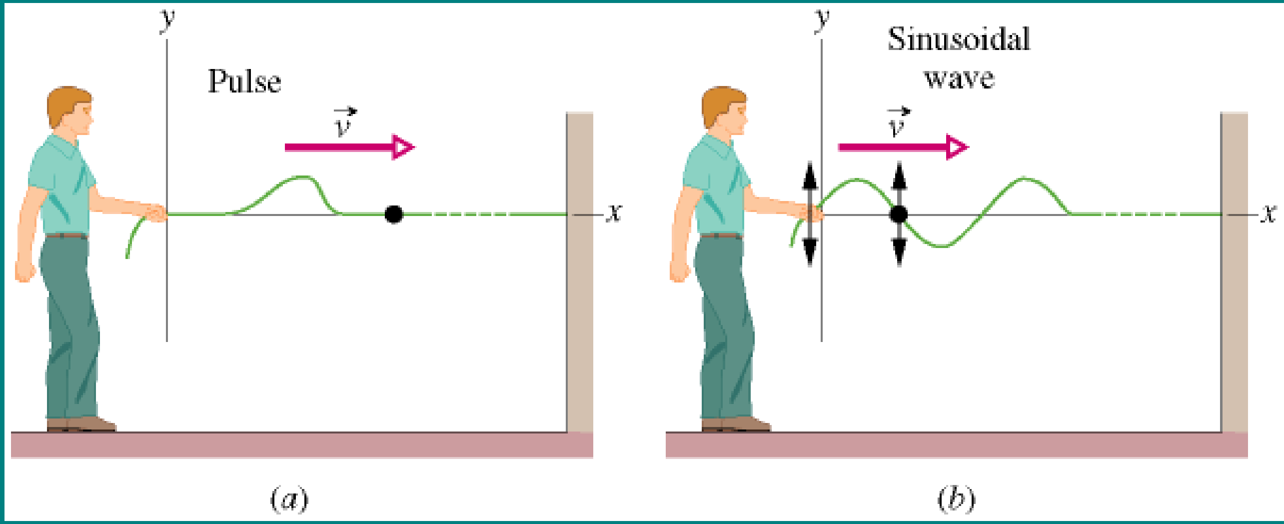
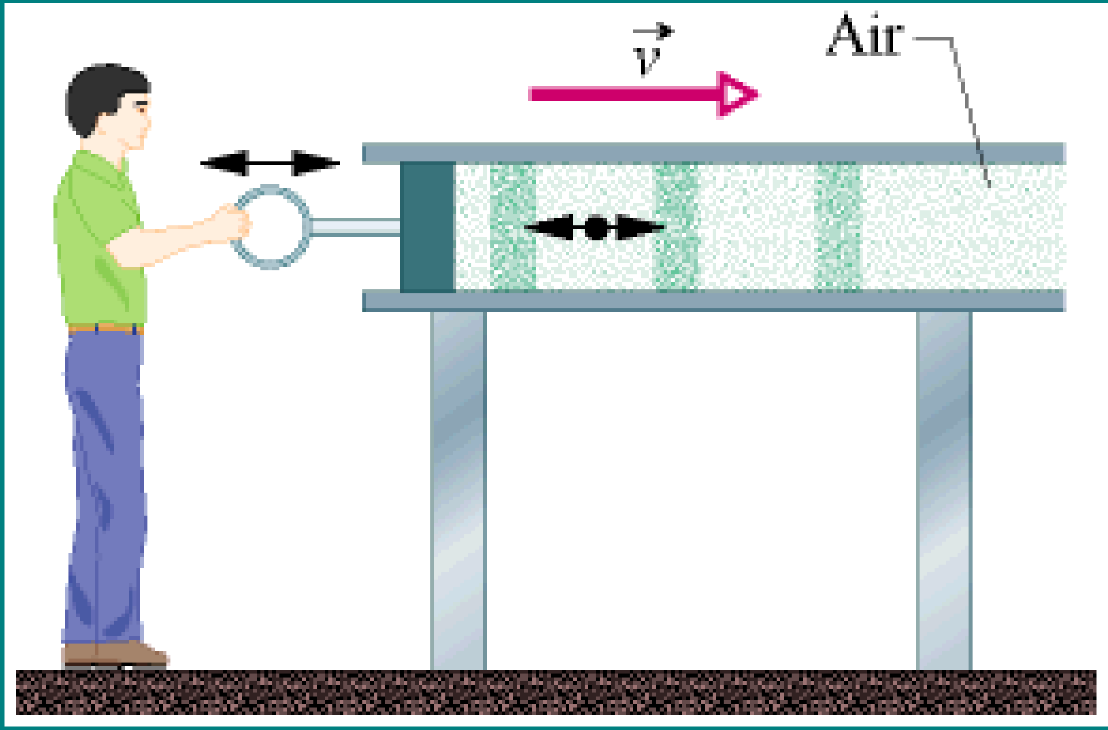
Mehanički talasi

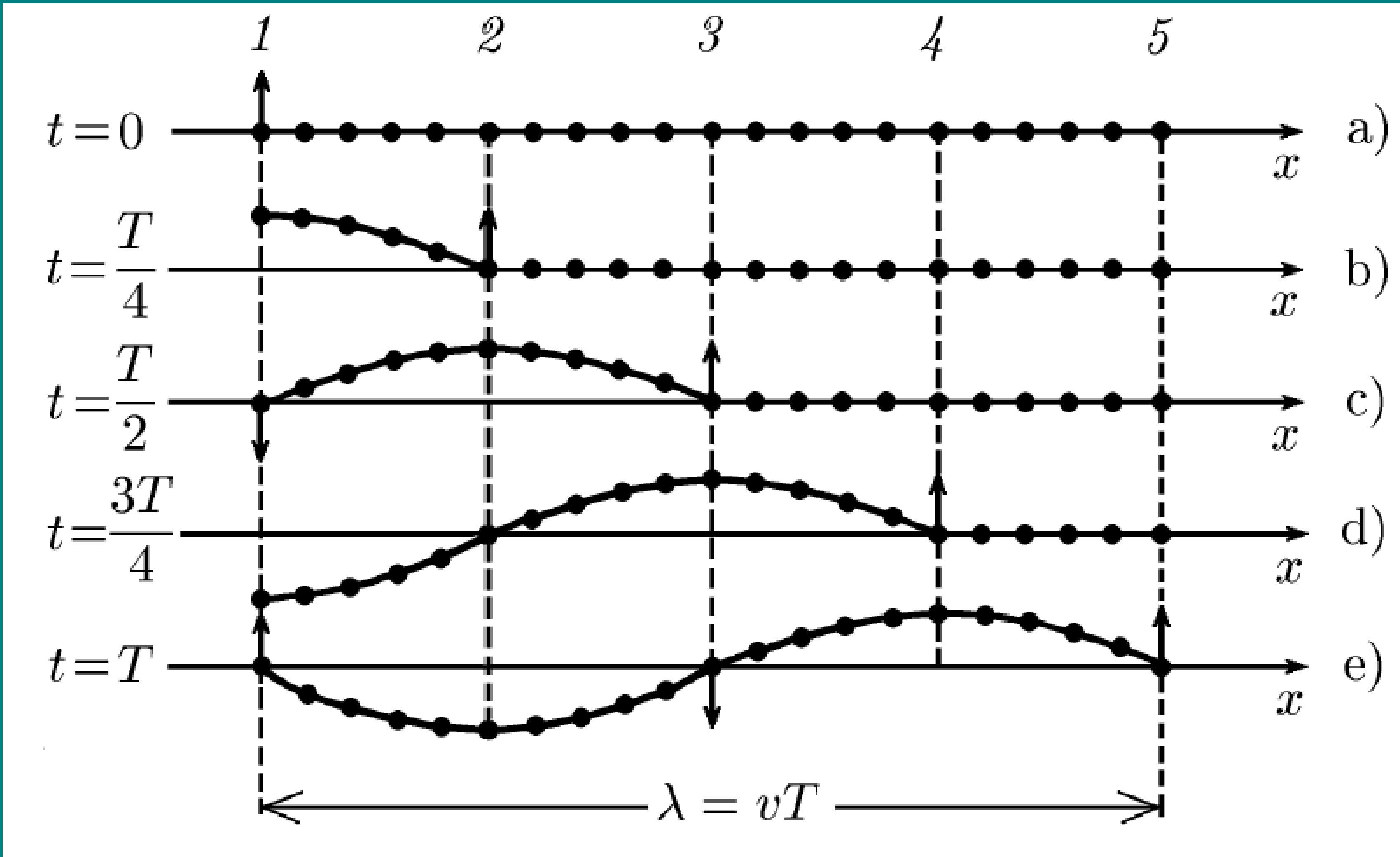
- Proces prenošenja, širenja oscilovanja kroz čvrstu, tečnu ili gasovitu elastičnu sredinu naziva se elastični talas.
- Pri prenošenju oscilovanja čestica na česticu deluje periodičnom prinudnom silom sa frekvencijom jednakom frekvenciji talasa.
- Sve čestice sredine osciluju oko svojih ravnotežnih položaja istom frekvencijom, koja je jednaka frekvenciji izvora talasa.

Prostiranje talasa kroz elastičnu sredinu

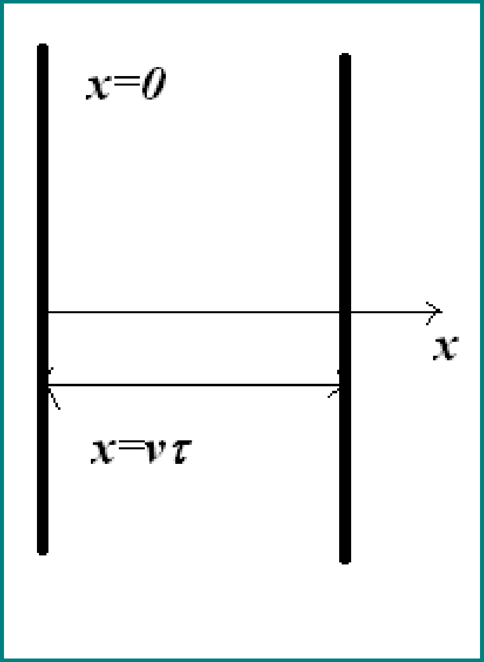
- Elastični talasi: longitudinalni i transverzalni.
- Transverzalni osciluju u pravcu koji je normalan na pravac širenja talasa.
- Longitudinalni osciluju u pravcu u kojem se širi talas.






1
2
3
4
5
$t=0$
a)
$t=\frac{T}{4}$
b)
$t=\frac{T}{2}$
c)
$t=\frac{3T}{4}$
d)
$t=T$
e)
x
$\lambda = vT$

Jednačina talasa



$$y(0,t) = y_0 \sin(\omega t + \alpha)$$

$$y(x,t) = y_0 \sin[\omega(t-\tau)+\alpha]$$

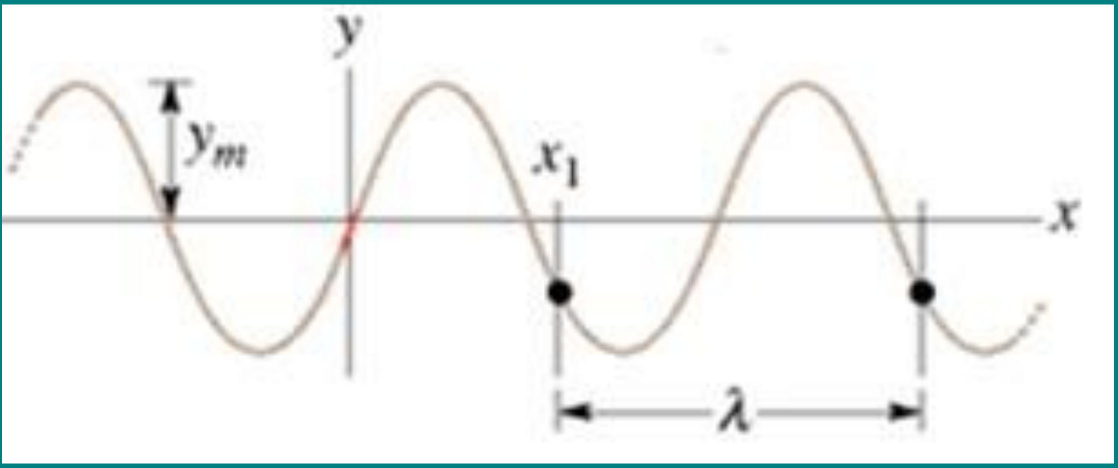
kako je $\tau = \frac{x}{v}$ i $\alpha = 0$

$$y = y_0 \sin\left[\omega\left(t - \frac{x}{v}\right)\right] = y_0 \sin\left(\omega t - \frac{\omega}{v}x\right)$$

$$y = y_0 \sin(\omega t - kx) = -y_0 \sin(kx - \omega t)$$

- y_0- amplituda, maksimalno udaljenje čestice od ravnotežnog položaja
- *sin(kx-wt) – oscilujući član*
- *kx-wt - faza talasa*
- *k - ugaoni talasni broj* $k = 2\pi/\lambda$
- *w – ugaona frekvencija* $\omega = 2\pi\nu$
- *x – položaj čestice*
- *t - vreme*
- y(x,t) - elongacija, udaljenje čestice od ravnotežnog položaja

$$\mathbf{y(x,t) = y_0 \sin(kx - \omega t)}$$



U trenutku t=0.

$$y(x,0) = y_m \sin kx$$

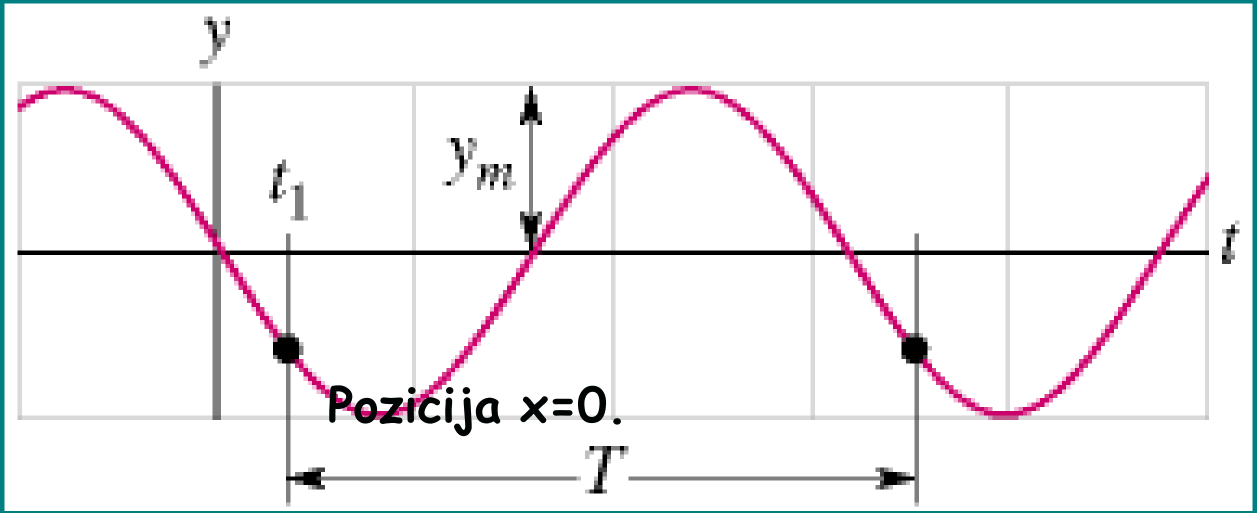
$$y_m \sin kx_1 = y_m \sin k(x_1 + \lambda)$$
$$= y_m \sin(kx_1 + k\lambda).$$

$$k\lambda = 2\pi$$

$$k = \frac{2\pi}{\lambda}$$

k je talasni broj

Talasna dužina λ je rastojanje izmedju dva najbliža delića koji osciluju u istoj fazi.



Period je vreme za koje delići sredine izvrše jednu punu oscilaciju.

$$y(0,t) = y_m \sin(-\omega t)$$
$$= -y_m \sin \omega t \qquad (x = 0).$$

$$-y_m \sin \omega t_1 = -y_m \sin \omega(t_1 + T)$$
$$= -y_m \sin(\omega t_1 + \omega T).$$

$$\omega = \frac{2\pi}{T}$$

ω je ugaona frekvencija

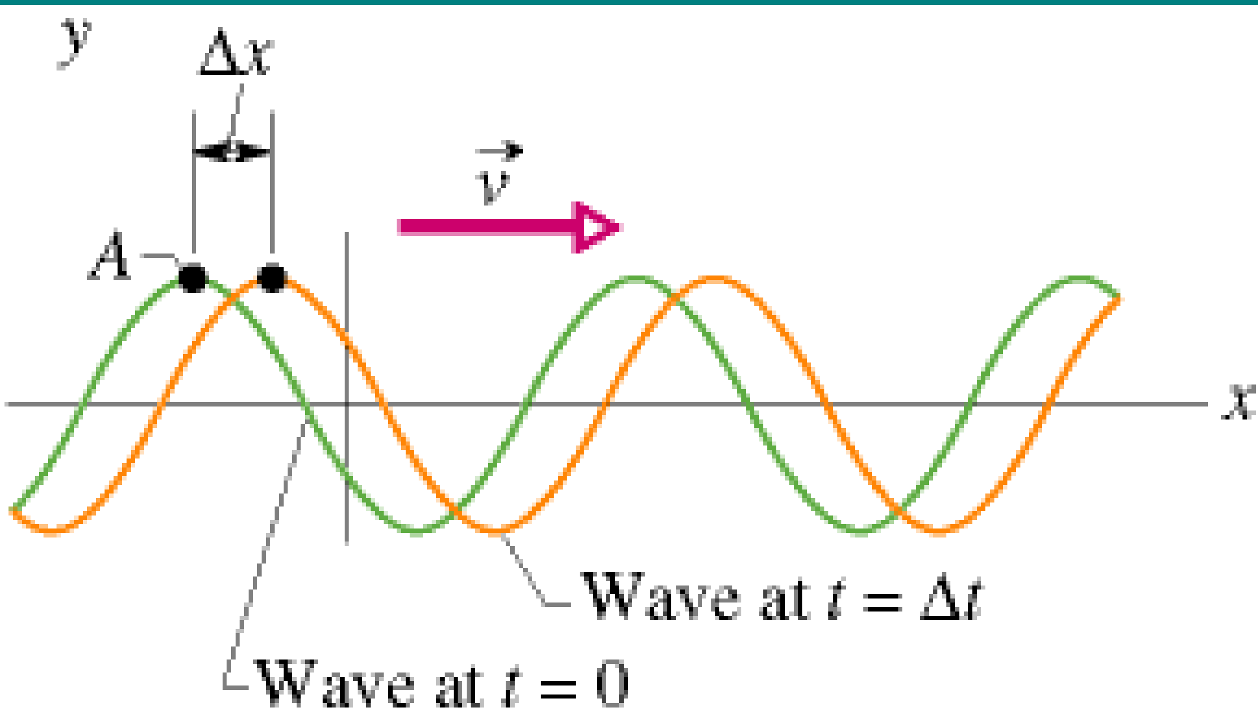
$$\omega T = 2\pi,$$

$$f = \frac{1}{T} = \frac{\omega}{2\pi}$$

f je frekvencija

Brzina talasa

kx - wt = const.

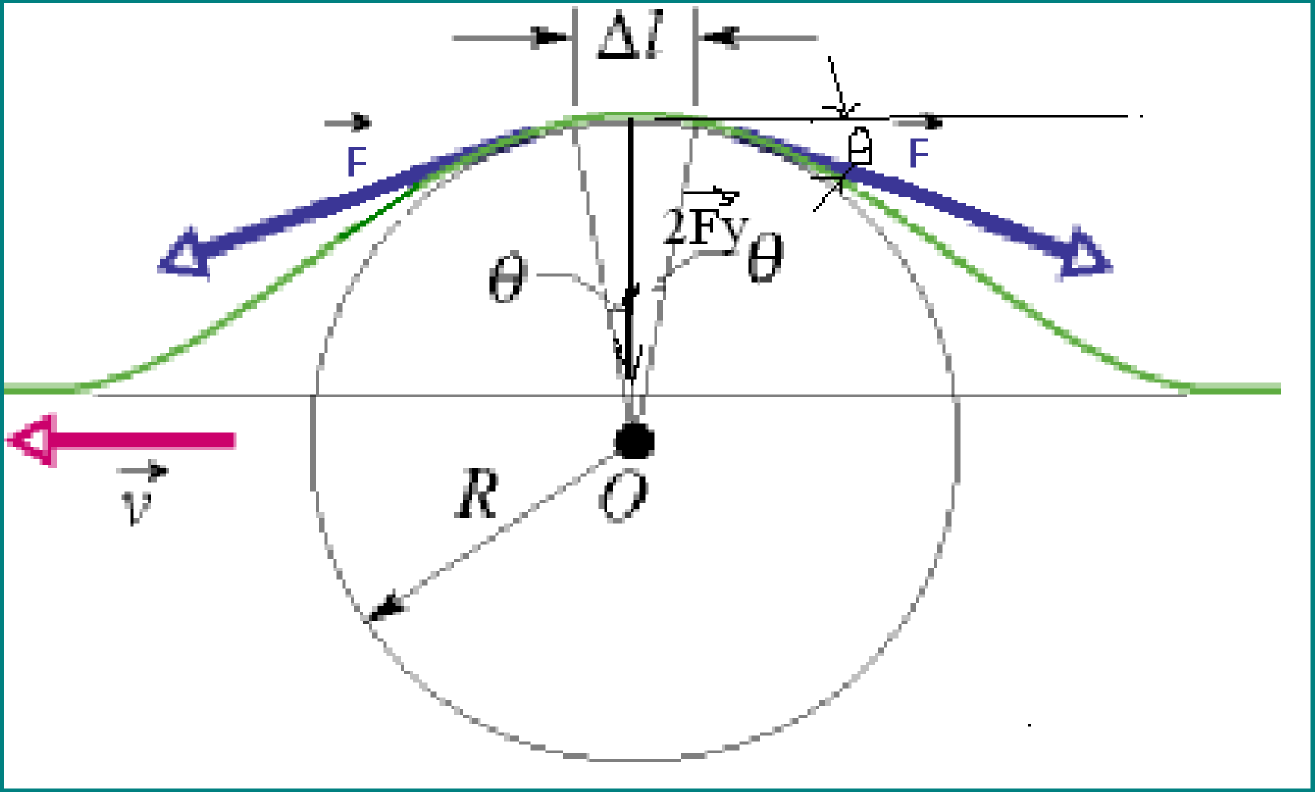


$$k\frac{dx}{dt} - \omega = 0$$

$$\frac{dx}{dt} = v = \frac{\omega}{k}.$$

$$v = \frac{\omega}{k} = \frac{\lambda}{T} = \lambda f$$

Brzina transverzalnih talasa na zategnutoj žici



$$F_y = 2F\sin\Theta \approx 2F\Theta = F\frac{\Delta l}{R}$$

jer je $2\theta = \Delta l / R$

$$F_y = \Delta m \cdot a$$

$$\Delta m = \mu \Delta l$$

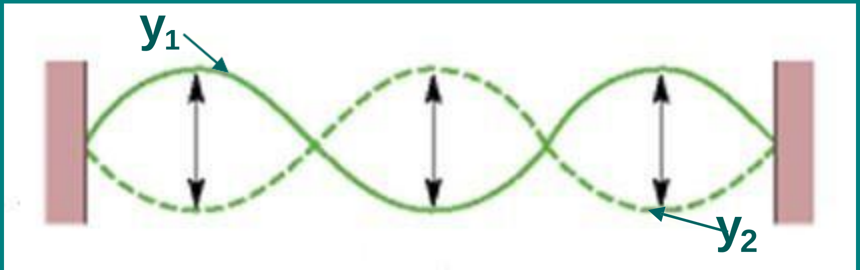
$$a = \frac{v^2}{R}$$

$$F\frac{\Delta l}{R} = (\mu \Delta l)\frac{v^2}{R} \Rightarrow \quad v = \sqrt{\frac{F}{\mu}}$$

Brzina prostiranja tranverzalnih talasa ne zavisi od amplitude ili talasne dužine talasa, već samo od sile zatezanja žice i njene mase po jedinici dužine.

Stojeći talasi

Nastaju interferencijom dva progresivna talasa istih amplituda i talasnih dužina koji se kreću istim pravcem, a suprotnim smerom.
Ti uslovi se postižu refleksijom progresivnog talasa.



$$y_1(x,t) = y_m \sin(kx - \omega t)$$

$$y_2(x,t) = y_m \sin(kx + \omega t).$$

$$y'(x,t) = y_1(x,t) + y_2(x,t) = y_m \sin(kx - \omega t) + y_m \sin(kx + \omega t).$$

$$\sin\alpha + \sin\beta = 2\sin\tfrac{1}{2}(\alpha+\beta)\cos\tfrac{1}{2}(\alpha-\beta).$$

$$y'(x,t) = \left[2y_m \sin kx\right]\cos\omega t$$

Čvorovi i trbusi stojećeg talasa

$$y'(x,t) = [2y_m \sin kx] \cos \omega t,$$

$$kx = n\pi, \qquad \text{for } n = 0, 1, 2, \ldots$$

$$k = \frac{2\pi}{\lambda}$$

$$x = n\frac{\lambda}{2} \qquad \text{for } n = 0, 1, 2, \ldots$$

čvorovi

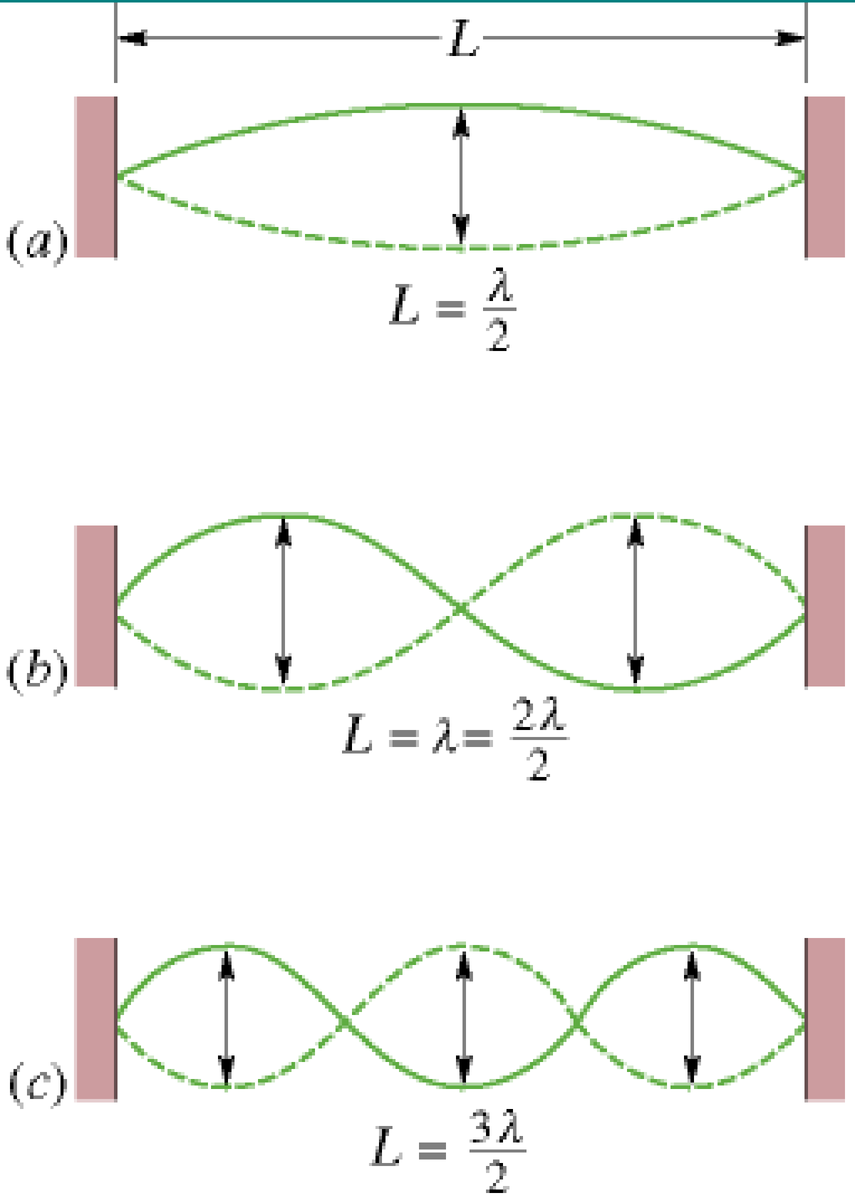
$$kx = \tfrac{1}{2}\pi, \tfrac{3}{2}\pi, \tfrac{5}{2}\pi \ldots$$

$$= (n + \tfrac{1}{2})\pi, \qquad \text{for } n = 0, 1, 2, \ldots$$

$$x = \left(n + \frac{1}{2}\right)\frac{\lambda}{2}, \qquad \text{for } n = 0, 1, 2, \ldots$$

trbusi

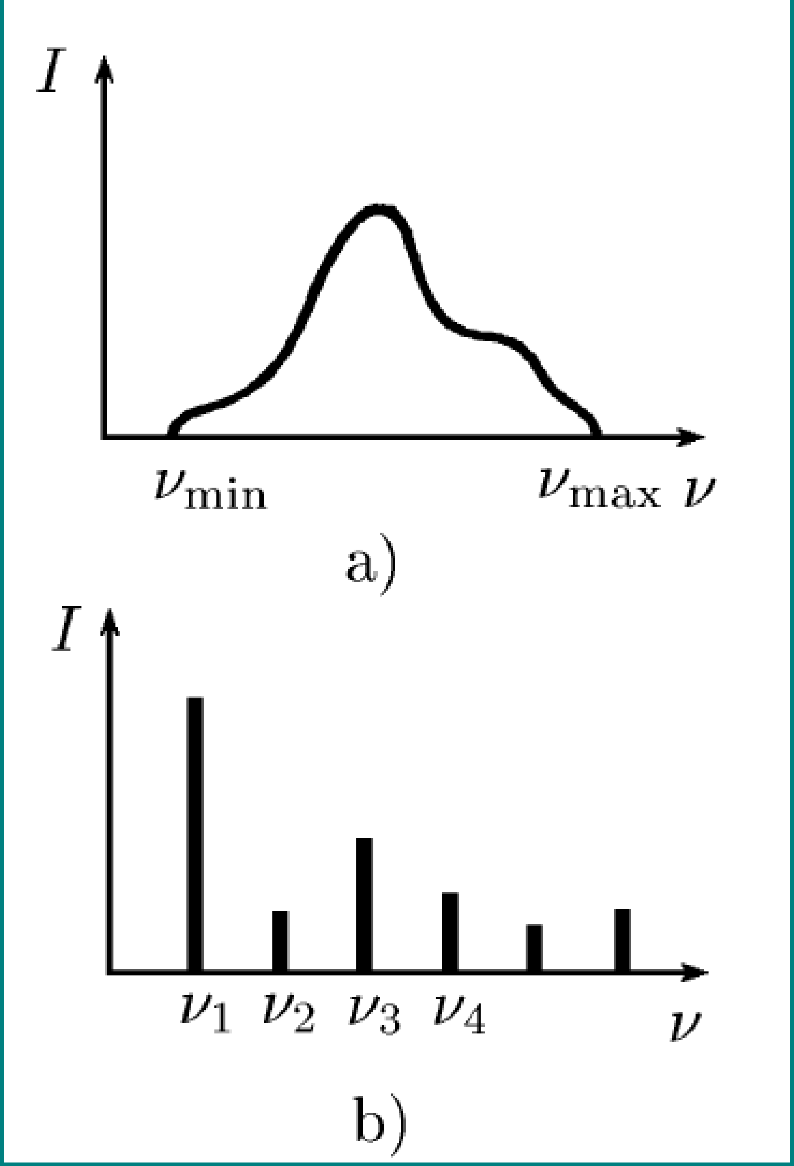
Oscilovanje žice učvršćene na krajevima



$$\lambda = \frac{2L}{n}, \qquad \text{for } n = 1, 2, 3, \ldots$$

$$f = \frac{v}{\lambda} = n\frac{v}{2L}, \qquad \text{for } n = 1, 2, 3, \ldots$$

Zvuk



Zvuk je svaki mehanički talas sa frekvencijom od 20 Hz do 20 kHz.
Ultrazvuk ima frekvenciju veću od 20 kHz, a infrazvuk manju od 20 Hz.

Ton ili tonalni zvuk ima:
visinu, boju i jačinu.
--visina (osnovna frekvencija)
--boja (viši harmonici)

Brzina zvuka

$$v = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{\text{elasticna svojstva}}{\text{inertna svojstva}}}$$

$$B = -\frac{\Delta p}{\Delta V / V}$$

Definicija modula stišljivosti

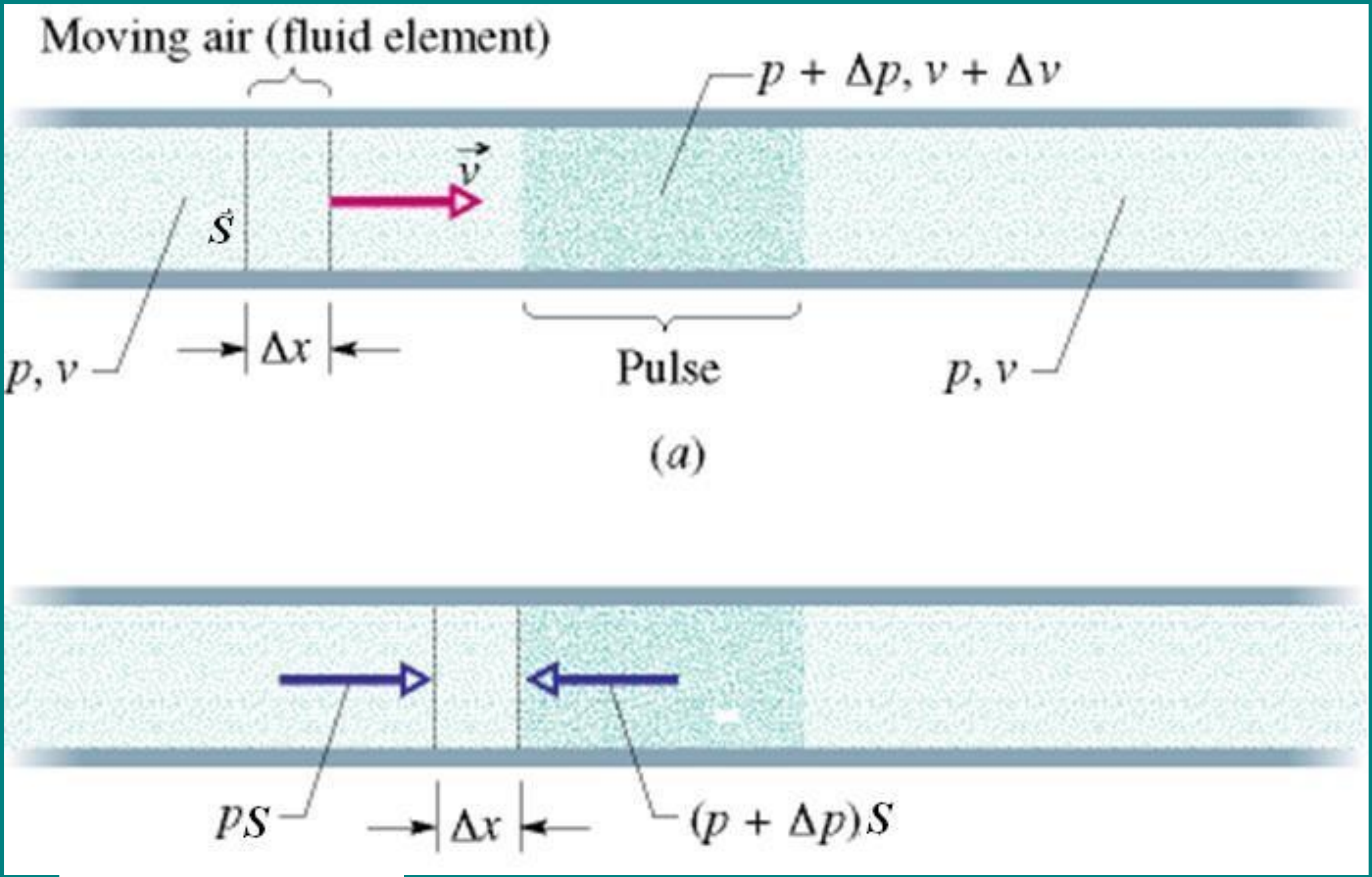
$$v = \sqrt{\frac{B}{\rho}}$$

Brzina zvuka za fluide

$$v = \sqrt{\frac{E}{\rho}}$$

E je Jungov moduo elastičnosti

Brzina zvuka za čvrsta tela

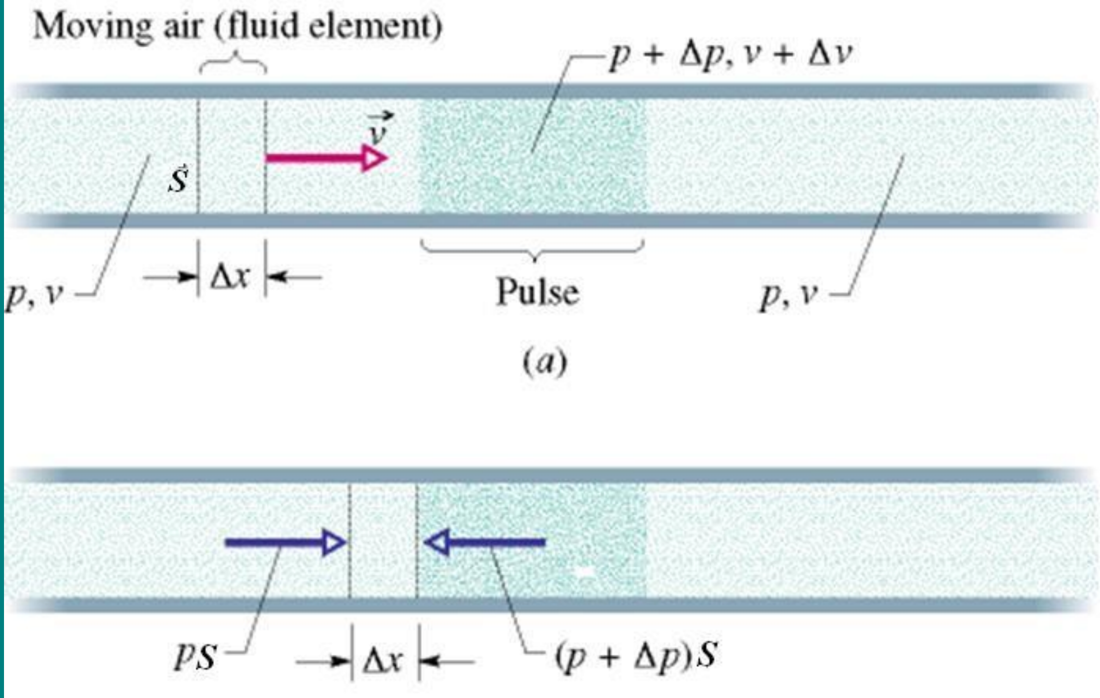


$$\Delta t = \frac{\Delta x}{v}.$$

$$F = pS - (p + \Delta p)S$$
$$= -\Delta p S$$

$$\Delta m = \rho S \Delta x = \rho S v \Delta t$$

$$a = \frac{\Delta v}{\Delta t} \qquad \text{(acceleration).}$$



$$-\Delta p S = (\rho S v \Delta t)\frac{\Delta v}{\Delta t}$$

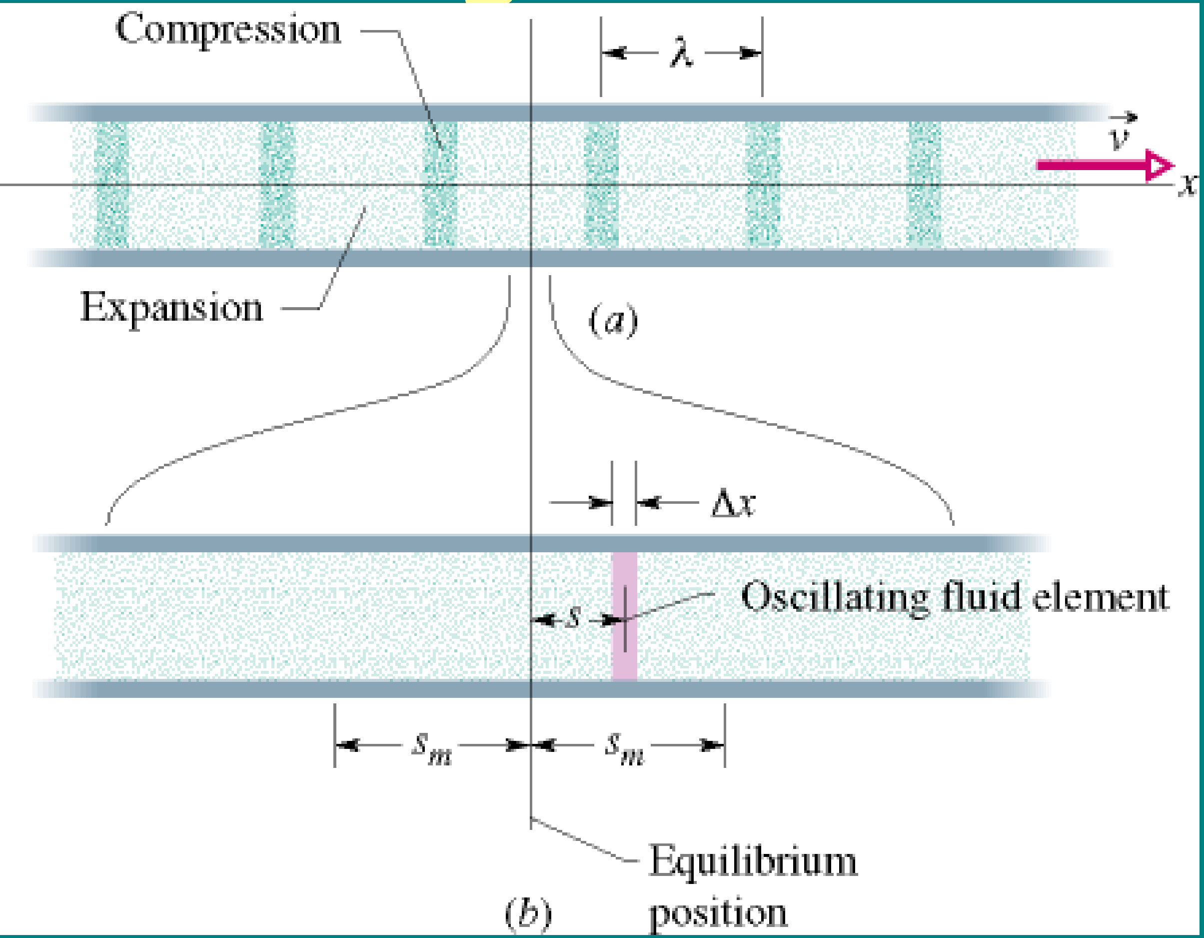
$$\rho v^2 = -\frac{\Delta p}{\Delta v / v}.$$

$$\rho v^2 = -\frac{\Delta p}{\Delta v / v} = -\frac{\Delta p}{\Delta V / V} = B.$$

$$\frac{\Delta V}{V} = \frac{S \Delta v \Delta t}{S v \Delta t} = \frac{\Delta v}{v}$$

$$v = \sqrt{\frac{B}{\rho}}$$

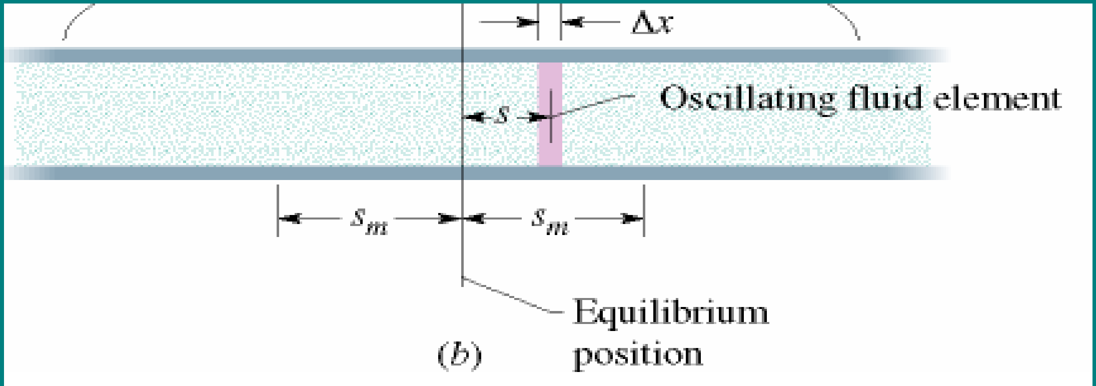
Progresivni zvučni talasi



$$\Delta p_m = (v\rho\omega)s_m.$$

$$s(x,t) = s_m \cos(kx - \omega t).$$

$$\Delta p(x,t) = \Delta p_m \sin(kx - \omega t).$$



$$V = S\,\Delta x$$

$$\Delta V = S\,\Delta s$$

$$s(x,t) = s_m \cos(kx - \omega t).$$

$$\Delta p = -B\frac{\Delta V}{V}.$$

$$\Delta p = -B\frac{\Delta s}{\Delta x} = -B\frac{\partial s}{\partial x}.$$

$$\frac{\partial s}{\partial x} = \frac{\partial}{\partial x}\left[s_m \cos(kx - \omega t)\right] = -ks_m \sin(kx - \omega t).$$

$$\Delta p = Bks_m \sin(kx - \omega t).$$

$$\Delta p_m = (Bk)s_m = (v^2 \rho k)s_m.$$

$$v = \frac{\omega}{k}.$$

$$\Delta p_m = (v\rho\omega)s_m.$$

Energija talasa

$$s(x,t) = s_m \cos(kx - \omega t)$$

$$\Delta K = \frac{1}{2}\rho\left(\frac{\partial s}{\partial t}\right)^2 \Delta V$$

$$\Delta U = \frac{1}{2}E\left(\frac{\partial s}{\partial x}\right)^2 \Delta V \text{, kako je}$$

$$\mathrm{E} = \rho v^2 \text{ to je}$$

$$\Delta U = \frac{1}{2}\rho v^2\left(\frac{\partial s}{\partial x}\right)^2 \Delta V$$

$$\Delta E = \Delta K + \Delta U = \frac{1}{2}\rho\left(\frac{\partial s}{\partial t}\right)^2 \Delta V + \frac{1}{2}\rho v^2\left(\frac{\partial s}{\partial x}\right)^2 \Delta V = \frac{1}{2}\rho\left[\left(\frac{\partial s}{\partial t}\right)^2 + v^2\left(\frac{\partial s}{\partial x}\right)^2\right]\Delta V$$

$$s(x,t) = s_m \cos(kx - \omega t)$$

$$\frac{\partial s}{\partial t} = \omega s_m \sin(kx - \omega t)$$

$$\frac{\partial s}{\partial x} = -k s_m \sin(kx - \omega t)$$

$$\omega^2 = k^2 v^2$$

$$\Delta E = \Delta K + \Delta U = \frac{1}{2}\rho\left(\frac{\partial s}{\partial t}\right)^2 \Delta V + \frac{1}{2}\rho v^2 \left(\frac{\partial s}{\partial x}\right)^2 \Delta V = \frac{1}{2}\rho\left[\left(\frac{\partial s}{\partial t}\right)^2 + v^2\left(\frac{\partial s}{\partial x}\right)^2\right]\Delta V$$

$$= \frac{1}{2}\rho\left[\omega^2 s_m^2 \sin^2(kx - \omega t) + k^2 v^2 s_m^2 \sin^2(kx - \omega t)\right]\Delta V$$

$$w = \frac{\Delta E}{\Delta V} = \rho \omega^2 s_m^2 \sin^2(kx - \omega t)$$

$$w_{sr} = \frac{1}{2}\rho\omega^2 s_m^2 \qquad I = w_{sr} v = \frac{1}{2}\rho\omega^2 s_m^2 v$$

Jačina ili intenzitet zvuka

Jačina ili intenzitet zvuka definiše se kao energija koju u jedinici vremena prenese zvučni talas kroz jedinicu površine normalnu na pravac prostiranja talasa.

$$I = \frac{P_s}{S} = \frac{P_s}{4\pi r^2}$$

Jedinica je W/m².

$$I = \frac{(\Delta p_m)^2}{2\rho v} = \frac{(v\rho\omega s_m)^2}{2\rho v} = \frac{1}{2} v\rho\omega^2 s_m^2$$

$$F_x = S\Delta p = S\Delta p_m \sin(kx - \omega t)$$

$$P = u_x F_x = u_x S\Delta p_m \sin(kx - \omega t)$$

kako je $s = s_m \cos(kx - \omega t)$ to je

$$u_x = \frac{\partial s}{\partial t} = s_m \omega \sin(kx - \omega t)$$

$$P = s_m \omega S\Delta p_m \sin^2(kx - \omega t)$$

kako je $s_m = \frac{\Delta p_m}{v\rho\omega}$ to je

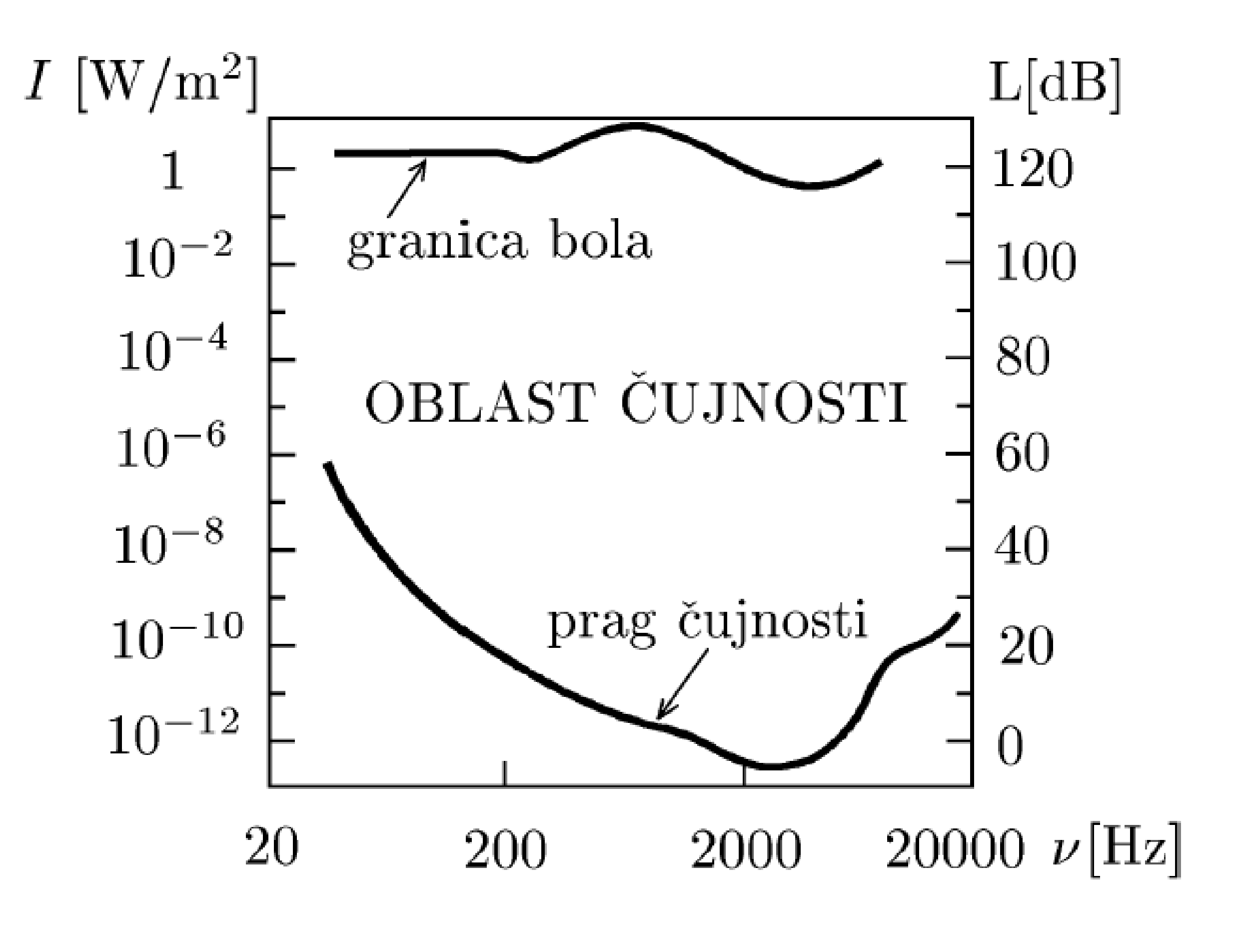
$$P = \frac{\Delta p_m}{v\rho\omega} \omega S\Delta p_m \sin^2(kx - \omega t)$$

$$P = \frac{(\Delta p_m)^2}{v\rho} S \sin^2(kx - \omega t)$$

$$P_s = \frac{(\Delta p_m)^2}{2\rho v} S$$

$$I = \frac{P_s}{S} = \frac{(\Delta p_m)^2}{2\rho v}$$

Intenzitet zvuka i nivo šuma



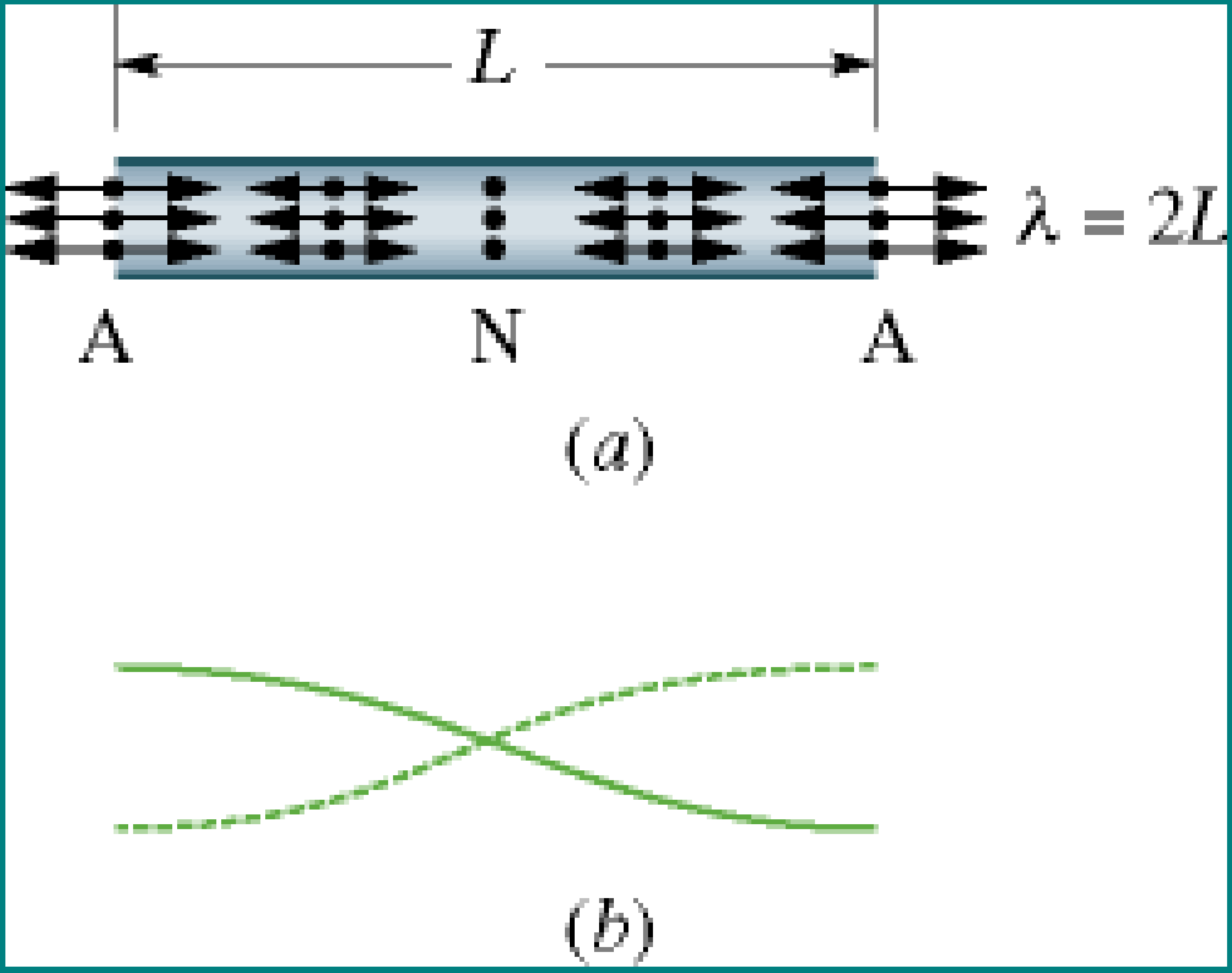
prag čujnosti $I_{min} = 10^{-12}\ \mathrm{W/m^2}$, $I_{max} = (1\text{–}10)\ \mathrm{W/m^2}$

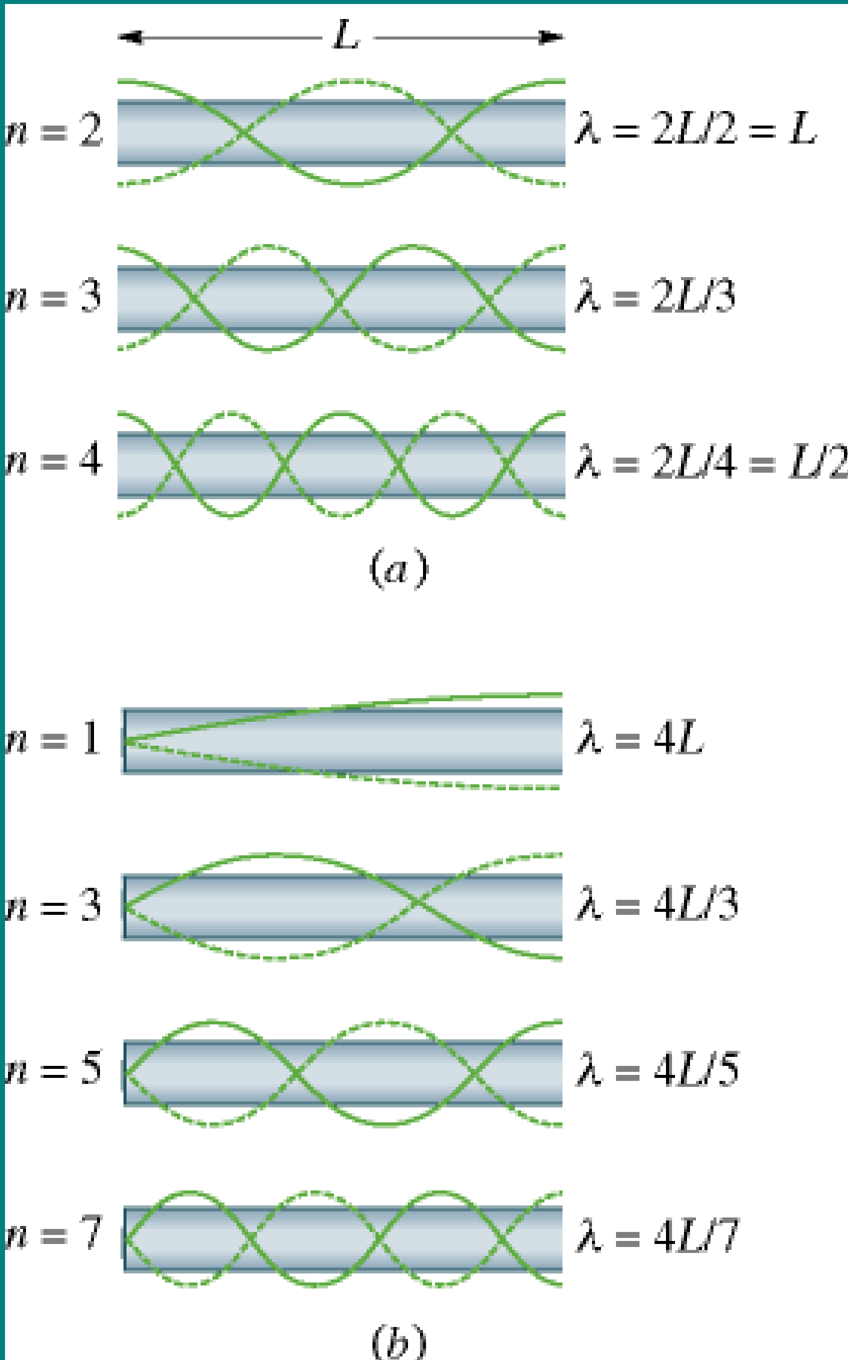
$$L = 10 \log \frac{I}{I_{min}}$$

Nivo šuma ili subjektivna jačina zvuka

$L = (0\text{–}130)\ \mathrm{dB}$

Oscilovanje vazdušnih stubova





$$\lambda = \frac{2L}{n}, \qquad \text{for } n = 1, 2, 3, \ldots,$$

$$f = \frac{v}{\lambda} = \frac{nv}{2L}, \qquad \text{for } n = 1, 2, 3, \ldots.$$

$$\lambda = \frac{4L}{n}, \qquad \text{for } n = 1, 3, 5, \ldots,$$

$$f = \frac{v}{\lambda} = \frac{nv}{4L}, \qquad \text{for } n = 1, 3, 5, \ldots.$$