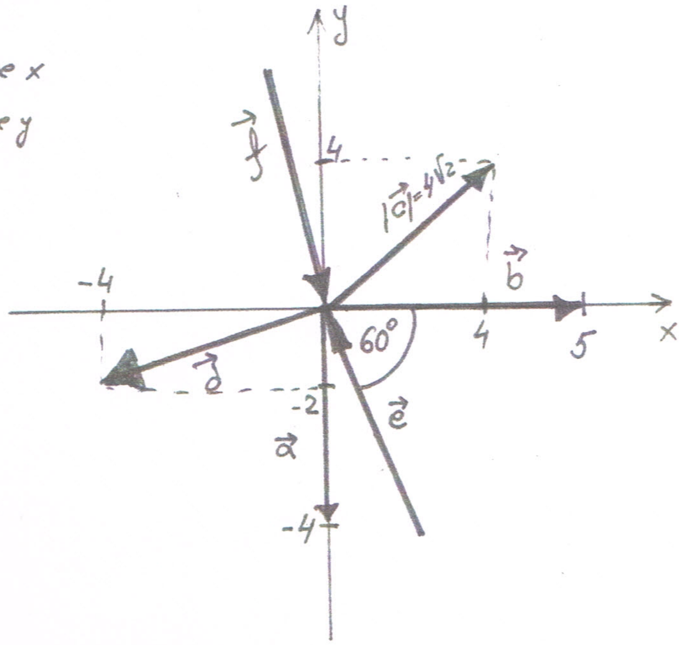
Statika, Mašinstvo

I, II i III čas vježbi, 28.09, 05.10, 12.10.2017

1. Vektore sa slike izraziti u Dekartovim koordinatama:

$\vec{i}$ - jedin. vektor ose x

$\vec{j}$ - jedin. vektor ose y



$$\vec{a} = 0\cdot\vec{i} - 4\cdot\vec{j}$$

$$\vec{b} = 5\cdot\vec{i} + 0\cdot\vec{j}$$

$$\vec{c} = (4\sqrt{2}\cdot\cos 45^\circ)\cdot\vec{i} + (4\sqrt{2}\cdot\sin 4$$

$$= 4\vec{i} + 4\vec{j}$$

$$\vec{d} = -4\vec{i} + (-2)\cdot\vec{j}$$

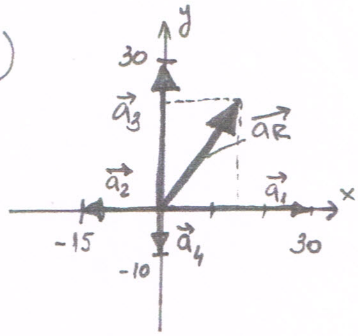
$$\vec{e} = -e\cdot\cos 60^\circ\cdot\vec{i} + e\sin 60^\circ\cdot\vec{j}$$

$$= -\frac{1}{2}e\cdot\vec{i} + \frac{e\sqrt{3}}{2}\cdot\vec{j}$$

$$\vec{f} = f\cos\theta\cdot\vec{i} - f\sin\theta\cdot\vec{j}$$

2. Za date vektore naći:
   - rezultujući vektor
   - intenzitet rezultujućeg vektora

a)



$$\vec{a_1} = 30\cdot\vec{i} + 0\cdot\vec{j}$$

$$\vec{a_2} = -15\cdot\vec{i} + 0\cdot\vec{j}$$

$$\vec{a_3} = 0\cdot\vec{i} + 30\cdot\vec{j}$$

$$\vec{a_4} = 0\cdot\vec{i} + (-10\cdot\vec{j})$$

$$\vec{a_R} = \vec{a_1} + \vec{a_2} + \vec{a_3} + \vec{a_4} =$$

$$= (30\vec{i} + 0\cdot\vec{j}) +$$

$$+ (-15\vec{i} + 0\cdot\vec{j}) +$$

$$+ (0\cdot\vec{i} + 30\cdot\vec{j}) +$$

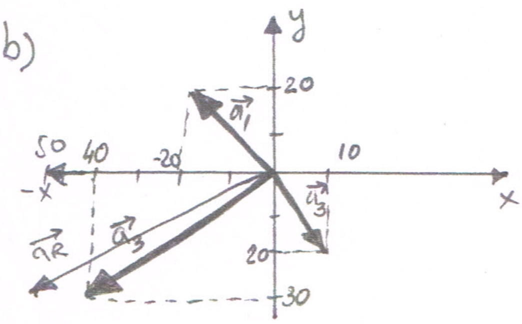
$$+ (0\cdot\vec{i} + (-10\cdot\vec{j})) =$$

$$= 15\vec{i} + 20\cdot\vec{j}$$

$$|\vec{a_R}| = \sqrt{a_{R_x}^2 + a_{R_y}^2} =$$

$$= \sqrt{15^2 + 20^2} = 25$$

b)



$$\vec{a_1} = -20\vec{i} + 20\vec{j}$$

$$\vec{a_2} = 10\vec{i} - 20\vec{j}$$

$$\vec{a_3} = -40\vec{i} - 30\vec{j}$$

$$\vec{a_R} = \vec{a_1} + \vec{a_2} + \vec{a_3} =$$

$$= (-20\vec{i} + 20\vec{j}) +$$

$$+ (10\vec{i} - 20\vec{j}) +$$

$$+ (-40\vec{i} - 30\vec{j}) =$$

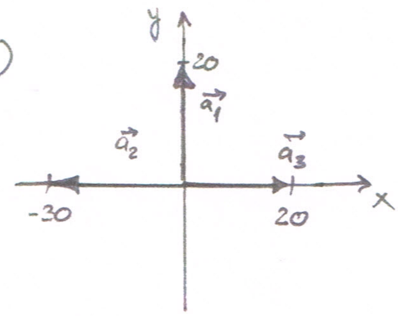
$$= -50\vec{i} - 30\vec{j}$$

$$a_{R_x} = -50\ ;\ a_{R_y} = -30$$

$$a_R = \sqrt{a_{R_x}^2 + a_{R_y}^2} =$$

$$= \sqrt{(-50)^2 + (-30)^2} = 10\sqrt{34}$$

c)



$$\vec{a_1} = 0\cdot\vec{i} + 20\cdot\vec{j}$$

$$\vec{a_2} = -30\cdot\vec{i} + 0\cdot\vec{j}$$

$$\vec{a_3} = 20\cdot\vec{i} + 0\cdot\vec{j}$$

$$\vec{a_R} = \vec{a_1} + \vec{a_2} + \vec{a_3} =$$

$$= (0\cdot\vec{i} + 20\cdot\vec{j}) +$$

$$+ (-30\vec{i} + 0\cdot\vec{j}) +$$

$$+ (20\vec{i} + 0\cdot\vec{j}) =$$

$$= -10\vec{i} + 20\vec{j}$$

$$|\vec{a}| = \sqrt{(-10)^2 + 20^2}$$

$$= \sqrt{500} = 10\sqrt{5}$$

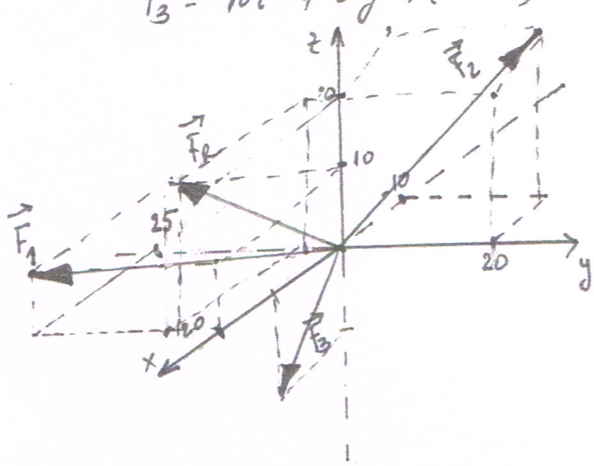
3. Zadate vektore nacrtati u Dekartovom koordinatnom sistemu, a zatim odrediti:

- intenzitet rezultante
- kosinuse uglova koje rezultanta zaklapa sa koord. osama

a) $\vec{F_1} = 20\vec{i} - 25\vec{j} + 10\vec{k}$

$\vec{F_2} = -10\vec{i} + 20\vec{j} + 20\vec{k}$

$\vec{F_3} = 10\vec{i} + 0 \cdot \vec{j} + (-10 \cdot \vec{k})$



$\vec{F_R} = \vec{F_1} + \vec{F_2} + \vec{F_3} =$

$= (20\vec{i} - 25\vec{j} + 10\vec{k}) +$

$+ (-10\vec{i} + 20\vec{j} + 20\vec{k}) +$

$+ (10\vec{i} + 0 \cdot \vec{j} + (-10\vec{k})) =$

$= 20\vec{i} - 5\vec{j} + 20\vec{k}$

$F_R = \sqrt{F_{Rx}^2 + F_{Ry}^2 + F_{Rz}^2}$

$F_{Rx} = 20; \quad F_{Ry} = -5; \quad F_{Rz} = 20$

$F_R = \sqrt{20^2 + (-5)^2 + 20^2} =$

$= 28{,}72$

$\cos\alpha = \frac{F_{Rx}}{F_R} = \frac{20}{28{,}72} = 0{,}69$

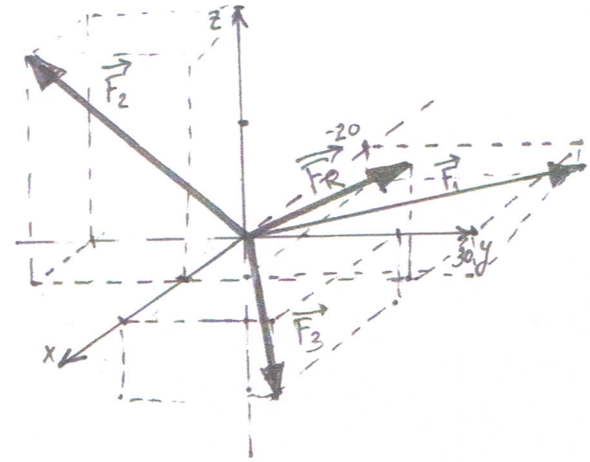
$\cos\beta = \frac{F_{Ry}}{F_R} = \frac{-5}{28{,}72} = 0{,}52$

$\cos\gamma = \frac{F_{Rz}}{F_R} = \frac{20}{28{,}72} = 0{,}69$

b) $\vec{F_1} = -20\vec{i} + 30\vec{j} - 5\vec{k}$

$\vec{F_2} = 10\vec{i} - 20\vec{j} + 30\vec{k}$

$\vec{F_3} = 20\vec{i} + 20\vec{j} - 10\vec{k}$



$\vec{F_R} = \vec{F_1} + \vec{F_2} + \vec{F_3} =$

$= (-20\vec{i} + 30\vec{j} - 5\vec{k}) +$

$+ (10\vec{i} - 20\vec{j} + 30\vec{k}) +$

$+ (20\vec{i} + 20\vec{j} - 10\vec{k}) =$

$= 10\vec{i} + 30\vec{j} + 15\vec{k}$

$F_{Rx} = 10; \quad F_{Ry} = 30; \quad F_{Rz} = 15$

$F_R = \sqrt{F_{Rx}^2 + F_{Ry}^2 + F_{Rz}^2} =$

$= \sqrt{10^2 + 30^2 + 15^2} =$

$= 35$

$\cos\alpha = \frac{F_{Rx}}{F_R} = \frac{10}{35} = 0{,}28$

$\cos\beta = \frac{F_{Ry}}{F_R} = \frac{30}{35} = 0{,}86$

$\cos\gamma = \frac{F_{Rz}}{F_R} = \frac{15}{35} = 0{,}43$

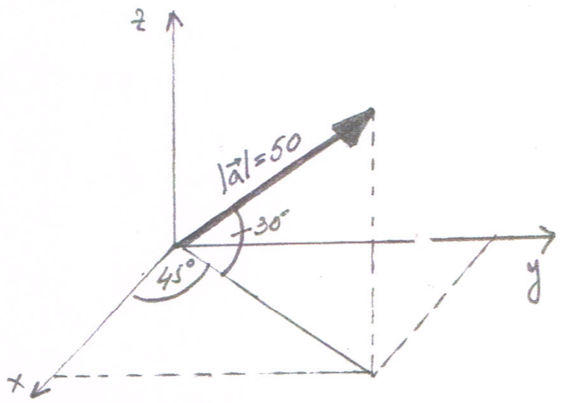
4. Odrediti projekciju datog vektora :

a)- na osu x

b)- na osu y

a) dato je: $\theta = 30^\circ$; $\alpha = 45^\circ$

$|\vec{a}| = 50$



$a_{xy} = a \cdot \cos 30^\circ$

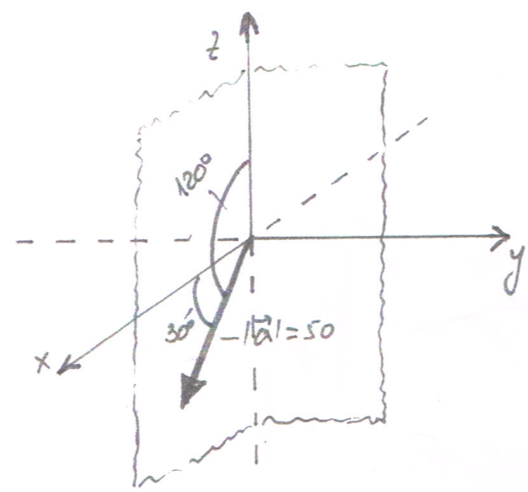
$a_x = a_{xy} \cdot \cos 45^\circ$

$a_x = a \cdot \cos 30^\circ \cdot \cos 45^\circ$

$= 50 \cdot \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} =$

$= 28{,}93$

b) dato je : $\alpha = 30^\circ$; $\gamma = 120^\circ$; $|\vec{a}| = 50$



$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$\cos^2\beta = 1 - \cos^2\alpha - \cos^2\gamma$

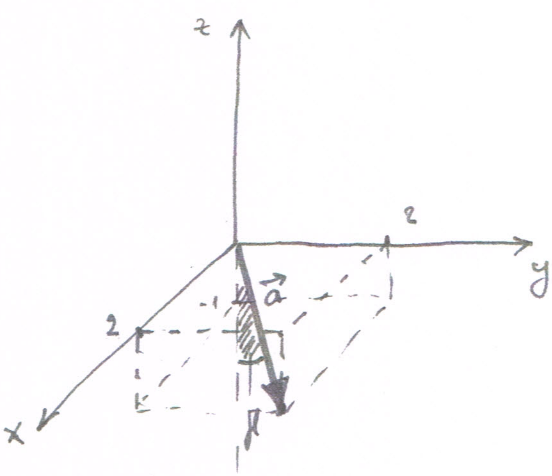
$\cos^2\beta = 1 - \cos^2 30^\circ - \cos^2 120^\circ$

$\cos\beta = \sqrt{1 - 0{,}75 - 0{,}25}$

$\cos\beta = 0$; $\cos\beta = \frac{a_y}{a}$

$a_y = a \cdot \cos\beta = 0$

5. Odrediti $\cos\gamma$. (Ugao između vektora $\vec{a}$ i ose z)

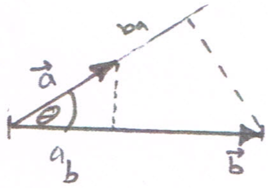


$a_x = 2$; $a_y = 2$; $a_z = -1$

$|\vec{a}| = \sqrt{a_x^2 + a_y^2 + a_z^2} = \sqrt{2^2 + 2^2 + (-1)^2} = 3$

$\cos\gamma = \frac{a_z}{|\vec{a}|} = \frac{-1}{3} = -0{,}33$

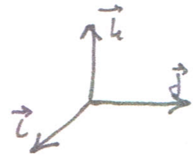
⊛ skalarni proizvod dva vektora



$$\vec{a}\cdot\vec{b} = |\vec{a}|\cdot|\vec{b}|\cdot\cos\theta$$

$$a_b = |\vec{a}|\cdot\cos\theta \;;\; b_a = |\vec{b}|\cdot\cos\theta$$

$$\vec{a}\cdot\vec{b} = |\vec{b}|\cdot a_b = \vec{a}\cdot b_a$$



$$\cos 90^\circ = 0$$
$$\cos 0^\circ = 1$$

$$\vec{i}\cdot\vec{j} = \vec{j}\cdot\vec{k} = \vec{k}\cdot\vec{i} = 0$$
$$\vec{i}\cdot\vec{i} = \vec{j}\cdot\vec{j} = \vec{k}\cdot\vec{k} = 1$$

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6. Izračunati skalarni proizvod vektora

a) $(5\vec{i})\cdot(3\vec{j}) = 5\cdot 3\cdot\vec{i}\cdot\vec{j} = 15\cdot 0 = 0$

b) $(-3\vec{j})\cdot(-2\vec{k}) = (-3)\cdot(-2)\cdot\vec{j}\cdot\vec{k} = 6\cdot 0 = 0$

c) $7\vec{i}\cdot 3\vec{i} = 7\cdot 3\cdot\vec{i}\cdot\vec{i} = 21\cdot 1 = 21$

d) $(3\vec{i}-2\vec{j}+4\vec{k})(2\vec{i}-4\vec{j}+3\vec{k}) = 6\,\vec{i}\cdot\vec{i} \cancel{-4\,\vec{i}\cdot\vec{j}} \cancel{+8\,\vec{k}\cdot\vec{i}} \cancel{-12\,\vec{i}\cdot\vec{j}} + 8\,\vec{j}\cdot\vec{j} \cancel{-16\,\vec{j}\cdot\vec{k}} \cancel{+9\,\vec{i}\cdot\vec{k}} \cancel{-6\,\vec{j}\cdot\vec{k}} + 12\,\vec{k}\cdot\vec{k} =$

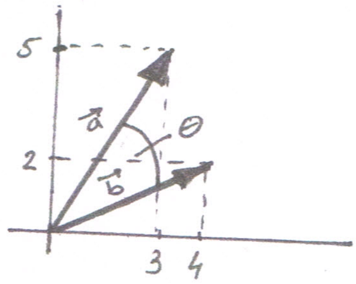
$= 6+8+12 = 26$

7. Izračunati ugao između vektora $\vec{a}$ i $\vec{b}$

a) $\vec{a} = 3\vec{i}+5\vec{j}$
$\vec{b} = 4\vec{i}+2\vec{j}$

$|\vec{a}| = \sqrt{3^2+5^2} = \sqrt{34}$

$|\vec{b}| = \sqrt{4^2+2^2} = \sqrt{20}$



$$\vec{a}\cdot\vec{b} = |\vec{a}|\cdot|\vec{b}|\cdot\cos\theta$$

$$\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}|\cdot|\vec{b}|}$$

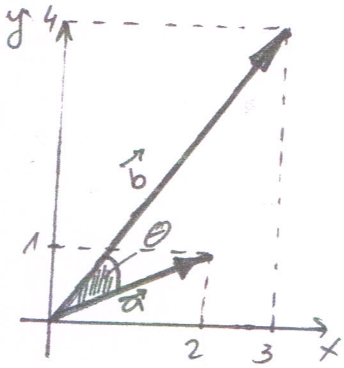
$$\cos\theta = \frac{(3\vec{i}+5\vec{j})\cdot(4\vec{i}+2\vec{j})}{\sqrt{34}\cdot\sqrt{20}}$$

$$\cos\theta = \frac{12+10}{\sqrt{640}} \;;\; \theta = \arccos\frac{22}{\sqrt{640}} = 3$$

b) $\vec{a} = 2\vec{i}+\vec{j}$
$\vec{b} = 3\vec{i}+4\vec{j}$

$|\vec{a}| = \sqrt{2^2+1^2} = \sqrt{5}$

$|\vec{b}| = \sqrt{3^2+4^2} = 5$



$$\vec{a}\cdot\vec{b} = |\vec{a}|\cdot|\vec{b}|\cdot\cos\theta$$

$$\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}|\cdot|\vec{b}|} = \frac{(2\vec{i}+\vec{j})(3\vec{i}+4\vec{j})}{\sqrt{5}\cdot 5} =$$

$$= \frac{10}{5\sqrt{5}} = \frac{2\sqrt{5}}{5} \;;\; \theta = \arccos\frac{2\sqrt{5}}{5} = \ldots$$

8. Izračunati projekciju vektora $\vec{a}$ na pravac $\vec{OA}$

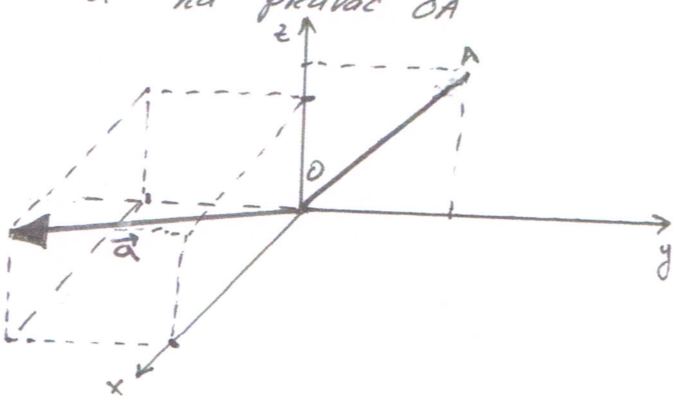
$$\vec{a} = 5\vec{i} - 4\vec{j} + 3\vec{k}$$

$a_{OA} = \vec{a}\cdot\vec{e}_{OA} \;;\; |\vec{OA}| = \sqrt{4^2+4^2}$

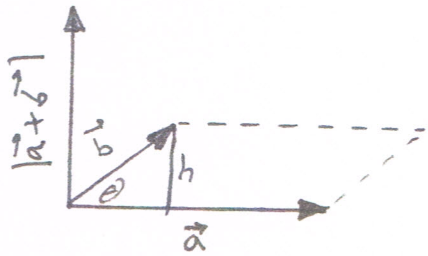
$$\vec{e}_{OA} = \frac{\vec{OA}}{|\vec{OA}|} = \frac{4\vec{j}+4\vec{k}}{\sqrt{4^2+4^2}} = \frac{\sqrt{2}}{2}\vec{j} + \frac{\sqrt{2}}{2}\vec{k}$$

$$a_{OA} = (5\vec{i}-4\vec{j}+3\vec{k})\cdot\left(\frac{\sqrt{2}}{2}\vec{j} + \frac{\sqrt{2}}{2}\vec{k}\right)$$

$$= -2\sqrt{2} + \frac{3\sqrt{2}}{2} = -\frac{\sqrt{2}}{2}$$



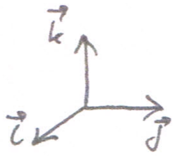
⊛ vektorski proizvod



$|\vec{a} \times \vec{b}| = |\vec{a}| \cdot |\vec{b}| \cdot \sin\theta$; $h = |\vec{b}| \cdot \sin\theta$

$|\vec{a} \times \vec{b}| = |\vec{a}| \cdot h$

$|\vec{a} \times \vec{b}| = P_{\square}$; $|\vec{a} \times \vec{b}| \perp \vec{a}$; $\vec{a} \times \vec{b} \perp \vec{b}$



$\sin 0° = 0$
$\sin 90° = 1$

$\vec{i} \times \vec{j} = \vec{k}$; $\vec{j} \times \vec{i} = -\vec{k}$; $\vec{i} \times \vec{i} = 0$

$\vec{j} \times \vec{k} = \vec{i}$; $\vec{k} \times \vec{j} = -\vec{i}$; $\vec{j} \times \vec{j} = 0$

$\vec{k} \times \vec{i} = \vec{j}$; $\vec{i} \times \vec{k} = -\vec{j}$; $\vec{k} \times \vec{k} = 0$

Izračunati vektorski proizvod i skalarni proizvod vektora.

a) $\vec{a_1} = \{10, 20, 30\}$

$\vec{a_2} = \{-15, 5, -10\}$

$\vec{a_1} = 10\vec{i} + 20\vec{j} + 30\vec{k}$

$\vec{a_2} = -15\vec{i} + 5\vec{j} - 10\vec{k}$

$\vec{a_1} \cdot \vec{a_2} = (10\vec{i} + 20\vec{j} + 30\vec{k}) \cdot (-15\vec{i} + 5\vec{j} - 10\vec{k}) =$

$= -150 + 100 - 300 = -350$

$$|\vec{a_1} \times \vec{a_2}| = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 10 & 20 & 30 \\ -15 & 5 & -10 \end{vmatrix} =$$

$= (20 \cdot (-10)) - (30 \cdot 5) \cdot \vec{i}$

$- (10 \cdot (-10)) - (30 \cdot (-15)) \cdot \vec{j}$

$+ (5 \cdot 10) - (20 \cdot (-15)) \cdot \vec{k} =$

$= -350\vec{i} - (-100 + 450) \cdot \vec{j} + 350\vec{k}$

$= -350\vec{i} - 350\vec{j} + 350\vec{k}$

$$\vec{a_2} \times \vec{a_1}| = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -15 & 5 & -10 \\ 10 & 20 & 30 \end{vmatrix} =$$

$= 350\vec{i} + 350\vec{j} - 350\vec{k}$

b) $\vec{a_1} = \vec{i} + \vec{j} + \vec{k}$

$\vec{a_2} = -\vec{i} - \vec{j} - \vec{k}$

$\vec{a_1} \cdot \vec{a_2} = (\vec{i} + \vec{j} + \vec{k})(-\vec{i} - \vec{j} - \vec{k}) = -3$

$$\vec{a_1} \times \vec{a_2} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 1 & 1 \\ -1 & -1 & -1 \end{vmatrix} = 0 \cdot \vec{i} - 0 \cdot \vec{j} + 0 \cdot \vec{k}$$

$\vec{a_2} \times \vec{a_1} = 0 \cdot \vec{i} - 0 \cdot \vec{j} + 0 \cdot \vec{k}$

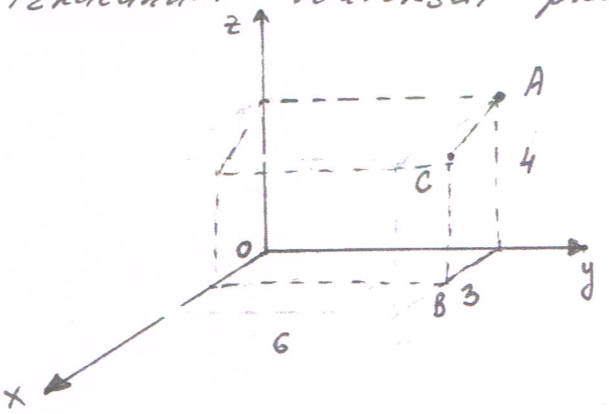
c) $\vec{a_1} = 4\vec{i} - 2\vec{j}$

$\vec{a_2} = 2\vec{i} + 3\vec{j} - 2\vec{k}$

$\vec{a_1} \cdot \vec{a_2} = (4\vec{i} - 2\vec{j})(2\vec{i} + 3\vec{j} - 2\vec{k}) = 8 - 6 = 2$

$$|\vec{a_1} \times \vec{a_2}| = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 4 & -2 & 0 \\ 2 & 3 & -2 \end{vmatrix} = 4\vec{i} + 8\vec{j} + 16\vec{k}$$

2. Vektor $\vec{a}$ ima intenzitet 5 i leži duž pravca OA. Naći jedinični vektor pravca OA. Napisati vektor $\vec{a}$ kao proizvod intenziteta i jediničnog vektora. Naći komponente vektora $\vec{a}$ u pravcu koordinatnih osa. Odrediti ugao koji vektor $\vec{a}$ zaklapa sa osom y. Izračunati vektorski proizvod $\vec{r}_{co}$ i $\vec{a}$.



a) $\vec{e}_{OA} = \dfrac{\vec{OA}}{|\vec{OA}|} = \dfrac{6\vec{j}+4\vec{k}}{\sqrt{6^2+4^2}} = \dfrac{6\vec{j}+4\vec{k}}{\sqrt{52}}$

$\vec{e}_{OA} = \dfrac{6}{\sqrt{52}}\vec{j} + \dfrac{4}{\sqrt{52}}\cdot\vec{k}$

b) $\vec{a} = |\vec{a}|\cdot\vec{e}_{OA} = 5\left(\dfrac{6}{\sqrt{52}}\cdot\vec{j} + \dfrac{4}{\sqrt{52}}\vec{k}\right)$

$= \dfrac{30}{\sqrt{52}}\cdot\vec{j} + \dfrac{20}{\sqrt{52}}\cdot\vec{k}$

c) $\vec{a}_x = 0\cdot\vec{i}\ ;\ \vec{a}_y = \dfrac{30}{\sqrt{52}}\cdot\vec{j}\ ;\ \vec{a}_z = \dfrac{20}{\sqrt{52}}\cdot\vec{k}$

$a_x = 0\ ;\ a_y = \dfrac{30}{\sqrt{52}}\ ;\ a_z = \dfrac{20}{\sqrt{52}}$

d) $\cos\beta = \dfrac{a_y}{|\vec{a}|} = \dfrac{\frac{30}{\sqrt{52}}}{5} = \dfrac{6}{\sqrt{52}}$

$\beta = \arccos\left(\dfrac{6}{\sqrt{52}}\right) = \ldots$

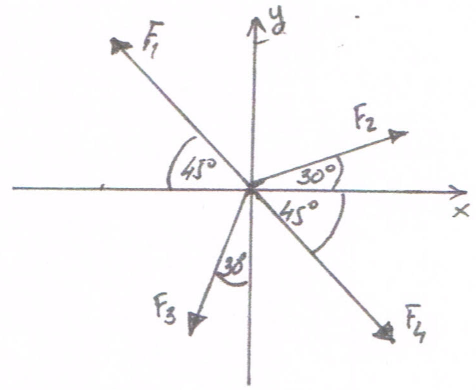
e) $\vec{r}_{co} = -3\vec{i} - 6\vec{j} - 4\vec{k}$

$\vec{a} = \dfrac{30}{\sqrt{52}}\vec{j} + \dfrac{20}{\sqrt{52}}\vec{k}$

$$|\vec{r}_{co}\times\vec{a}| = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -3 & -6 & -4 \\ 0 & \frac{30}{\sqrt{52}} & \frac{20}{\sqrt{52}} \end{vmatrix} = \left(\frac{-120}{\sqrt{52}} + \frac{120}{\sqrt{52}}\right)\cdot\vec{i} - \left(\frac{-60}{\sqrt{52}} + 0\right)\cdot\vec{j} + \left(\frac{-90}{\sqrt{52}}\right)\vec{k} =$$

$$= \frac{60}{\sqrt{52}}\cdot\vec{j} - \frac{90}{\sqrt{52}}\cdot\vec{k}$$

3. Odrediti rezultantu datih sila



$F_1 = 14,2 = 10\sqrt{2}$ kN

$F_2 = 15$ kN

$F_3 = 8$ kN

$F_4 = 18$ kN

$\vec{F}_1 = -14,2\cdot\cos 45°\cdot\vec{i} + 14,2\cdot\sin 45°\cdot\vec{j} =$

$= -10\vec{i} + 10\vec{j}$

$\vec{F}_2 = 15\cdot\cos 30°\cdot\vec{i} + 15\cdot\sin 30°\cdot\vec{j} =$

$= 13\vec{i} + 7,5\vec{j}$

$\vec{F}_3 = -8\cdot\sin 30°\cdot\vec{i} - 8\cdot\cos 30°\cdot\vec{j} =$

$= -4\vec{i} - 7\vec{j}$

$\vec{F}_4 = 18\cos 45°\cdot\vec{i} - 18\cdot\sin 45°\cdot\vec{j} = 12,72\vec{i} - 12,72\vec{j}$

$\vec{F}_R = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_4 =$

$= (-10\vec{i} + 10\vec{j}) +$

$+ (13\vec{i} + 7,5\vec{j}) +$

$+ (-4\vec{i} - 7\vec{j}) +$

$+ (12,72\vec{i} - 12,72\vec{j}) =$

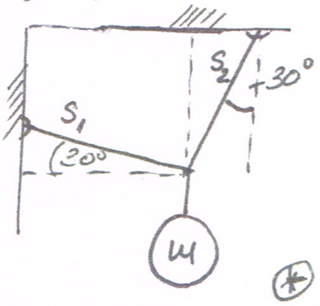
$= 11,72\vec{i} - 2,22\vec{j}$

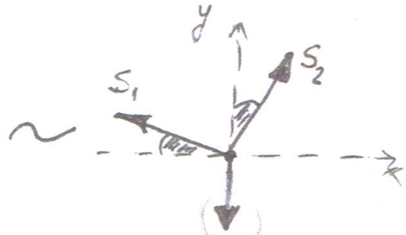
$F_{Rx} = 11,72\ ;\ F_{Ry} = -2,22$

$F_R = \sqrt{F_{Rx}^2 + F_{Ry}^2} = 12$ kN ; $\tan\Theta = \dfrac{F_{Ry}}{F_{Rx}} = \dfrac{11,7}{-2,22}$

4. Kugla mase 10 kg se održava u ravnoteži pomoću dva užeta S_1 i S_2 koja se nalaze pod uglom od 30° redom u odnosu na vertikalu tj. horizontalu (slika). Naći sile u užadima S_1 i S_2

$m = 10\,kg \; ; \; g = 9{,}81\ m/s^2$





$$S_{1x} = -S_1 \cdot \cos 30°$$
$$S_{1y} = S_1 \cdot \sin 30°$$
$$S_{2x} = S_2 \cdot \sin 30°$$
$$S_{2y} = S_2 \cdot \cos 30°$$

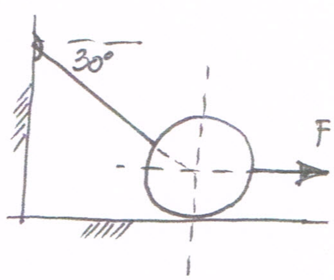
⊛ da bi tijelo bilo u ravnoteži, rezultanta datih sila mora biti jednaka nuli, tj: $\vec{S_1} + \vec{S_2} + m\vec{g} = \vec{0}$. To znači da projekcije rezultante na ose x i y moraju biti jednake nuli, pa je:

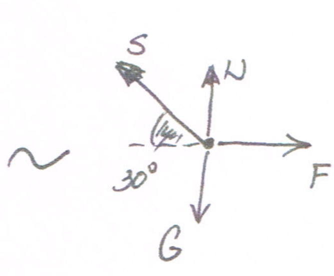
$$-S_{1x} + S_{2x} = 0 \qquad ; \qquad S_{1y} + S_{2y} - m \cdot g = 0$$

$$S_{1x} = S_{2x}$$
$$S_1 \cdot \cos 30° = S_2 \cdot \sin 30°$$
$$S_1 = S_2 \cdot tg 30°$$

$$S_1 \cdot \sin 30° + S_2 \cos 30° - mg = 0$$
$$S_2 \cdot tg 30° \cdot \sin 30° + S_2 \cdot \cos 30° = m \cdot g$$
$$S_2 \left(\sin 30° \cdot tg 30° + \cos 30° \right) = mg$$
$$S_2 = \frac{m \cdot g}{\sin 30° \cdot tg 30° + \cos 30°} = \frac{98{,}1}{0{,}5 \cdot 0{,}57 + 0{,}86}$$
$$S_2 = 86{,}05\,N \quad ; \quad S_1 = 49{,}05\,N$$

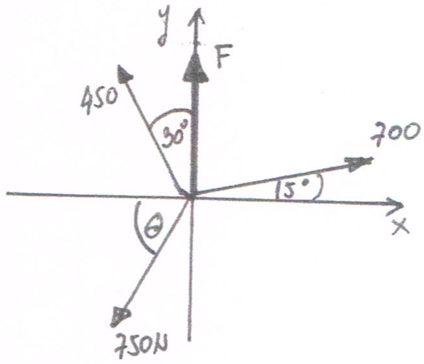
5. Kugla mase 50 kg miruje na glatkoj vodoravnoj podlozi. Ako na kuglu djeluje sila 500 N kao na slici, koliko iznosi normalna reakcija podloge N.





$$F - S \cdot \cos 30° = 0$$
$$N + S \cdot \sin 30° - G = 0$$
$$S = \frac{F}{\cos 30°} = 581{,}4\,N$$
$$N = G - S \cdot \sin 30° =$$
$$= 500 - 290{,}69 =$$
$$= 209{,}30\,N$$

1. Odrediti intenzitet sile F i ugao θ prikazan na slici, ako je sistem u ravnoteži



$\sum F_x = 0;$

$$700 \cdot \cos 15 - 450 \cdot \sin 30^\circ - 750 \cdot \cos\theta = 0$$

$$\cos\theta = \frac{450 \cdot \sin 30^\circ - 700 \cdot \cos 15^\circ}{-750} = 0{,}601$$

$$\boxed{\theta = 53{,}03^\circ}$$

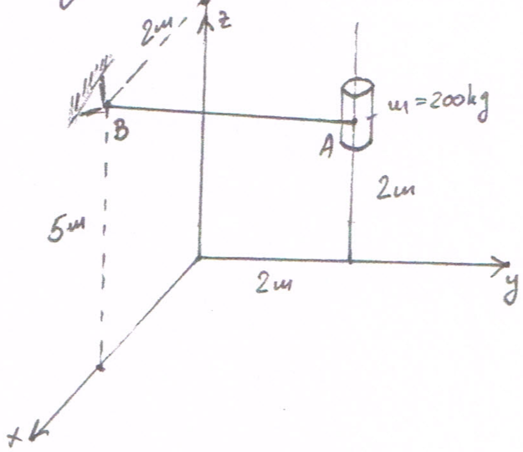
$\sum F_y = 0;$

$$F + 450 \cdot \cos 30^\circ + 700 \cdot \sin 15^\circ - 750 \cdot \sin\theta = 0$$

$$F = 750 \cdot \sin 53{,}03^\circ - 700 \cdot \sin 15^\circ - 450 \cdot \cos 30^\circ$$

$$\boxed{F = 28{,}28\,N}$$

2. Klizač mase 200kg pridržava se na vertikalnoj vodici pomoću užeta kao što je prikazano na slici. Odrediti silu u užetu i normalnu reakciju vodice.



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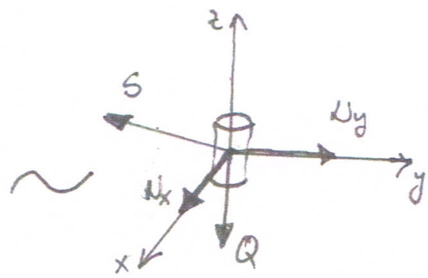


$m = 200kg$

$g = 9{,}81\ m/s^2$

$N_x = ?;\ N_y = ?;\ S = ?$

---

3 nepoznate, 3 uslova ravnot.

$Q = m \cdot g = 1962\,N$

$$\vec{N_x} = N_x \cdot \vec{i}$$
$$\vec{N_y} = N_y \cdot \vec{j}$$
$$\vec{Q} = -Q \cdot \vec{k}$$

$$\vec{S} = S \cdot \vec{e_{AB}} = S \cdot \frac{\vec{AB}}{|\vec{AB}|} =$$

$$= S \cdot \frac{2\vec{i} - 2\vec{j} + 3\vec{k}}{\sqrt{2^2 + (-2)^2 + 3^2}} =$$

$$= \frac{2S}{\sqrt{17}} \cdot \vec{i} - \frac{2S}{\sqrt{17}} \cdot \vec{j} + \frac{3S}{\sqrt{17}} \cdot \vec{k}$$

(*) postavljamo uslove ravnoteže za ose x, y, z

(1) $\sum F_x = 0;\quad N_x + \frac{2S}{\sqrt{17}} = 0$

(2) $\sum F_y = 0;\quad N_y - \frac{2S}{\sqrt{17}} = 0$

(3) $\sum F_z = 0;\quad -Q + \frac{3S}{\sqrt{17}} = 0$

---

(3) $\Rightarrow \quad Q = \frac{3S}{\sqrt{17}} \Rightarrow S = \frac{Q \cdot \sqrt{17}}{3}$

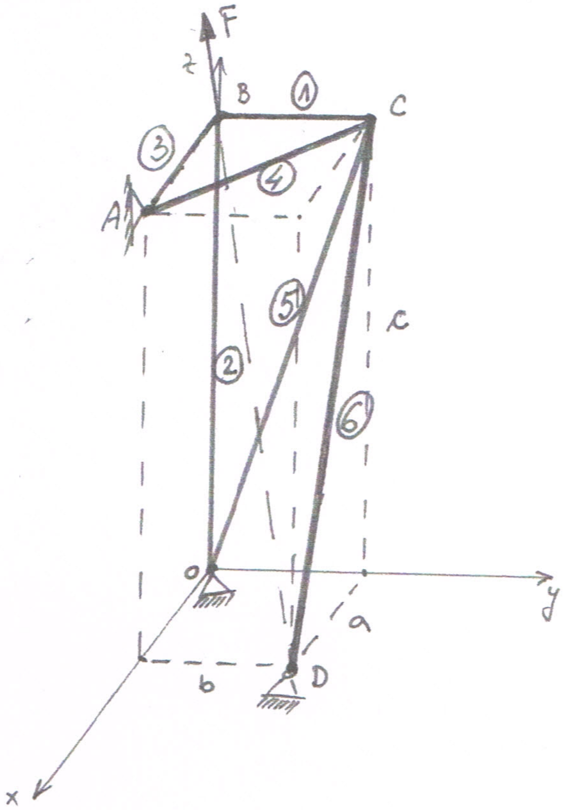
$$\boxed{S = 2695{,}5\,N}$$

zamjenom S u (2) i u (1) dobijamo:

$N_y = \frac{2S}{\sqrt{17}};\quad \boxed{N_y = 1308\,N}$

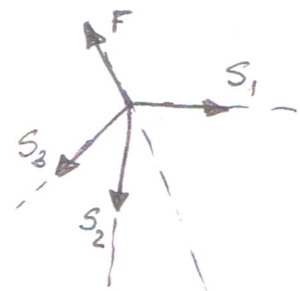
$N_x = -\frac{2S}{\sqrt{17}};\quad \boxed{N_x = -1308\,N}$ – znak "–" znači da je pogrešno pretpostavljen smjer sile N_x

3. Odrediti sile u štapovima date konstrukcije sastavljene od 6 zglobno vezanih, lakih štapova. Na konstrukciju djeluje sila $\vec{F}$, u tački B. Dato je: $a=3m$; $b=4m$; $c=12m$; $F=13\,kN$

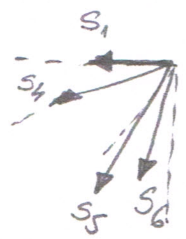


Sistem je u ravnoteži ako su sile koje djeluju na čvorove B i C u ravnoteži

Čvor B:



Čvor C:



Čvor B:

$$\vec{F} = F\cdot\vec{e}_{DB} = F\cdot\frac{\vec{DB}}{|\vec{DB}|} =$$

$$= F\cdot\frac{-3\vec{i}-4\vec{j}+12\vec{k}}{\sqrt{(-3)^2+(-4)^2+12^2}} =$$

$$= -3\vec{i}-4\vec{j}+12\vec{k}$$

$$\vec{S}_1 = S_1\cdot\vec{j}$$

$$\vec{S}_2 = -S_2\cdot\vec{k}$$

$$\vec{S}_3 = S_3\cdot\vec{i}$$

uslovi ravnoteže:

$\Sigma F_x = 0;\quad -3+S_3=0$

$\Sigma F_y = 0;\quad S_1-4=0$

$\Sigma F_z = 0;\quad -S_2+12=0$

$S_1 = 4\,kN$; $S_2 = 12\,kN$

$S_3 = 3\,kN$

Čvor C:

$$\vec{S}_1 = -S_1\cdot\vec{j} = -4\cdot\vec{j}$$

$$\vec{S}_4 = S_4\cdot\vec{e}_{CA} = S_4\cdot\frac{\vec{CA}}{|\vec{CA}|} = S_4\cdot\frac{3\vec{i}-4\vec{j}}{\sqrt{3^2+4^2}} = \frac{3S_4}{5}\cdot\vec{i}-\frac{4\cdot S_4}{5}\cdot\vec{j}$$

$$\vec{S}_5 = S_5\cdot\vec{e}_{CO} = S_5\cdot\frac{\vec{CO}}{|\vec{CO}|} = S_5\cdot\frac{-4\vec{j}-12\vec{k}}{\sqrt{4^2+12^2}} = -\frac{\sqrt{10}}{10}\cdot S_5\cdot\vec{j}-\frac{3\sqrt{10}}{10}\cdot S_5\cdot\vec{k}$$

$$\vec{S}_6 = S_6\cdot\vec{e}_{CD} = S_6\cdot\frac{\vec{CD}}{|\vec{CD}|} = S_6\cdot\frac{3\vec{i}-12\vec{k}}{\sqrt{3^2+(-12)^2}} = \frac{\sqrt{17}}{17}\cdot S_6\cdot\vec{i}-\frac{4\sqrt{17}}{17}S_6\cdot\vec{k}$$

uslovi ravnoteže:

(4) $\Sigma F_x = 0;\quad \frac{3}{5}S_4+\frac{\sqrt{17}}{17}S_6=0$

(5) $\Sigma F_y = 0;\quad -4-\frac{4S_4}{5}-\frac{\sqrt{10}}{10}\cdot S_5=0$

(6) $\Sigma F_z = 0;\quad -\frac{3\sqrt{10}}{10}\cdot S_5-\frac{4\sqrt{17}}{17}\cdot S_6=0$

Rješavanjem sistema jednačina dobijamo:

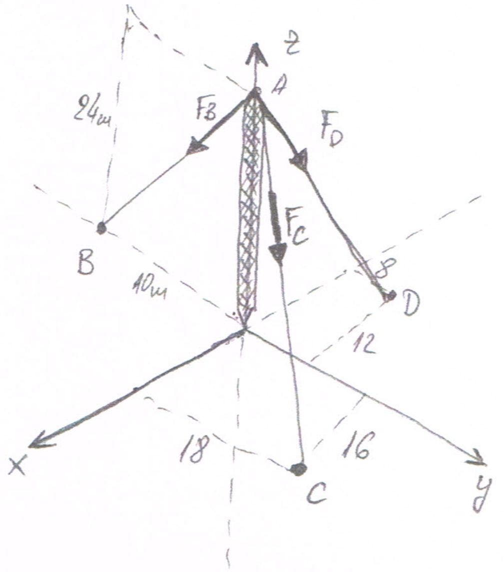
$S_6 = \frac{3\sqrt{17}}{2}\,kN$

$S_5 = -2\sqrt{10}\,kN$

$S_4 = -2{,}5\,kN$

Znak minus u proračunu znači da su smjerovi sila u štapovima 4 i 5 pogrešno pretpostavljeni, što znači da se treba promijeniti smjer. To dalje znači da su štapovi 4 i 5 opterećeni na istezanje, a štapovi 1, 2, 3, 6 su opterećeni na pritisak.

4. Antenski stub je učvršćen za podlogu pomoću tri užeta kako te pokazuje slika. Ako sile u užadima iznose: $F_B = 520N$, $F_C = 680N$, $F_D = 56$ odrediti rezultantu sila po intenzitetu, kao i uglove koje ona zaklap sa koordinatnim osama.



$$\vec{F}_B = F_B \cdot \vec{e}_{AB} = F_B \cdot \frac{\vec{AB}}{|\vec{AB}|} = F_B \cdot \frac{-10\vec{j} - 24\vec{k}}{\sqrt{10^2+24^2}} = -\frac{5}{13} F_B \vec{j} - \dots$$

$$\vec{F}_B = -200\vec{j} - 480 \cdot \vec{k}$$

$$\vec{F}_C = F_C \cdot \vec{e}_{AC} = F_C \cdot \frac{\vec{AC}}{|\vec{AC}|} = F_C \cdot \frac{16\vec{i} + 18\vec{j} - 24\vec{k}}{\sqrt{16^2+18^2+24^2}} =$$

$$= 170\vec{i} + 360\vec{j} - 480\vec{k}$$

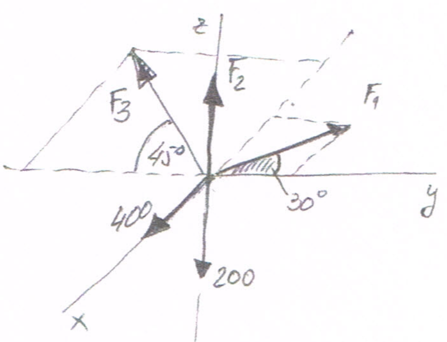
$$\vec{F}_D = F_D \cdot \vec{e}_{AD} = F_D \cdot \frac{\vec{AD}}{|\vec{AD}|} = F_D \cdot \frac{-12\vec{i} + 8\vec{j} - 24\vec{k}}{\sqrt{12^2+8^2+24^2}} =$$

$$= -24\vec{i} + 16 \cdot \vec{j} - 48\vec{k}$$

$$\vec{F}_R = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 = 146\vec{i} - 54 \cdot \vec{j} - 1008\vec{k}$$

$$|\vec{F}_R| = 1020 N$$

5. Odrediti intenzitet sila F_1, F_2 i F_3 iz uslova ravnoteže tačke A.



(1) $\sum F_x = 0; \quad 400 - F_3 \cdot \sin 45° - F_1 \cdot \sin 30° = 0$

(2) $\sum F_y = 0; \quad -F_3 \cdot \cos 45° + F_1 \cdot \cos 30° = 0$

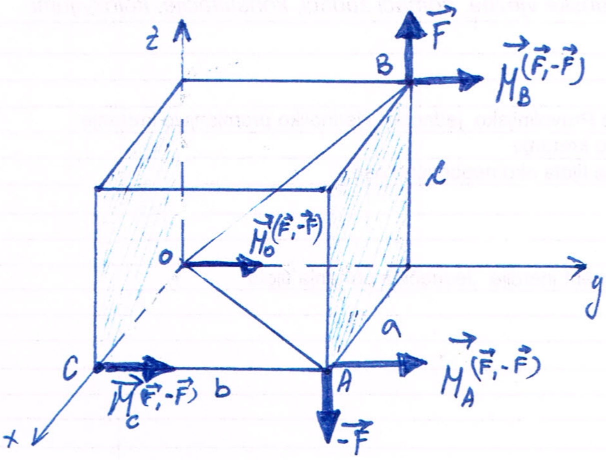
(3) $\sum F_z = 0; \quad F_2 - 200 = 0$

$F_2 = 200 N$; $F_1 = 292,8 N$; $F_3 = 358,6 N$

Statika, Mašinstvo, IV čas vježbi

1. Za kvadar prikazan na slici, opterećen silama $\vec{F}$ i $-\vec{F}$ naći:

a) moment sprega sila $\vec{F}$ i $-\vec{F}$ za tačku O
b) moment sprega sila $\vec{F}$ i $-\vec{F}$ za tačku C
c) moment sprega sila $\vec{F}$ i $-\vec{F}$ za tačku A
d) moment sprega sila $\vec{F}$ i $-\vec{F}$ za tačku B



a) $\vec{M}_O^{(\vec{F},-\vec{F})} = \vec{M}_O^{\vec{F}} + \vec{M}_O^{-\vec{F}} = \vec{OB}\times\vec{F} + \vec{OA}\times(-\vec{F}) = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & b & c \\ 0 & 0 & F \end{vmatrix} + \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a & b & 0 \\ 0 & 0 & -F \end{vmatrix} =$

$= \cancel{(F\cdot b)}^{0}\cdot\vec{i} + 0\cdot\vec{j} + 0\cdot\vec{k} + \cancel{(-F\cdot b)}^{0}\vec{i} + (F\cdot a)\cdot\vec{j} + 0\cdot\vec{k} = F\cdot a\cdot\vec{j}$

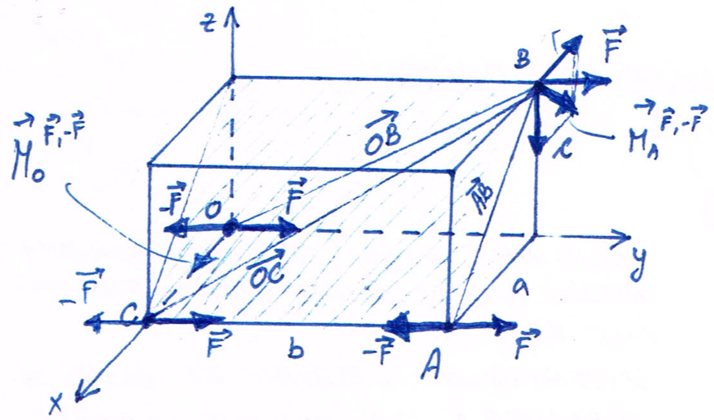
b) $\vec{M}_C^{(\vec{F},-\vec{F})} = \vec{M}_C^{\vec{F}} + \vec{M}_C^{(-\vec{F})} = \vec{CB}\times\vec{F} + \vec{CA}\times(-\vec{F}) = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -a & b & c \\ 0 & 0 & F \end{vmatrix} + \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & b & 0 \\ 0 & 0 & -F \end{vmatrix} =$

$= \cancel{(F\cdot b)}^{0}\cdot\vec{i} + F\cdot a\cdot\vec{j} + 0\cdot\vec{k} - \cancel{(F\cdot b)}^{0}\vec{i} + 0\cdot\vec{j} + 0\cdot\vec{k} = F\cdot a\cdot\vec{j}$

c) $\vec{M}_A^{(\vec{F},-\vec{F})} = \vec{AB}\times\vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a & 0 & c \\ 0 & 0 & F \end{vmatrix} = F\cdot a\cdot\vec{j}$

d) $\vec{M}_B^{(\vec{F},-\vec{F})} = \vec{BA}\times(-\vec{F}) = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -a & 0 & -c \\ 0 & 0 & -F \end{vmatrix} = (-F)\cdot(-a)\cdot\vec{j} = F\cdot a\cdot\vec{j}$

2. Za kvadar prikazan na slici, i opterećen silom $\vec{F}$:



a) Redukovati silu $\vec{F}$ na tačku A

b) Redukovati silu $\vec{F}$ na tačku O

c) Redukovati silu $\vec{F}$ na tačku C

a) Redukcijom sile $\vec{F}$ na tačku A dobijamo silu i moment sile:

- sila u tački A: $\vec{F} = F\cdot\vec{j}$;
- moment sile u tački A: $\vec{M}_A^{\vec{F}_1,-\vec{F}} = (\vec{AB}\times\vec{F}) = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -a & 0 & c \\ 0 & F & 0 \end{vmatrix} = -F\cdot c\cdot\vec{i} - F\cdot a\cdot$

b) Redukcija sile $\vec{F}$ na tačku O:

$$\vec{F} = F\cdot\vec{j}$$

$$\vec{M}_O^{(\vec{F}_1,-\vec{F})} = (\vec{OB}\times\vec{F}) = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & b & c \\ 0 & F & 0 \end{vmatrix} = -F\cdot c\cdot\vec{i}$$

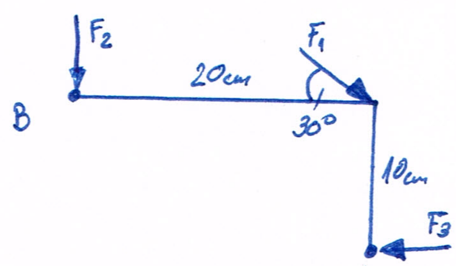
c) Redukcija sile $\vec{F}$ na tačku C:

$$\vec{F} = F\cdot\vec{j}$$

$$\vec{M}_C^{(\vec{F}_1,-\vec{F})} = (\vec{CB}\times\vec{F}) = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -a & b & c \\ 0 & F & 0 \end{vmatrix} = -F\cdot c\cdot\vec{i} - F\cdot a\cdot\vec{k}$$

3. Redukovati dati sistem sila na tačku B. Dato je:

$F_1 = 30N$; $F_2 = 20N$; $F_3 = 10N$



Redukcijom datog sistema sila na tačku B, kao rezultat dobijamo silu F_R i moment M_R.

$$F_R \begin{cases} \sum F_x = -F_3 + F_1\cdot\cos 30^\circ = -10 + 26 = 16N \\ \sum F_y = -F_2 - F_1\cdot\sin 30^\circ = -20 - 15 = -35N \end{cases}$$

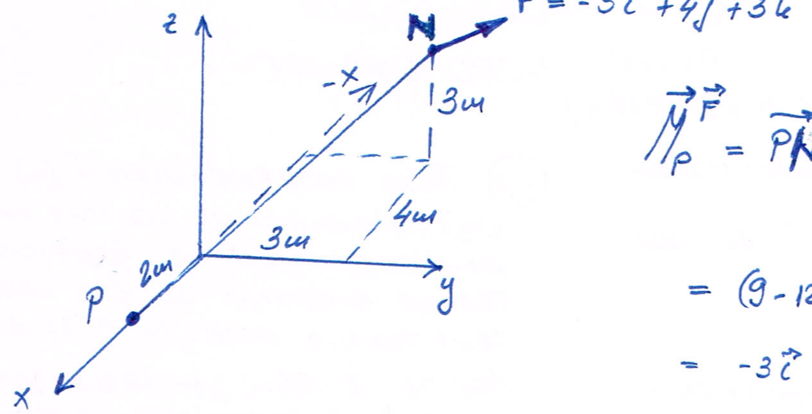
$$\sum M_B = F_3\cdot 10 + F_1\cdot\sin 30^\circ\cdot 20 =$$

$$= 10\cdot 10 + 15\cdot 20 = 400\ Ncm$$

$$F_R = \sqrt{F_x^2 + F_y^2} = \sqrt{16^2 + (-35)^2} = 38{,}48N$$

$$M_R = 400\ Ncm$$

4. Odrediti moment date sile za tačku P i za osu x



$$\vec{M}_P^{\vec{F}} = \vec{PN} \times \vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -6 & 3 & 3 \\ -5 & 4 & 3 \end{vmatrix} =$$

$$= (9-12)\cdot\vec{i} - (-18+15)\cdot\vec{j} + (-24+15)\cdot\vec{k}$$

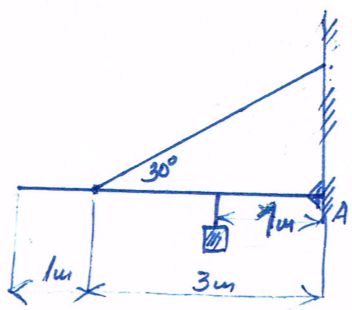
$$= -3\vec{i} + 3\vec{j} - 9\vec{k}$$

$\vec{PN} = -2\vec{i} + 3\vec{j} - 4\vec{i} + 3\vec{k} =$
$= -6\vec{i} + 3\vec{j} + 3\vec{k}$

$\underline{M_x^F = -3}$ – moment sile F za osu x

(*) Projekciju momenta sile F na osu x možemo da izračunamo i tako što ćemo izračunati proizvod projekcije sile F na osu y i najkraćeg rastojanja do ose x (4,3m) i proizvod sile F na osu z i najkraćeg rastojanja do ose x (3·1·3m). Projekcija sile F na osu y daje negativan moment za osu x.

5. O horizontalni štap zanemarljive težine obješen je teret težine 10 kN. Odrediti silu u užetu i reakciju zgloba A u položaju ravnoteže



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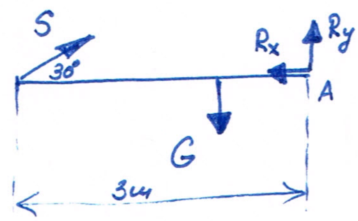


$G = 10\,kN$

$$\left.\begin{aligned} &(1) \quad \Sigma F_x = 0; \quad S\cdot\cos 30° - R_x = 0 \\ &(2) \quad \Sigma F_y = 0; \quad S\cdot\sin 30° + R_y - G = 0 \\ &(3) \quad \Sigma M_A = 0; \quad S\cdot\sin 30°\cdot 3 - G\cdot 1 = 0 \end{aligned}\right\}$$

$(3) \Rightarrow \quad S\cdot\frac{1}{2}\cdot 3 = G$

$S = \frac{2G}{3} = 6{,}67N \; ; \; \boxed{S = 6{,}67N}$

$(2) \Rightarrow R_y = G - S\cdot\sin 30° = 10 - 6{,}67\cdot\frac{1}{2}$

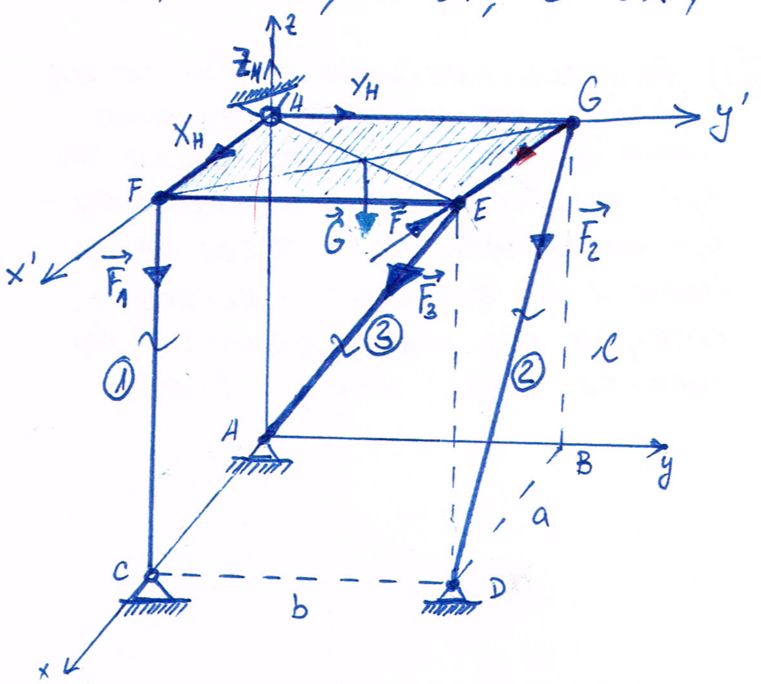
$\boxed{R_y = 6{,}665\,N}$

$(1) \Rightarrow \; R_x = S\cdot\cos 30° = 6{,}67\cdot\frac{\sqrt{3}}{2}$

$\boxed{R_x = 5{,}77N}$

6. Homogena pravougaona ploča težine G održava se u ravnotežnom položaju pomoću sfernog ležišta H i tri štapa CF, EA, i GD. Na ploču u tački E djeluje sila F u pravcu EG. Odrediti reakcije, ako je dato

$a = 3m$; $b = 4m$; $C = 12m$; $G = 8KN$, $F = 10KN$



$$\vec{R_H} = \{X_H, Y_H, Z_H\}$$

$$\vec{F} = \{-F, 0, 0\}$$

$$\vec{G} = \{0, 0, -G\}$$

$$\vec{F_1} = \{0, 0, -F_1\}$$

$$\vec{F_2} = \left\{\frac{F_2 \cdot a}{\sqrt{a^2+c^2}}, 0, \frac{-F_2 \cdot c}{\sqrt{a^2+c^2}}\right\}$$

$$\vec{F_3} = \left\{\frac{-F_3 \cdot a}{\sqrt{a^2+b^2+c^2}}, \frac{-F_3 \cdot b}{\sqrt{a^2+b^2+c^2}}, \frac{-F_3 \cdot c}{\sqrt{a^2+b^2+c^2}}\right\}$$

$$\vec{F_2} = F_2 \cdot \vec{e}_{GD} = F_2 \cdot \frac{\vec{GD}}{|\vec{GD}|} = F_2 \cdot \frac{-c \cdot \vec{k} + a \cdot \vec{i}}{\sqrt{(-c)^2 + a^2}} =$$

$$= \frac{-F_2 \cdot c}{\sqrt{a^2+c^2}} \cdot \vec{k} + \frac{a \cdot F_2}{\sqrt{a^2+c^2}} \cdot \vec{i} =$$

$$\vec{F_2} = \frac{F_2 \cdot a}{\sqrt{a^2+c^2}} \cdot \vec{i} - \frac{F_2 \cdot c}{\sqrt{a^2+c^2}} \cdot \vec{k}$$

$$\vec{F_3} = F_3 \cdot \vec{e}_{EA} = F_3 \cdot \frac{\vec{EA}}{|\vec{EA}|} = F_3 \cdot \frac{-a \cdot \vec{i} - b \cdot \vec{j} - c \cdot \vec{k}}{\sqrt{(-a)^2 + (-b)^2 + (-c)^2}} =$$

$$= -\frac{F_3 \cdot a}{\sqrt{a^2+b^2+c^2}} \vec{i} - \frac{F_3 \cdot b}{\sqrt{a^2+b^2+c^2}} \vec{j} - \frac{F_3 \cdot c}{\sqrt{a^2+b^2+c^2}} \vec{k}$$

uslovi ravnoteže:

(1) $\sum F_x = 0$; $X_H - F - \frac{F_3 \cdot a}{\sqrt{a^2+b^2+c^2}} = 0$

(2) $\sum F_y = 0$; $Y_H - \frac{F_3 \cdot b}{\sqrt{a^2+b^2+c^2}} = 0$

(3) $\sum F_z = 0$; $Z_H - G - F_1 - \frac{F_2 \cdot c}{\sqrt{a^2+c^2}} - \frac{F_3 \cdot c}{\sqrt{a^2+b^2+c^2}} = 0$

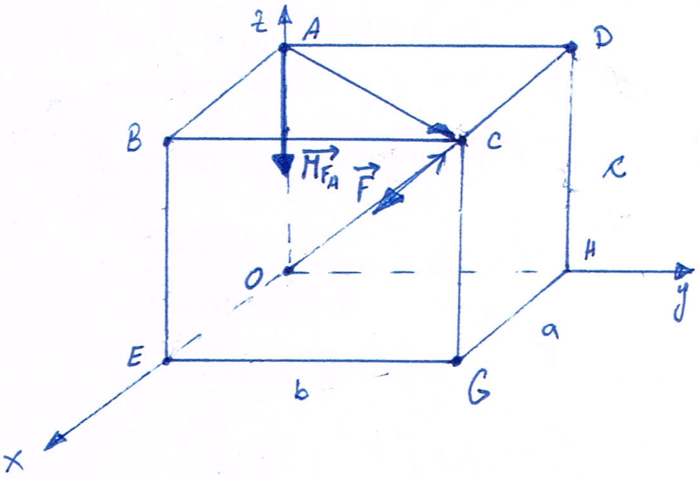
(4) $\sum M_{x'} = 0$; $G \cdot \frac{b}{2} + \frac{F_2 \cdot c}{\sqrt{a^2+c^2}} \cdot b + \frac{F_3 \cdot c}{\sqrt{a^2+b^2+c^2}} \cdot b = 0$

(5) $\sum M_{y'} = 0$; $F_1 \cdot a + G \cdot \frac{a}{2} + \frac{F_3 \cdot c}{\sqrt{a^2+b^2+c^2}} \cdot a = 0$

(6) $\sum M_z' = 0$; $F \cdot b - F_2 \cdot \frac{a \cdot b}{\sqrt{a^2+c^2}} = 0$

4. Na kvadar prikazan na slici djeluje sila F. Odrediti:

- moment sile F za tačku A
- moment sile F za tačku O
- moment sile F za tačku G
- moment sile F za osu GA. Dato je F, a, b, c



$$\vec{AC} = a\cdot\vec{i} + b\cdot\vec{j}$$

$$\vec{OC} = a\cdot\vec{i} + b\cdot\vec{j} + c\cdot\vec{k}$$

$$\vec{\lambda}_{GA} = \frac{\vec{GA}}{|\vec{GA}|} = \frac{-a\cdot\vec{i} - b\cdot\vec{j} + c\cdot\vec{k}}{\sqrt{a^2+b^2+c^2}} = \frac{-a}{\sqrt{a^2+b^2+c^2}}\cdot\vec{i} + \frac{-b}{\sqrt{a^2+b^2+c^2}}\cdot\vec{j} + \frac{+c}{\sqrt{a^2+b^2+c^2}}\cdot\vec{k}$$

a) $$\vec{M}_{F_A} = \vec{AC}\times\vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a & b & 0 \\ F & 0 & 0 \end{vmatrix} = 0\cdot\vec{i} - 0\cdot\vec{j} - F\cdot b\cdot\vec{k}$$

$M_{F_{Ax}} = 0$; $M_{F_{Ay}} = 0$; $M_{F_{Az}} = -F\cdot b$

b) $$\vec{M}_{F_O} = \vec{OC}\times\vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a & b & c \\ F & 0 & 0 \end{vmatrix} = 0\cdot\vec{i} + F\cdot c\cdot\vec{j} - F\cdot b\cdot\vec{k}$$

$M_{F_{Ox}} = 0$; $M_{F_{Oy}} = F\cdot c$; $M_{F_{Oz}} = -F\cdot b$

c) $$\vec{M}_{F_G} = \vec{GC}\times\vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 0 & c \\ F & 0 & 0 \end{vmatrix} = 0\cdot\vec{i} + F\cdot c\cdot\vec{j} + 0\cdot\vec{k}$$

$M_{F_{Gx}} = 0$; $M_{F_{Gy}} = F\cdot c$; $M_{F_{Gz}} = 0$

d) $$M_{GA}^{F} = \vec{M}_A^F\cdot\vec{\lambda}_{GA} = \vec{M}_G^F\cdot\vec{\lambda}_{GA} =$$

$$= (-F\cdot b\cdot\vec{k})\left(\frac{-a}{\sqrt{a^2+b^2+c^2}}\cdot\vec{i} + \frac{-b}{\sqrt{a^2+b^2+c^2}}\cdot\vec{j} + \frac{c}{\sqrt{a^2+b^2+c^2}}\cdot\vec{k}\right) =$$

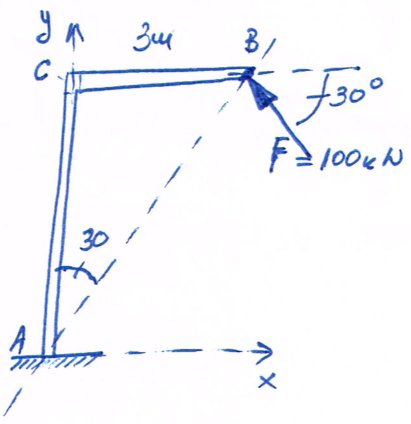
$$= \frac{-F\cdot b\cdot c}{\sqrt{a^2+b^2+c^2}}$$

(*) Intenzitet momenta može se izračunati i kao proizvod sile i najkraćeg rastojanja do tačke za koju tražimo moment. Naprimjer: $|\vec{M}_{F_A}| = F\cdot\overline{AD} = F\cdot b$

(**) moment sile F za osu $\overline{GA}$ bi dobili isti da smo tražili skalarni proizvod $\vec{M}_G^F\cdot\vec{\lambda}_{GA} = (F\cdot c\cdot\vec{j})\left(\frac{-a}{\sqrt{a^2+b^2+c^2}}\cdot\vec{i} + \frac{-b}{\sqrt{a^2+b^2+c^2}}\cdot\vec{j} + \frac{c}{\sqrt{a^2+b^2+c^2}}\cdot\vec{k}\right)$

$$= \frac{-F\cdot c\cdot b}{\sqrt{a^2+b^2+c^2}}$$

5. Na nosač prikazan na slici djeluje sila F. Naći Moment sile F za tačku A.



$$\vec{M}_{F_A} = \vec{AB} \times \vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3 & 3\sqrt{3} & 0 \\ -\frac{F\cdot\sqrt{3}}{2} & \frac{F}{2} & 0 \end{vmatrix} =$$

$$= 0\cdot\vec{i} + 0\cdot\vec{j} + \left(50\cdot 3 - (-50\sqrt{3}\cdot 3\sqrt{3})\right)\cdot\vec{k}$$

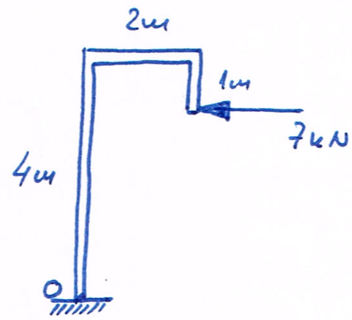
$$= 600\,\vec{k}$$

$$M_{F_A} = 100\cdot 6 \quad (F\cdot \overline{AB})$$

$$tg\,30° = \frac{CB}{AC} \Rightarrow AC = CB\cdot\frac{1}{tg\,30°} = 3\sqrt{3}$$

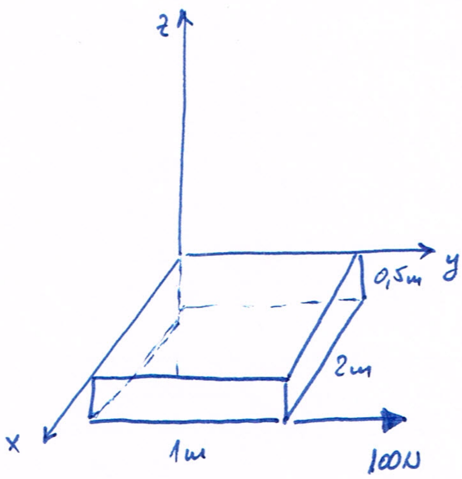
$$\vec{F} = -F\cdot\cos 30°\cdot\vec{i} + F\cdot\sin 30°\cdot\vec{j}$$

6. Odrediti moment date sile za tačku O.



$$M_{F_O} = 7\cdot 3 = 21\ kNm$$

7. Odrediti moment date sile od 100N za sve tri koordinatne ose.



$$M_x = 50\,Nm$$

$$M_y = 0\,Nm$$

$$M_z = 200\,Nm$$