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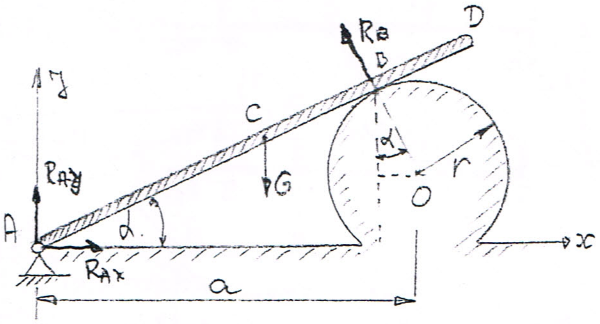
PRIMER: ŠTAP AD, TEŽINE G i DUŽINE l, ZGLOBNO JE VEZAN U TAČKI A A U TAČKI B SE OSLANJA NA GLATKI VALJAK POLUPREČNIKA r. AKO JE UGAO IZMEĐU PRAVCA ŠTAPA AD I HORIZONTALNE OSE JEDNAK α I AKO JE RASTOJANJE IZMEĐU TAČAKA A I O JEDNAKO a ODREDITI REAKCIJE VEZA.

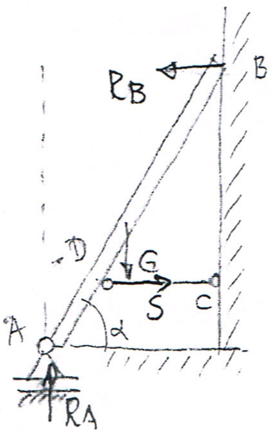


$$\sum M_A = 0:\ R_B\frac{(a - r\sin\alpha)}{\cos\alpha} - G\cdot\frac{l}{2}\cos\alpha = 0 \Rightarrow \boxed{R_B = G\cdot\frac{1}{2}\,\frac{l\cos^2\alpha}{a - r\sin\alpha}}$$

$$\sum F_{ix} = 0:\ R_{Ax} - R_B\sin\alpha = 0 \Rightarrow R_{Ax} = G\,\frac{1}{2}\,\frac{l\cos^2\alpha\cdot\sin\alpha}{a - r\sin\alpha}$$

$$\sum F_{iy} = 0:\ R_{Ay} - G + R_B\cos\alpha = 0 \Rightarrow R_{Ay} = G\left(1 - \frac{1}{2}\,\frac{l\cos^3\alpha}{a - r\sin\alpha}\right)$$

PRIMJER: Homogeni štap AB težine G i dužine l oslanja se na vertikalni zid u tački B a za horizontalni pod je vezan pokretnim osloncem A. Pomoću horizontalnog užeta BC štap je vezan za zid tako da je $AD = l/6$.
Odrediti reakcije u vezama ako je $\alpha = 60°$. Zid smatrati idealno glatkim.



R: $R_A = G,\ R_B = S = 10\sqrt{3}\,G$.

$R_B = S.$

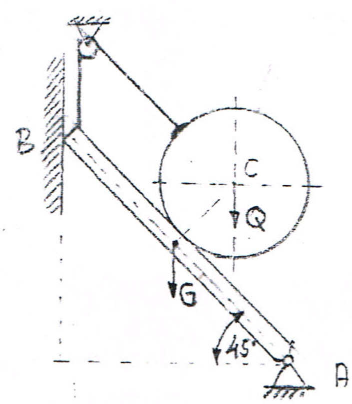
$$S\cdot\frac{l}{6}\cdot\frac{\sqrt{3}}{2} + G\cdot\frac{l}{2}\cdot\frac{1}{2} - R_B\cdot l\cdot\frac{\sqrt{3}}{2} = 0$$

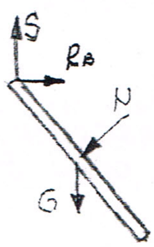
$$-S\,l\left(\frac{\sqrt{3}}{12} - \frac{\sqrt{3}}{2}\right) = \frac{G\cdot l}{4}$$

$$S\,\frac{5\sqrt{3}}{12} = \frac{Gl}{4} \Rightarrow S = \frac{3\cdot G}{5\sqrt{3}}\,\frac{\sqrt{3}}{\sqrt{3}} = \frac{G\sqrt{3}}{5}$$

$$\boxed{S = \frac{G\sqrt{3}}{5}} = R_B$$

PRIMJER: Homogeni valjak C, težine Q, pridržava se užetom prebačenim preko nepomičnog kotura E zanemarljive težine, vezanim u tački B. Valjak se oslanja na štap AB, težine G i dužine l u tački na sredini štapa. Odrediti reakcije u tačkama A i B i silu u užetu.



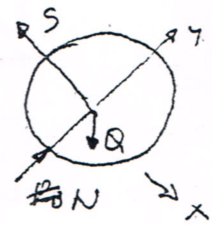


R: $R_B = Q\left(1 - \frac{\sqrt{2}}{2}\right)$

$R_{Ax} = -\frac{Q}{2}(\sqrt{2} - 1)$

$R_{Ay} = \frac{Q}{2}(3 - \sqrt{2})$

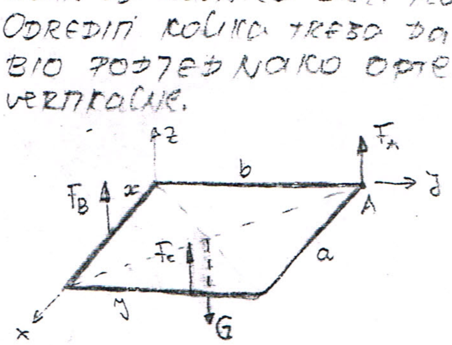
$S = Q\cdot\frac{\sqrt{2}}{2} = N.$



$$\Sigma M_A = 0.\quad R_B\cdot\frac{l\sqrt{2}}{2} - Q\frac{\sqrt{2}}{2}\cdot\frac{l\sqrt{2}}{2} + G\cdot\frac{l}{2}\cdot\frac{\sqrt{2}}{2} - Q\frac{\sqrt{2}}{2}\cdot\frac{l}{2} = 0$$

$$R_B + Q\frac{\sqrt{2}}{2} - \frac{G}{2} - \frac{Q}{2} = 0 \Rightarrow R_B = \frac{Q}{2}(\sqrt{2} - 1)$$

PRIMJER: Pri podizanju homogene pravougaone ploče dimenzija a i b (sl.), jedan od radnika drži ploču u tački A, drugi u tački B a treći u tački C. Odrediti kolika treba da su rastojanja x i y da bi svaki od radnika bio podjednako opterećen. Smatrati da su sile u tačkama A, B i C vertikalne.



$\Sigma M_z = 0$

$\Sigma M_x = 0:\ F_A\cdot b - G\cdot\frac{b}{2} + F_C\cdot y = 0$

$\Sigma M_y = 0:\ F_B\cdot x - G\cdot\frac{a}{2} + F_C\cdot a = 0$

$\Sigma F_{ix} = 0$

$\Sigma F_{iy} = 0$

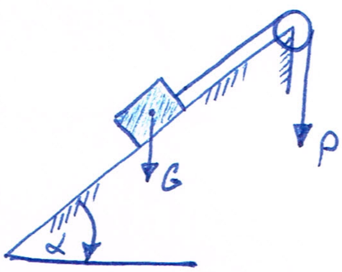
$\Sigma F_{iz} = 0:\ F_A + F_B + F_C - G = 0$

Pošto je $F_A = F_B = F_C$, to je $F = \frac{G}{3},\ x = \frac{a}{2};\ y = \frac{b}{2}$

, Mašinstvo, V čas vježbi

09. 11. 2017

1. Odrediti interval vrijednosti intenziteta sile P tako da tijelo težine G koje se nalazi na strmoj ravni nagiba α bude u ravnoteži. Nagib strme ravni je takav da bi se tijelo pod dejstvom sopstvene težine kretalo niz strmu ravan. Koeficijent trenja f je poznat.



Da bi tijelo bilo u ravnoteži treba da je

$$P_{min} \leq P \leq P_{max},$$

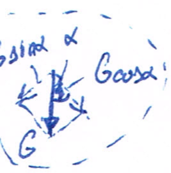
ako je $P < P_{min}$, tijelo se kreće niz strmu ravan

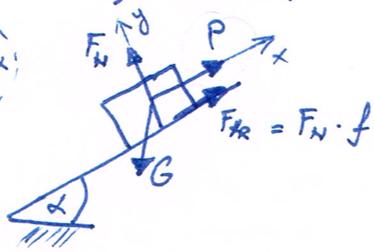
a ako je $P > P_{max}$, tijelo se kreće uz strmu ravan

Razmatraćemo slučajeve kada je $P = P_{min}$; $P = P_{max}$

1° Tijelo ima tendenciju da se kreće niz strmu ravan

a) $P = P_{min}$





$$\sum F_x = 0; \quad P + F_{tR} - G \cdot \sin\alpha = 0$$

$$\sum F_y = 0; \quad F_N - G \cdot \cos\alpha = 0$$

$$F_N = G \cdot \cos\alpha$$

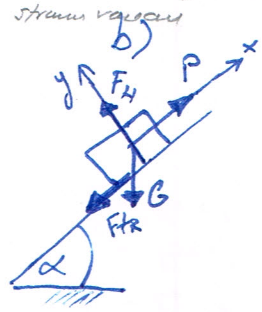
$$P = G \cdot \sin\alpha - F_{tR}$$

$$P = G \cdot \sin\alpha - G \cdot \cos\alpha \cdot f$$

$$P_{min} = P = G(\sin\alpha - \cos\alpha \cdot f)$$

2° Tijelo ima tendenciju da se kreće uz strmu ravan

b) $P = P_{max}$



$$F_{tR} = F_N \cdot f$$

$$\sum F_x = 0; \quad P - F_{tR} - G \cdot \sin\alpha = 0$$

$$\sum F_y = 0; \quad F_N - G \cdot \cos\alpha = 0$$

$$F_N = G \cdot \cos\alpha$$

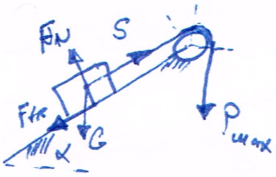
$$P_{max} = P = F_{tR} + G \cdot \sin\alpha$$

$$P_{max} = G \cdot \cos\alpha \cdot f + G \cdot \sin\alpha$$

$$P_{max} = G(\sin\alpha + \cos\alpha \cdot f)$$

$$G(\sin\alpha - \cos\alpha \cdot f) \leq P \leq G(\sin\alpha + \cos\alpha \cdot f)$$

Naći maksimalnu silu P_{max}, ako postoji trenje užeta o cilindričnu površ, koeficijenta k



ugao zahvata θ:

$$\theta = 90 + \alpha$$

$$P_{max} = S \cdot e^{k\theta}$$

$$F_N = G\cos\alpha$$

$$S = F_{tR} + G\sin\alpha$$

$$S = G\cos\alpha \cdot f + G\sin\alpha$$

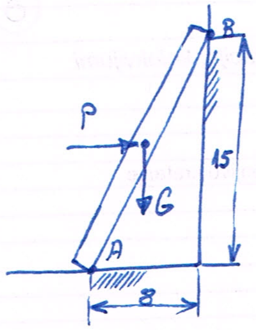
$$P_{max} = (G(\cos\alpha \cdot f + \sin\alpha)) \cdot e^{k\theta} =$$

$$P_{max} = G(\cos\alpha \cdot f + \sin\alpha) \cdot e^{k(90+\alpha)}$$

2\. Greda AB težine $G=10kN$ održava se u položaju prikazanom na slici pod dejstvom sile P. Ako koeficijent trenja f na mjestima kontakta ima vrijednost 0,2 odrediti:

a) minimalnu vrijednost sile P da bi greda AB bila u ravnoteži

b) maksimalnu vrijednost sile P da bi greda AB bila u ravnoteži



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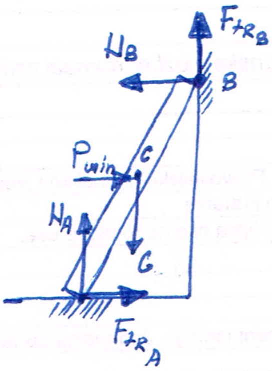

1° Greda ima tendenciju da se spušta pod dejstvom sile G

$$\Sigma F_x=0;\quad P_{min}+F_{trA}-N_B=0 \quad \dots (1)$$

$$\Sigma F_y=0;\quad N_A+F_{trB}-G=0 \quad \dots (2)$$

$$\Sigma M_C=0;\quad N_B\cdot 7{,}5+F_{trB}\cdot 4-N_A\cdot 4+F_{trA}\cdot 7{,}5=0 \quad \dots (3)$$

$$F_{trB}=N_B\cdot f \qquad F_{trA}=N_A\cdot f$$

$$(1)\Rightarrow P_{min}=N_B-N_A\cdot f$$

$$(2)\Rightarrow N_A=G-N_B\cdot f$$

$$(3)\Rightarrow N_B(7{,}5+f\cdot 4)-N_A(4-f\cdot 7{,}5)=0$$

$$N_B=N_A\cdot\left(\frac{4-7{,}5\cdot f}{7{,}5+4f}\right)=0{,}3\,N_A$$

$$N_A=G-N_A\cdot\left(\frac{4-7{,}5f}{7{,}5+4f}\right)$$

$$N_A=\frac{G}{1+\left(\frac{4-7.5\cdot f}{7.5+4f}\cdot f\right)}=9{,}43\ kN$$

$$N_B=2{,}8\ kN;\quad \boxed{P_{min}=1\ kN}$$

$$P_{min}=N_B-N_A\cdot f=2{,}8-9{,}43\cdot 0{,}2=0{,}91$$

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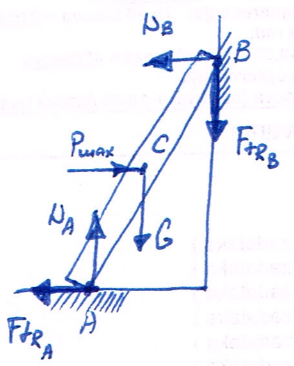


2° Greda ima tendenciju da se podiže pod dejstvom sile P

$$\Sigma F_x=0;\quad P_{max}-F_{trA}-N_B=0 \quad \dots (1)$$

$$\Sigma F_y=0;\quad N_A-G-F_{trB}=0 \quad \dots (2)$$

$$\Sigma M_C=0;\quad N_B\cdot 7{,}5-F_{trB}\cdot 4-N_A\cdot 4-F_{trA}\cdot 7{,}5=0 \quad \dots (3)$$

$$N_B(7{,}5-4\cdot f)-N_A(4+7{,}5\cdot f)=0$$

$$N_B=N_A\cdot\frac{4+7.5f}{7{,}5-4\cdot f}$$

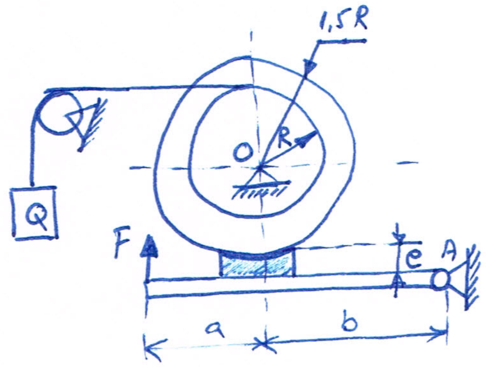
$$N_A=G+N_B\cdot f$$

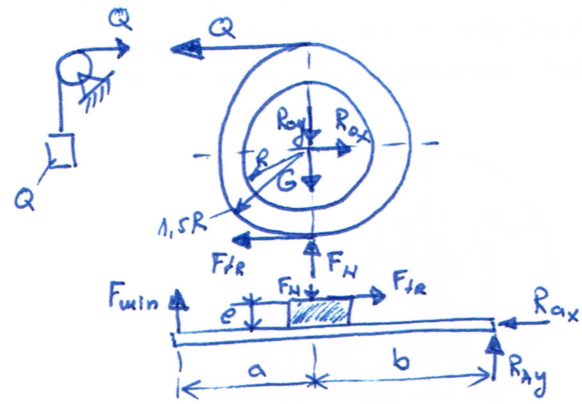
$$N_A=\frac{G}{1-\frac{4+7{,}5f}{7{,}5-4\cdot f}\cdot f}=12\ kN$$

$$N_B=10{,}0\ kN;\quad \boxed{P_{max}=12{,}4\ kN}$$

$$N_B=\frac{N_A-G}{f};\quad P_{max}=N_B+N_A\cdot f$$

3. Odrediti minimalnu vrijednost sile F, Reakcije u osloncima A i O i silu međusobnog pritiska između papuče kočnice i koaksijalnog doboša kočnice prikazane na slici. Težina štapa AB se zanemaruje, dok su težine tereta i doboša Q, odnosno G. Koeficijent trenja između papuče i doboša je f_0. Ostali potrebni podaci dati su na slici.





DOBOŠ:

$$\Sigma M_o=0;\quad F_{tr}\cdot 1,5\cdot R - Q\cdot R = 0 \;\Rightarrow\; F_{tr}=F_N\cdot f_0;\; \boxed{F_N=\frac{Q}{1,5\cdot f_0}}$$

$$\Sigma F_x=0;\quad R_{ox}-Q-F_{tr}=0;\;\Rightarrow\; R_{ox}=Q+F_N\cdot f_0;\; \boxed{R_{ox}=\frac{5}{3}Q}$$

$$\Sigma F_y=0;\quad F_N-R_{oy}-G=0;\; R_{oy}=F_N-G \;\Rightarrow\; \boxed{R_{oy}=\frac{2\cdot Q}{3\cdot f_0}-G}$$

POLUGA:

$$\Sigma M_A=0;\quad F_{min}(a+b)+F_{tr}\cdot e-F_N\cdot b=0;\;\Rightarrow\; F_{min}=(F_N\cdot b-F_{tr}\cdot e)\cdot\frac{1}{a+b}=\frac{2}{3}Q\cdot\frac{b-e\cdot f_0}{(a+b)\cdot f_0}$$

$$\Sigma F_x=0;\quad F_{tr}-R_{Ax}=0 \;\Rightarrow\; \boxed{R_{Ax}=\frac{2}{3}Q}$$

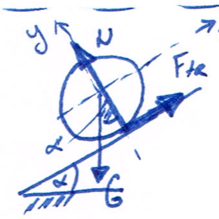
$$\Sigma F_y=0;\quad F_{min}+R_{Ay}-F_N=0 \;\Rightarrow\; \boxed{R_{Ay}=\frac{2}{3}Q\cdot\frac{a+e\cdot f_0}{(a+b)\cdot f_0}}$$

4. Točak poluprečnika r, se nalazi na strmoj ravni nagibnog ugla α i koeficijenta trenja μ. Odrediti:

a) ugao α, tako da dođe do klizanja točka

b) krak trenja kotrljanja e, tako da dođe do kotrljanja to

a)



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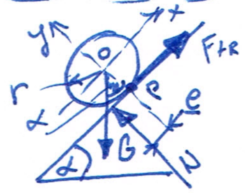
$$(1)\;\Sigma F_x=0;\quad F_{tr}-G\cdot\sin\alpha=0;\; F_{tr}=N\cdot\mu$$

$$(2)\;\Sigma F_y=0;\quad N-G\cdot\cos\alpha=0$$

$$\left.\begin{array}{l}(1)\Rightarrow N=\dfrac{G\cdot\sin\alpha}{\mu}\\ (2)\Rightarrow N=G\cdot\cos\alpha\end{array}\right\}\; \cancel{G}\cdot\cos\alpha\cdot\mu=\cancel{G}\cdot\sin\alpha \quad \boxed{\mu=tg\alpha}$$

b)



$$(1)\ldots\;\Sigma F_y=0;\quad N-G\cdot\cos\alpha=0;\;\Rightarrow N=G\cdot\cos\alpha$$

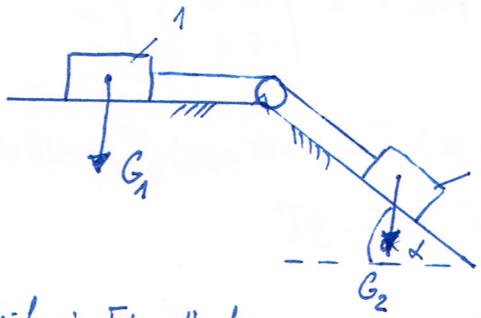
$$(2)\ldots\;\Sigma M_P=0;\quad N\cdot e-G\cdot\sin\alpha\cdot r=0$$

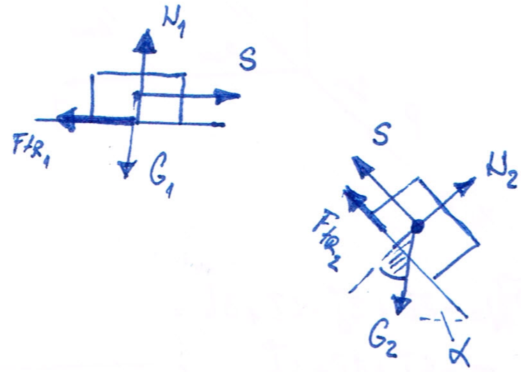
$$\cancel{G}\cdot\cos\alpha\cdot e=\cancel{G}\cdot\sin\alpha\cdot r$$

$$\boxed{e=tg\alpha\cdot r}$$

5. Dva tijela težina G_1 i G_2 vezana su nerastegljivim užetom. Odrediti težinu tijela na strmoj ravni, da bi sistem bio u stanju mirovanja. Dato je: $G_1 = 10\,kN$; $f_1 = 0{,}1$; $f_2 = 0{,}2$; $\alpha = 30^\circ$

Trenje užeta o cilindričnu površ zanemariti.





$F_{tr_1} = N_1 \cdot f_1$; $F_{tr_2} = N_2 \cdot f_2$

tijelo 1:

$\sum F_y = 0$; $N_1 - G_1 = 0$

$N_1 = G_1$

$\sum F_x = 0$;

$S - F_{tr_1} = 0$

$S = F_{tr_1}$

$\boxed{S = N_1 \cdot f_1 = G_1 \cdot f_1}$

tijelo 2:

$\sum F_y = 0$; $N_2 - G_2 \cdot \cos\alpha = 0$

$N_2 = G_2 \cdot \cos\alpha$

$\sum F_x = 0$; $G_2 \cdot \sin\alpha - S - F_{tr_2} = 0$

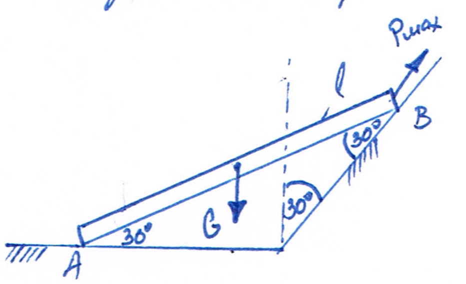
$G_2 \cdot \sin\alpha - G_1 \cdot f_1 - N_2 \cdot f_2 = 0$

$G_2 \cdot \sin\alpha - G_1 \cdot f_1 - G_2 \cdot \cos\alpha \cdot f_2 = 0$

$G_2 (\sin\alpha - \cos\alpha \cdot f_2) = G_1 \cdot f_1$

$$G_2 = \frac{G_1 \cdot f_1}{\sin\alpha - \cos\alpha \cdot f_2} = \frac{10 \cdot 0{,}1}{\sin 30^\circ - \cos 30^\circ \cdot 0{,}2} \approx 3\,kN$$

6. Za gredu AB dužine l i težine G, u položaju prikazanom na slici, odrediti maksimalnu vrijednost sile P da bi tijelo bilo u stanju mirovanja. Koeficijent trenja između grede i horizontalne podloge je f.

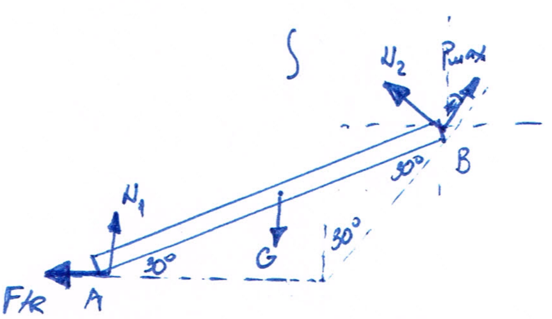


$\cancel{\sum F_x = 0;\ P_{max} \cdot \sin 30^\circ - F_{tr} - N_2 \cdot \cos 30^\circ = 0}$

$\cancel{\sum F_y = 0;\ N_1 + P_{max} \cdot \cos 30^\circ - G + N_2 \cdot \sin 30^\circ = 0}$

(1) $\sum M_B = 0$; $N_1 \cdot \frac{l\sqrt{3}}{2} + N_1 \cdot f \cdot \frac{l}{2} - G \cdot \frac{l\sqrt{3}}{2} \cdot \frac{1}{2} = 0$

(2) $\sum M_C = 0$; $P \cdot l - N_1 \cdot f \cdot l - G \cdot \frac{l\sqrt{3}}{2} \cdot \frac{1}{2} = 0$



(1) $\Rightarrow$ $\frac{N_1}{2}(f + \sqrt{3}) = \frac{G\sqrt{3}}{4}$; $N_1 = \frac{\sqrt{3}}{2(\sqrt{3}+f)} \cdot G$

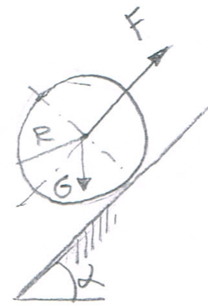
(2) $\Rightarrow$ $P = N_1 \cdot f + \frac{G\sqrt{3}}{4}$

$$P = \frac{G\sqrt{3} \cdot f}{2 \cdot (\sqrt{3}+f)} + \frac{G\sqrt{3}}{4} = \frac{G\sqrt{3}}{4}\left(\frac{2f}{\sqrt{3}+f} + 1\right) = \frac{G\sqrt{3}}{4}\left(\frac{2f+\sqrt{3}+f}{\sqrt{3}+f}\right)$$

$$\boxed{P = \frac{G\sqrt{3}}{4}\left(\frac{3f+\sqrt{3}}{f+\sqrt{3}}\right)}$$

$F_{tr} = N_1 \cdot f$

TOČAK POLUPREČNIKA R[m] I MASE m[kg] SE NALAZI NA STRMOJ RAVNI NAGIBNOG UGLA α. KOEFICIJENTI TRENJA KLIZANJA I KOTRLJANJA SU: f i e[m].
ODREDITI VRIJEDNOSTI SILE F TAKO DA TOČAK KLIZA A NE KOTRLJA UZ STRMU RAVAN.
UZETI: $R = 0{,}4\,m$; $m = 80\,kg$; $\alpha = 45^\circ$; $f = 0{,}2$; $e = 0{,}3\,m$



$g \approx 10\ m/s^2$;

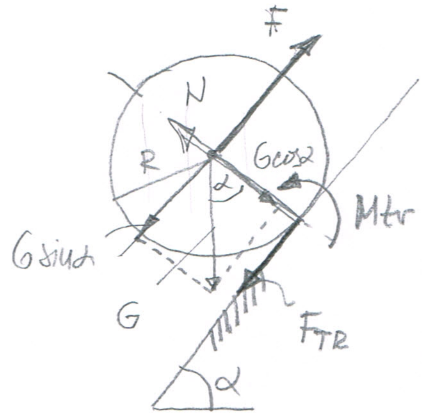
$G = 80 \cdot 10 = 800\,N$

$F_{TR} = N \cdot f$

$N = G\cos\alpha = 800 \cdot \frac{\sqrt{2}}{2} = 400\sqrt{2}\ N$

$F_{TR} = 400\sqrt{2} \cdot 0{,}2 = 80\sqrt{2}\ [N] = 113{,}1\ N$

$M_{TR} = N \cdot e = 400\sqrt{2}\ 0{,}3 = 120\sqrt{2}\ [N] = 169{,}7\ Nm$



KLIZA UZ

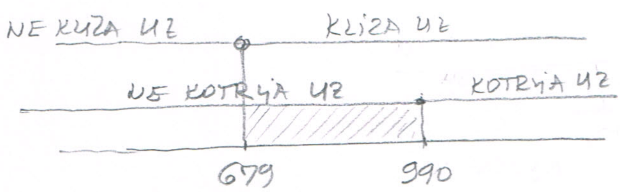
$F > G\sin\alpha + F_{TR}$

$F > 800 \cdot \frac{\sqrt{2}}{2} + 80\sqrt{2} = 480\sqrt{2} = 679\,N$

NE KOTRLJA UZ

$F \cdot R \leq G\sin\alpha \cdot R + M_{tr} = G\sin\alpha \cdot R + Ne$

$F \leq G\sin\alpha + N\frac{e}{R} = \dfrac{800 \cdot \frac{\sqrt{2}}{2} + 120\sqrt{2}}{0{,}4} = 700\sqrt{2} = 990\,N$



$F \in (679, 990]$ – KLIZA A NE KOTRLJA UZ STRMU RAVAN

$F \in [-\infty, 679]$ – NE KLIZA I NE KOTRLJA UZ STRMU RAVAN

$F \in (990, \infty]$ – KLIZA I KOTRLJA UZ STRMU RAVAN