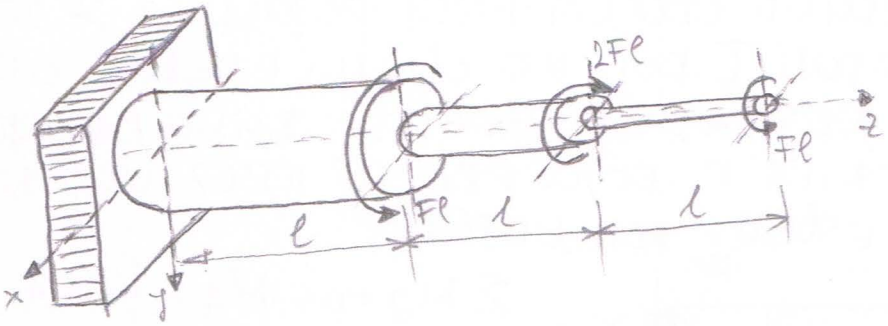
UNIVERZITET CRNE GORE
MAŠINSKI FAKULTET

OTPORNOST MATERIJALA I

PISANA PREDAVANJA

⑤ KOLOKVIJUM

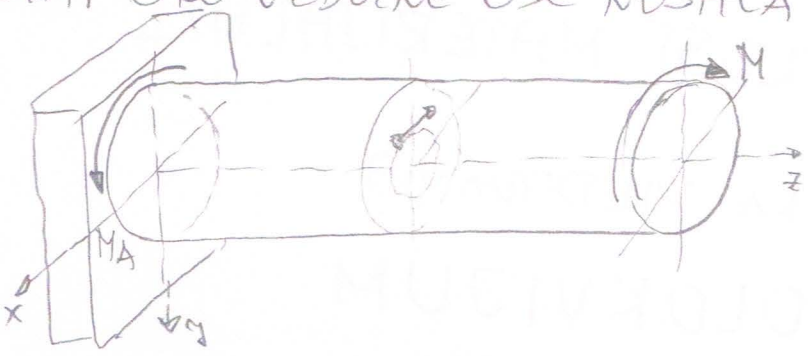


UVIJANJE
KOMBINOVANO NAPREZANJE: SAVIJANJE SA UVIJANJEM

PODGORICA, PLJEVLJA 2012 godi.

UVIJANJE GREDNOG NOSAČA KRUŽNOG I PRSTENASTOG POPREČNOG PRESJEKA

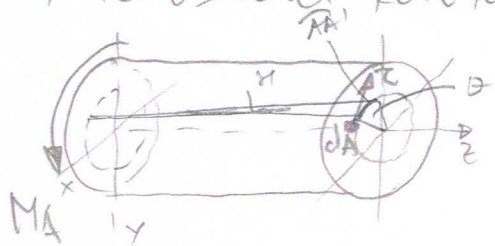
POD UVIJANJEM GREDNOG NOSAČA PODRAZUMIJEVA SE TAKVA VRSTA NJEGOVOG OPTEREĆIVANJA KADA JE NOSAČ OPTEREĆEN SA SPREGOM-MOMENTOM KOJI IMA OBRTNI EFEKAT OKO UZDUŽNE OSE NOSAČA (SL.)



U UKLJEŠTENJU SE POJAVLJUJE REAKTIVNI MOMENT KOJI IMA SUPROTNI OBRTNI EFEKAT U ODNOSU NA DATI MOMENT M.

$$\text{Iz } \Sigma M_z = 0: \quad M_A - M = 0 \Rightarrow M_A = M$$

U PROIZVOLJNO UOČENOM POPREČNOM PRESJEKU NOSAČA I U PROIZVOLJNO UOČENOJ TAČKI POJAVLJUJE SE TANGENCIJALNI NAPON τ KOJI IMA OBRTNI EFEKAT OKO UZDUŽNE OSE NOSAČA, ŠTO ZNAČI DA TANGIRA KRUŽNICU POLUPREČNIKA r KOJA PROLAZI KROZ UOČENU TAČKU (SL.) IZ USLOVA RAVNOTEŽE:



$$\Sigma M_z = 0: \quad M_A - \int \tau \, dA \cdot r = 0$$

Nalazimo napon τ.

NOSAČ OPTEREĆEN NA UVIJANJE DEFORMIŠE SE TAKO ŠTO SE POJEDINI POPREČNI PRESJECI ZAOKREĆU U ODNOSU NA POPREČNI PRESJEK KOJI PRIPADA UKLJEŠTENJU I KOJI JE NEPOKRETAN (SL.). TAJ UGAO ZAOKRETANJA SE ZOVE **UGAO UVIJANJA** I OZNAČAVAMO GA SA φ. UOČIĆEMO JEDNO PROIZVOLJNO VLAKNO IZ UNUTRAŠNJOSTI. PRIJE DEFORMISANJA NOSAČA TO VLAKNO JE PRAVA LINIJA PARALELNA SA OSOM z (SL.) NAKON DEFORMISANJA NOSAČA TO VLAKNO SE

ZAOKRUŽENJE ZA UGAO γ ZBOG TOGA ŠTO SE TAČKA A ZAROTIRA ZA UGAO θ JER JE SE KRAJNJI POPREČNI PRESJEK ZAOKRENUO ZA TAJ UGAO.

VAŽI: $l \cdot \gamma = r \cdot \theta = \widehat{AA'}$

DAKLE, $\gamma = \frac{r \cdot \theta}{l}$. POŠTO JE $\gamma = \frac{\tau}{G}$, TO IZ USLOVA RAVNOTEŽE: $M_A - \int_A \tau dA \cdot r = 0$ DOBIJAMO ($M_A = M$):

$$\int_A \gamma \cdot G \cdot r \cdot dA = \int_A \frac{r \cdot \theta G}{l} \cdot r \, dA = \frac{\theta G}{l} \int_A r^2 dA = M$$

odnosno

$$\frac{G\gamma}{r} \cdot I_0 = M$$

jer je $I_0 = \int_A r^2 dA$. Dakle,

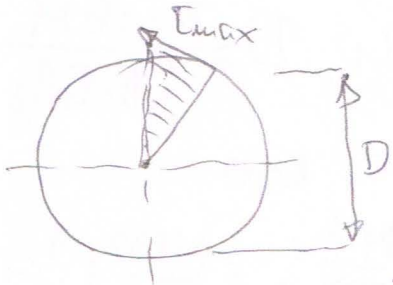
$$G \cdot \frac{\gamma}{r} = \frac{M}{I_0} \Rightarrow \frac{\tau}{r} = \frac{M}{I_0} \Rightarrow \boxed{\tau = \frac{M_t}{I_0} \cdot r} \quad (*)$$

Pošto je: $\theta = \frac{\gamma \cdot l}{r} = \frac{\tau}{G} \cdot \frac{l}{r} = \frac{Mr}{I_0 G} \cdot \frac{l}{r}$ dobijamo

$$\boxed{\theta = \frac{Ml}{GI_0}} \quad (**)$$

Najveći tangencijalni napon τ se pojavljuje u tačkama poprečnog presjeka koje imaju najveću udaljenost za r a to su tačke koje se nalaze na kružnici poluprečnika $r = \frac{D}{2}$. Najmanja udaljenost tangencijalnog napona je u centru kruga ($\tau = 0$ za $r = 0$).

$$\tau_{max} = \frac{M}{I_0} \cdot r_{max} = \frac{M}{I_0} \frac{D}{2}$$



Pošto je: $I_0 = \frac{\pi D^4}{32}$ to je:

$$\tau_{max} = \frac{M}{\frac{\pi D^3}{16}} = \frac{16M}{\pi D^3}$$

τ_{max} ne smije da pređe τ_{dop} pa je $\boxed{\tau_{max} = \frac{16M}{\pi D^3} \leq \tau_{dop}}$

ZADATAK: ZA GREDNI NOSAČ OPTEREĆEN NA UVIJANJE MOMENTOM M KAO NA SLICI

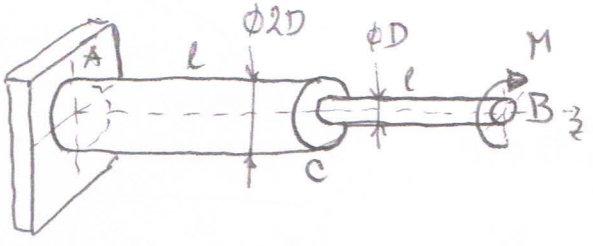
a) ODREDITI REAKTIVNI MOMENT I NACRTATI DIJAGRAM PROMJENE MOMENTA UVIJANJA.

b) IZRAČUNATI τ_{max} I DIMENZIONISATI NOSAČ

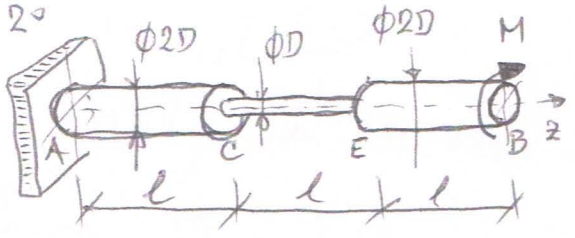
c) ODREDITI UGAO UVIJANJA PRESJEKA B U ODNOSU NA A TJ. $\theta_{B,A}$.

Dato je: F, ℓ, τ_{dz} i G.

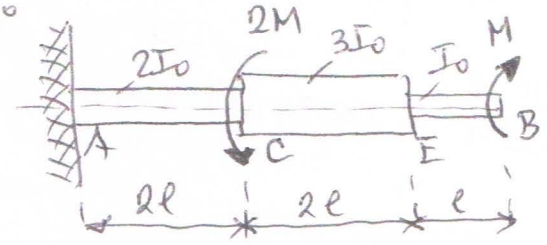
1°



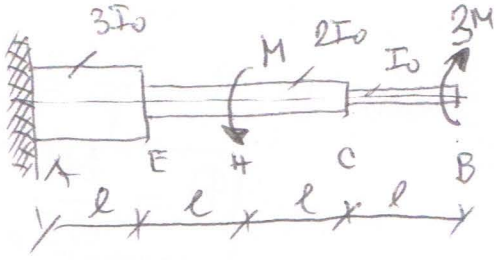
2°



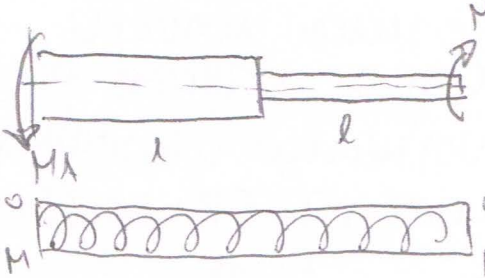
3°



4°



RJEŠENJE: **1°** Iz $\Sigma M_z = 0$ dobijamo $M_A - M = 0$ tj. $M_A = M$.



Dijagram promjene momenta uvijanja je dat na slici.

$$\tau_{max} = \frac{M}{I_0} \cdot \frac{D}{2} = \frac{M}{\frac{\pi D^4}{32}} \frac{D}{2} = \frac{16M}{\pi D^3}$$

Iz uslova

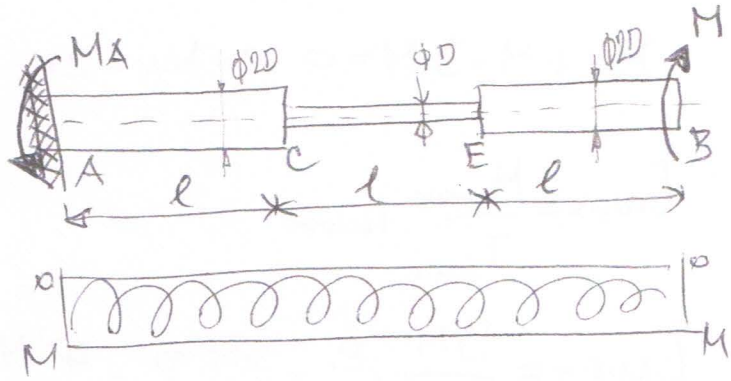
$$\tau_{max} = \frac{16M}{\pi D^3} \leq \tau_{dz}$$ dobijamo $$D \geq \sqrt[3]{\frac{16M}{\pi \cdot \tau_{dz}}}$$

Ugao uvijanja $\theta_{B,A}$ dobijamo kao

$$\theta_{B,A} = \theta_{C,A} + \theta_{B,C} = \frac{M \cdot \ell}{\frac{\pi (2D)^4}{32} \cdot G} + \frac{M \cdot \ell}{\frac{\pi D^4}{32} \cdot G} = \frac{M\ell 32}{\pi D^4 G}\left(\frac{1}{16} + 1\right)$$

$$\theta_{B,A} = \frac{34 M\ell}{\pi D^4 G}$$

2°



$M_A = M$

$$\tau_{max} = \frac{M}{\frac{\pi D^4}{32}} \cdot \frac{D}{2} = \frac{16M}{\pi D^3}$$

Iz USLOVA $\tau_{max} \le \tau_{dop}$

DOBIJAMO

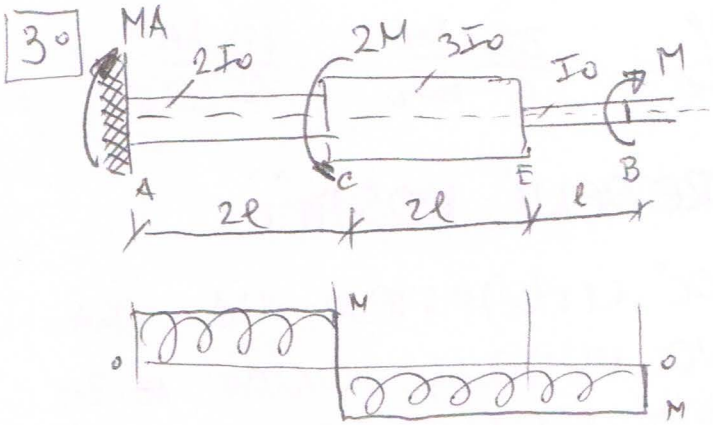
$$\frac{16M}{\pi D^3} \le \tau_{dop} \Rightarrow D \ge \sqrt[3]{\frac{16M}{\pi \tau_{dop}}}$$

Ugao uvijanja $\theta_{B,A}$ IP:

$$\theta_{B,A} = \theta_{C,A} + \theta_{E,C} + \theta_{B,E} = \frac{M\ell}{G \cdot \frac{\pi (2D)^4}{32}} + \frac{M \cdot \ell}{G \cdot \frac{\pi D^4}{32}} + \frac{M\ell}{G \cdot \frac{\pi (2D)^4}{32}}$$

$$\theta_{B,A} = \frac{32M\ell}{G \cdot \pi D^4}\left(\frac{1}{16} + 1 + \frac{1}{16}\right) = \frac{36 \cdot M \cdot \ell}{G \cdot \pi D^4}. \;//$$

3°



$M_A - 2M + M = 0$

$\Rightarrow M_A = M$

$M_{max} = M$

$$\tau_{max} = \frac{M_{max}}{I_{min}} \cdot r_{max}$$

$D_{EB} = D$

$I_{AC} = 2I_0 = 2I_{EB}$

$\frac{\pi D_{AC}^4}{32} = 2 \cdot \frac{\pi D^4}{32}$

$D_{AC} = \sqrt[4]{2} \cdot D$

$I_{CE} = 3I_0 = 3I_{EB}$

$\frac{\pi D_{CE}^4}{32} = 3 \cdot \frac{\pi D^4}{32}$

$D_{CE} = \sqrt[4]{3} D$

$\sqrt[4]{2} = 1{,}187$

$\sqrt[4]{3} = 1{,}315$

EB: $$\tau_{max} = \frac{M}{\frac{\pi D^4}{32}} \cdot \frac{D}{2} = \frac{16M}{\pi D^3} \le \tau_{dop} \rightarrow D \ge \sqrt[3]{\frac{16M}{\pi \tau_{dop}}}$$

AC: $\tau_{max} = \frac{M}{2 \cdot \frac{\pi d^4}{32}} \cdot \frac{\sqrt[4]{2}d}{2} = \frac{\sqrt[4]{2}}{2} \cdot \frac{16M}{\pi D^3} = 0{,}6 \cdot \frac{16M}{\pi D^3}$

CE $\tau_{max} = \ldots = \frac{\sqrt[4]{3}}{2} \cdot \frac{16M}{\pi D^3} = 0{,}66 \cdot \frac{16M}{\pi D^3}$

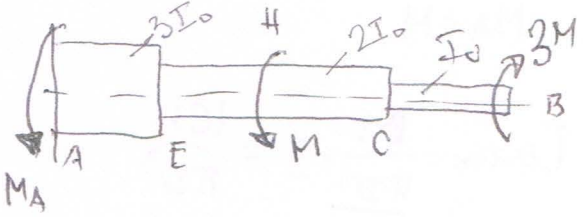
$\theta_{B,A} = \theta_{B,A}^{(M)} + \theta_{B,A}^{(2M)}$

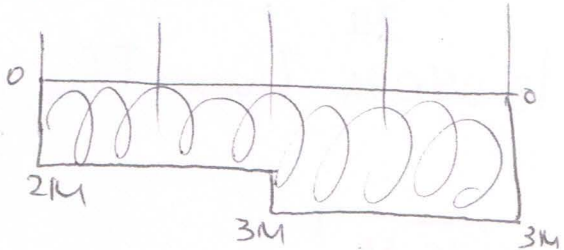
$$\theta_{B/A}^{(M)} = \theta_{CA}^{(M)} + \theta_{EC}^{(M)} + \theta_{BE}^{(M)} = \frac{M \cdot 2\ell}{G \cdot 2I_0} + \frac{M \cdot 2\ell}{G \cdot 3I_0} + \frac{M \cdot \ell}{G I_0} = \frac{M\ell}{G I_0}\left(1 + \frac{2}{3} + 1\right) = \frac{8M\ell}{3 G I_0}$$

$$\theta_{BA}^{(2M)} = \theta_{CA}^{(2M)} + \theta_{EC}^{(2M)} + \theta_{B,E}^{(2M)} = -\frac{2M \cdot 2\ell}{G \cdot 2I_0} + 0 + 0 = -\frac{2M\ell}{G I_0}$$

Dalje: $\theta_{B,A} = \frac{8M\ell}{3GI_0} - \frac{2M\ell}{GI_0} = \frac{2M\ell}{3GI_0}$

4º





$M_A + M - 3M = 0 \Rightarrow M_A = 2M$

$\tau_{max} = \frac{M_{max}}{I_{min}} \cdot r_{max}$

$\tau_{max} = \frac{3M}{I_0} \cdot \frac{D}{2} = \frac{3M}{\frac{\pi D^4}{32}} \cdot \frac{D}{2} = \frac{48M}{\pi D^3}$

$\tau_{max} \leq \tau_{adm} \Rightarrow D \geq \sqrt[3]{\frac{48 \cdot M}{\pi \cdot \tau_{adm}}}$

$\theta_{B,A} = \theta_{B,A}^{(M)} + \theta_{B,A}^{(3M)}$

$\theta_{B,A}^{(M)} = \frac{M \cdot \ell}{G \cdot 3I_0} + \frac{M \cdot \ell}{2GI_0} = \frac{5}{6} \frac{M\ell}{GI_0}$

$\theta_{B,A}^{(3M)} = -\frac{3M \cdot \ell}{3I_0 \cdot G} - \frac{3M \cdot \ell}{G \cdot 2I_0} - \frac{3M\ell}{G \cdot 2I_0} - \frac{3M\ell}{GI_0} = -\frac{M\ell}{GI_0}\left(1 + \frac{3}{2} + \frac{3}{2} + 3\right) = -\frac{7M\ell}{GI_0}$

Dando, $\theta_{B,A} = \frac{5}{6} \frac{M\ell}{GI_0} - \frac{7M\ell}{GI_0} = -\frac{38}{6} \frac{M\ell}{GI_0} = -\frac{19}{2} \frac{M\ell}{GI_0}$ //