7.2. *Exercises*

1. Suppose that $f(x)$ and $g(x)$ are effectively computable functions. Prove, using Church's thesis, that the function h given by

$$h(x) = \begin{cases} x & \text{if } x \in \text{Dom}(f) \cap \text{Dom}(g), \\ \text{undefined} & \text{otherwise} \end{cases}$$

is URM-computable.

2. Suppose that f is a total unary computable function. Prove, by Church's thesis, that the following function h is URM-computable

$$h(x) = \begin{cases} 1 & \text{if } x \in \text{Ran}(f), \\ \text{undefined} & \text{otherwise.} \end{cases}$$

3. Give a detailed proof by Church's thesis that the Ackermann function (example 2-5.5) is computable.

4. Prove by Church's thesis that the function g given by

   $g(n) = n$th digit in the decimal expansion of e

   is computable (where the number e is the basis for natural logarithms).