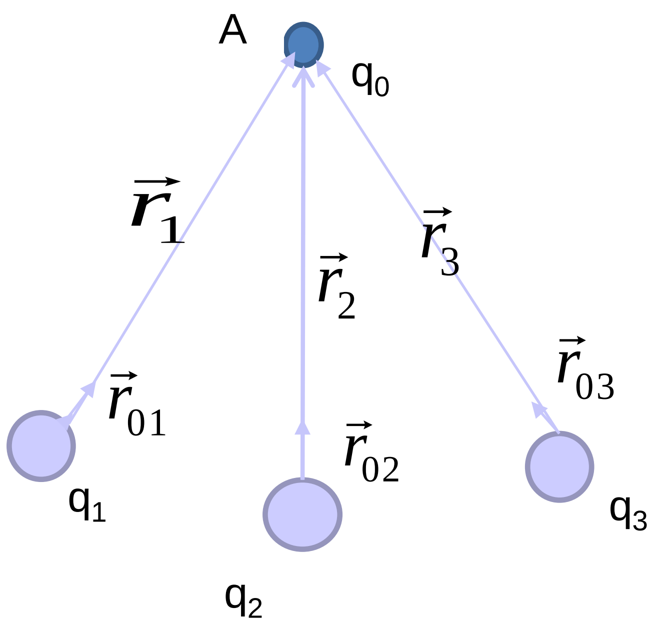


$$\vec{F} = \frac{1}{4\pi\varepsilon_0}\frac{qq_0}{r^2}\vec{r}_0$$

$$\vec{E} = \frac{\vec{F}}{q_0} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\vec{r}_0$$

$$E = \frac{F}{q_0} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$$

Polje sistema tačkastih naelektrisanja



$$\vec{F}_1 = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_0}{r_1^2}\vec{r}_{01}$$

$$\vec{F}_2 = \frac{1}{4\pi\varepsilon_0}\frac{q_2 q_0}{r_2^2}\vec{r}_{02}$$

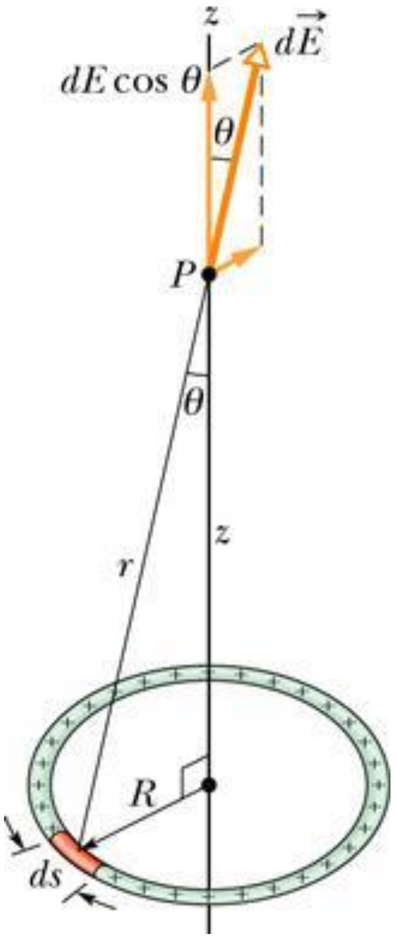
$$\vec{F}_3 = \frac{1}{4\pi\varepsilon_0}\frac{q_3 q_0}{r_3^2}\vec{r}_{03}$$

$$\vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 = \frac{q_0}{4\pi\varepsilon_0}\left(\frac{q_1}{r_1^2}\vec{r}_{01} + \frac{q_2}{r_2^2}\vec{r}_{02} + \frac{q_3}{r_3^2}\vec{r}_{03}\right)$$

$$\vec{E} = \frac{\vec{F}}{q_0} = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{r_1^2}\vec{r}_{01} + \frac{q_2}{r_2^2}\vec{r}_{02} + \frac{q_3}{r_3^2}\vec{r}_{03}\right) = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{3}\frac{q_i}{r_i^2}\vec{r}_{0i}$$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\frac{q_i}{r_i^2}\vec{r}_{0i} = \vec{E}_1 + \vec{E}_2 + \ldots + \vec{E}_n = \sum_{i=1}^{n}\vec{E}_i$$

Polje na osi ravnomerno naelektrisanog prstena



$$dq = \lambda ds.$$

λ je naelektrisanje po jedinici dužine

$$dE = \frac{1}{4\pi\epsilon_0}\frac{dq}{r^2} = \frac{1}{4\pi\epsilon_0}\frac{\lambda ds}{r^2}.$$

$$dE = \frac{1}{4\pi\epsilon_0}\frac{\lambda ds}{\left(z^2 + R^2\right)}.$$

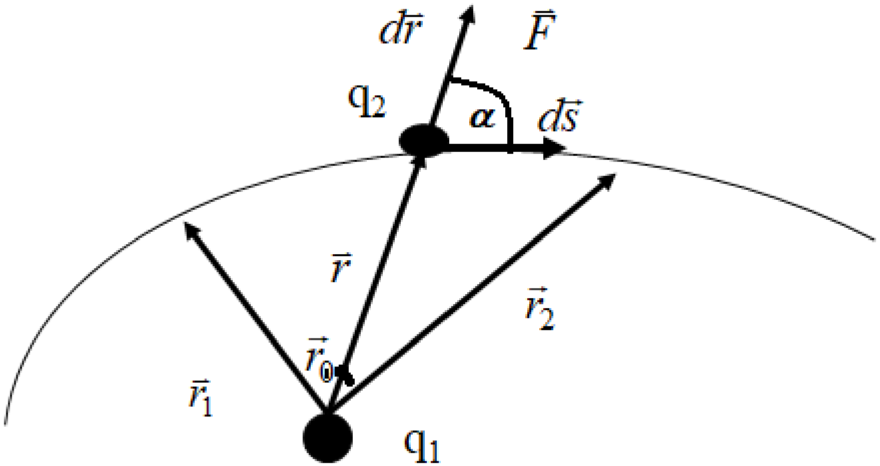
$$\cos\theta = \frac{z}{r} = \frac{z}{\left(z^2 + R^2\right)^{1/2}}.$$

$$dE\cos\theta = \frac{z\lambda}{4\pi\epsilon_0\left(z^2 + R^2\right)^{3/2}}ds.$$

$$E = \int dE\cos\theta = \frac{z\lambda}{4\pi\epsilon_0\left(z^2 + R^2\right)^{3/2}}\int_0^{2\pi R} ds = \frac{z\lambda(2\pi R)}{4\pi\epsilon_0\left(z^2 + R^2\right)^{3/2}}.$$

$$E = \frac{qz}{4\pi\epsilon_0\left(z^2 + R^2\right)^{3/2}}$$ (naelektrisani prsten).

Električna potencijalna energija



$$\vec{F} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}\vec{r}_0 = F(r)\vec{r}_0$$

$$A_{12} = \int_1^2 F(r)\cdot\vec{r}_0\cdot d\vec{s} = \int_1^2 F(r)\cdot dr =$$

$$= \frac{1}{4\pi\varepsilon_0} q_1 q_2 \int_1^2 \frac{dr}{r^2} = \frac{1}{4\pi\varepsilon_0} q_1 q_2 \left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

$$E_p = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r} + const.$$

Kad je $r = \infty$, E_p=0.

$$A_{12} = E_{p1} - E_{p2}$$

$$E_p = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r}$$

Električni potencijal V

Električna potencijalna energija po jedinici naelektrisanja u datoj tački polja ne zavisi od probnog naelektrisanja i predstavlja karakteristiku polja i zove se električni potencijal.

$$V = \frac{E_p}{q_2}$$

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$$

$$E_p = qV$$

$$V = \frac{A_\infty}{q} = \frac{1}{q}\int_C^\infty \vec{F}d\vec{r} = -\frac{1}{q}\int_\infty^C \vec{F}d\vec{r}$$

$$A_{12} = q(V_1 - V_2)$$

$$V = -\int_\infty^C \vec{E}d\vec{r}$$

Potencijal date tačke polja brojno je jednak radu koji izvrše sile polja na premeštanju jediničnog probnog naelektrisanja iz date tačke u beskonačnost.

Jedinica za električni potencijal je Volt (V=J/C).

Električni napon

$$V_A = -\frac{1}{q}\int_{\infty}^{A} \vec{F} d\vec{r} = -\int_{\infty}^{A} \vec{E} d\vec{r} = -\int_{\infty}^{A} E dr \cos\Theta$$

$$V_B - V_A = -\int_{\infty}^{B} \vec{E} d\vec{r} + \int_{\infty}^{A} \vec{E} d\vec{r} = -\int_{A}^{B} \vec{E} d\vec{r} = -\int_{A}^{B} E dr \cos\Theta$$

Razlika potencijala izmedju dve tačke polja ili napon je brojno jednak radu koji treba izvršiti da se jedinično naelektrisanje prenese iz tačke A u tačku B.

Električni potencijal i električno polje

$$dV = -\vec{E}d\vec{r} = -E\cos\Theta dr, \text{ kako je } \mathrm{E}\cos\Theta = \mathrm{E}_{\mathrm{x}}$$

$$E_x = -\frac{\partial V}{\partial x}, \quad E_y = -\frac{\partial V}{\partial y}, \quad E_z = -\frac{\partial V}{\partial z}$$

$$\vec{E} = E_x\vec{i} + E_y\vec{j} + E_z\vec{k} = -\left(\frac{\partial V}{\partial x}\vec{i} + \frac{\partial V}{\partial y}\vec{j} + \frac{\partial V}{\partial z}\vec{k}\right)$$

$$\nabla = \vec{i}\frac{\partial}{\partial x} + \vec{j}\frac{\partial}{\partial y} + \vec{k}\frac{\partial}{\partial z}, \text{ diferencij alni operator nabla,}$$

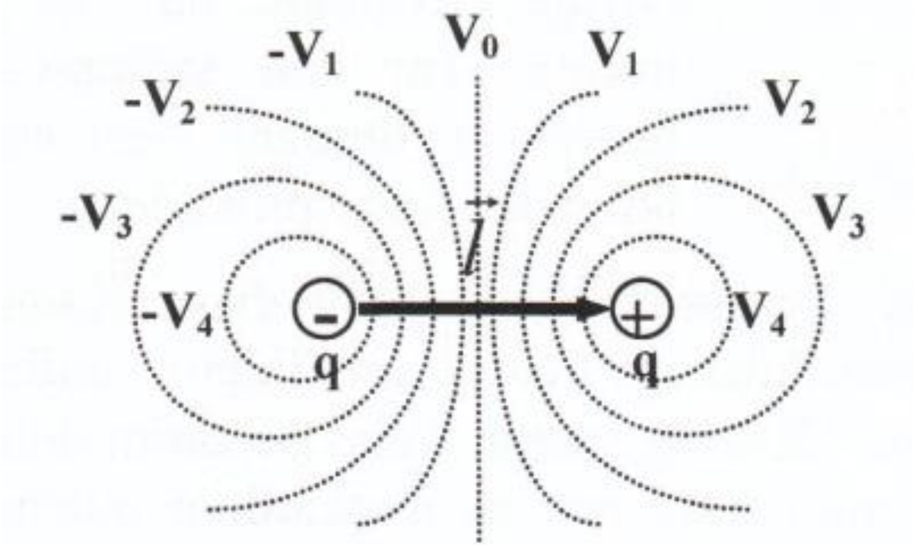
$$\vec{E} = -\nabla V(\vec{r}) = -\mathrm{grad}\, V(\vec{r})$$

Ekvipotencijalne površi

$$\vec{E} = -gradV(\vec{r}) \Rightarrow E_x = -\frac{\partial V}{\partial x}, E_y = -\frac{\partial V}{\partial y}, E_z = -\frac{\partial V}{\partial z}$$

$$E = -\frac{\Delta V}{\Delta r}$$

$$\Delta r = -\frac{1}{E}\Delta V$$



Polazeći od činjenice da je električno polje dato gradijentom potencijala uvodimo nov način prikazivanja polja pomoću *ekvipotencijalnih površina*.

Ekvipotencijalna površina u prostoru je skup svih tačaka u kojima je potencijal jednake vrednosti.

Kapacitet izolovanog provodnika

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r} \Rightarrow q = 4\pi\varepsilon_0 r V$$

$$q = CV$$

$$C = \frac{q}{V} = 4\pi\varepsilon_0 r$$

C je kapacitet, konstanta koja ne zavisi od q ili V.

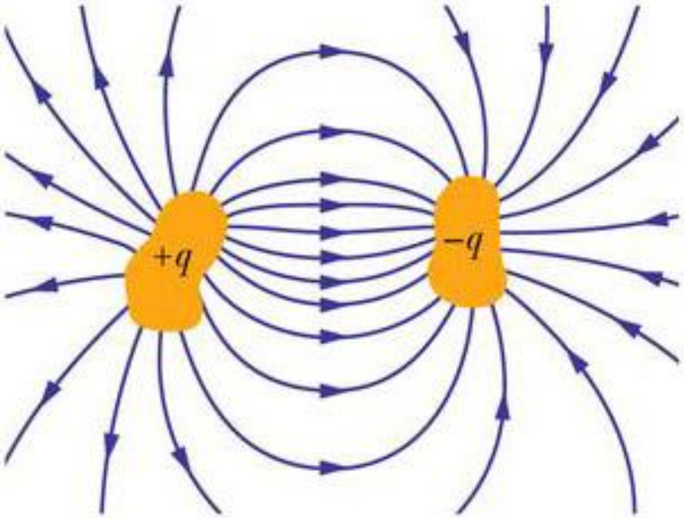
1 farad = 1 F = 1 coulomb per volt = 1 C / V.

1 μF = 10^{-6} F

1 pF = 10^{-12} F

Kondenzatori

Električni kondenzator je sistem dva provodnika postavljenih jedan pored drugog i naelektrisana jednakim količinama naelektrisanja suprotnog znaka.



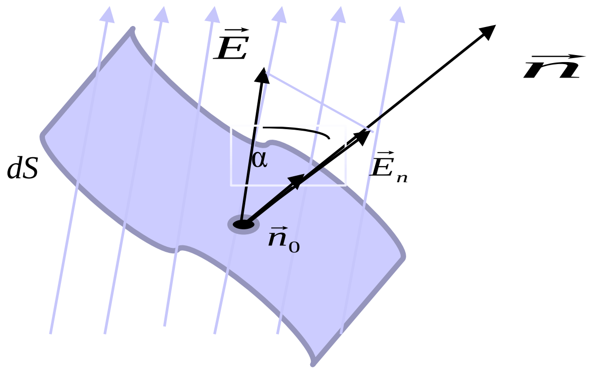
$$C = \frac{q}{V}$$

Kapacitet od jednog farada ima kondenzator kome se razlika potencijala promeni za 1V, kada se naelektrisanje promeni za 1C.

Električni fluks.Gausova teorema

$$d\Phi = \vec{E}d\vec{S} = EdS\cos\alpha = E_n dS, \text{gde je } d\vec{S} = dS\vec{n}_0$$

$$\Phi = \oint_S \vec{E}d\vec{S}$$

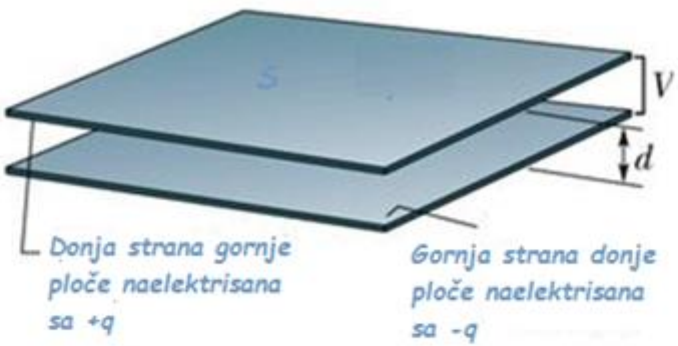


$$\Phi = \oint_S \vec{E}d\vec{S} = \frac{1}{\varepsilon_0}\sum_{i=1}^{n} q_i$$

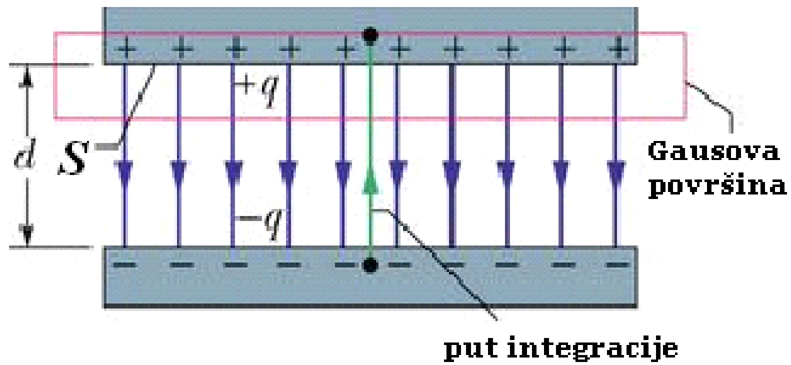
$$\varepsilon_0 \oint_S \vec{E}d\vec{S} = \sum_{i=1}^{n} q_i$$

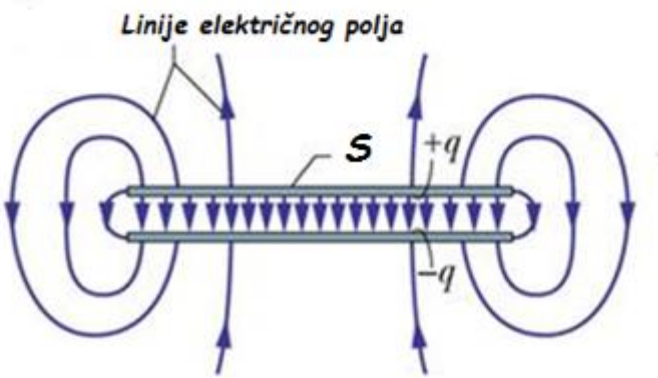
Gausova teorema glasi: Fluks vektora električnog polja kroz proizvoljnu zatvorenu površinu S jednak je algebarskom zbiru naelektrisanja unutar te površine, podeljenim sa ε_0.

Pločasti kondenzator



(a)





$$ES = \frac{q}{\varepsilon_0} \Rightarrow E = \frac{q}{\varepsilon_0 S}$$

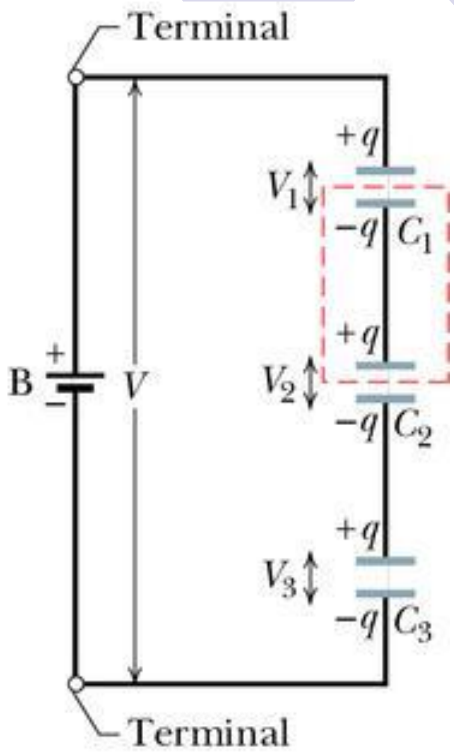
$$V = \int_{+}^{-} E dl = E \int_{0}^{d} dl = Ed = \frac{q}{\varepsilon_0 S} d$$

$$C = \frac{q}{V} = \frac{q}{\frac{q}{\varepsilon_0 S} d} = \frac{\varepsilon_0 S}{d}$$

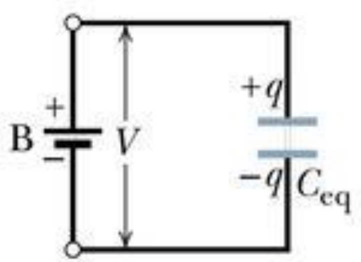
Vezivanje kondenzatora

- Paralelna veza
- Redna veza
- Kada imamo više kondenzatora u strujnom kolu možemo ih zameniti jednim ekvivalentnim kondenzatorom, koji ima ukupni kapacitet svih kondenzatora.

Redna veza kondenzatora



(a)



(b)

$$V_1 = \frac{q}{C_1}, V_2 = \frac{q}{C_2}, V_3 = \frac{q}{C_3}$$

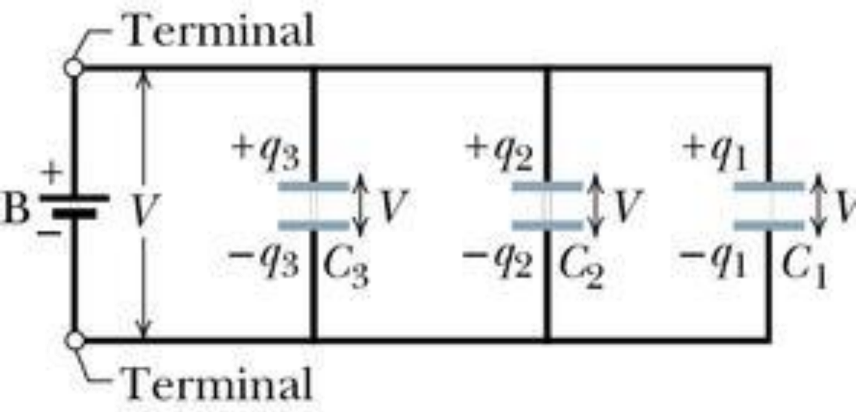
$$V = V_1 + V_2 + V_3 = q\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right).$$

$$C_{eq} = \frac{q}{V} = \frac{1}{1/C_1 + 1/C_2 + 1/C_3},$$

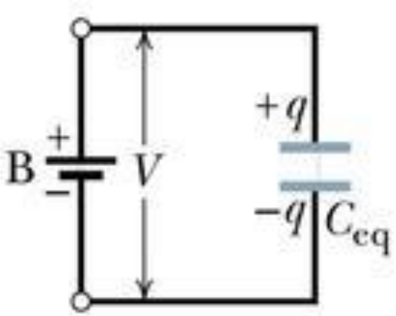
$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}.$$

$$\frac{1}{C_{eq}} = \sum_{j=1}^{n} \frac{1}{C_j}$$

Paralelna veza kondenzatora



(a)



(b)

$$q_1 = C_1 V, \quad q_2 = C_2 V, \quad q_3 = C_3 V$$

$$q = q_1 + q_2 + q_3 = (C_1 + C_2 + C_3)V.$$

$$C_{eq} = \frac{q}{V} = C_1 + C_2 + C_3,$$

$$C_{eq} = \sum_{j=1}^{n} C_j$$

Energija napunjenog kondezatora.Energija električnog polja

Proces punjenja kondenzatora je prenos naelektrisanja sa obloge nižeg na oblogu višeg potencijala. Pri tome se vrši rad nasuprot dejstvu elektrostatičkih sila.

$$dA = Vdq = \frac{q}{C}dq$$

$$A = \int_0^Q Vdq = \frac{1}{C}\int_0^Q qdq = \frac{Q^2}{2C}$$

$$E_p = \frac{Q^2}{2C} = \frac{QV}{2} = \frac{CV^2}{2}$$

$$W = \frac{CV^2}{2} = \frac{1}{2}\varepsilon_0 \frac{S}{d}V^2 = \frac{1}{2}\varepsilon_0 \frac{V^2}{d^2}Sd$$

$$W = \frac{1}{2}\varepsilon_0 E^2 Sd = \frac{1}{2}\varepsilon_0 E^2 V_{zapremina}$$

$$w = \frac{W}{V_z} = \frac{1}{2}\varepsilon_0 E^2$$