

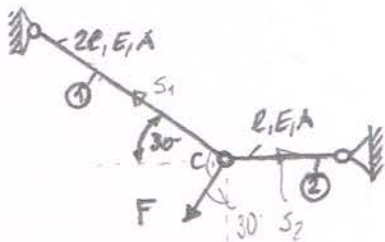
ZA SISTEM ŠTAPOVA POKAZATI NA SLICI NACI:

a) SILE U ŠTAPOVIMA I RUDŽENJA/SKRACENJA ŠTAPOVA

b) ODREDITI NOVI POLOŽAJ TAČKE C, TI HORIZONTALNO I VERTIKALNO POMIČANJE

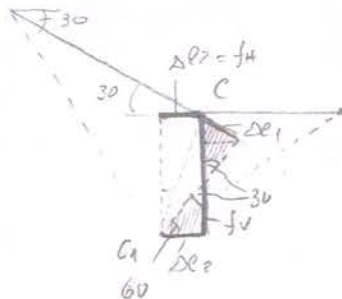
POZNATO JE: F, l, E, A

1°



$$\sum F_V = 0 \quad S_1 \cdot \frac{1}{2} - F \cdot \frac{\sqrt{3}}{2} = 0 \quad S_1 = F\sqrt{3}$$

$$\sum F_H = 0 \quad S_2 - F \cdot \frac{1}{2} - S_1 \cdot \frac{\sqrt{3}}{2} = 0 \quad S_2 = \frac{F}{2} + F\sqrt{3} \cdot \frac{\sqrt{3}}{2} = 2F = S_2$$



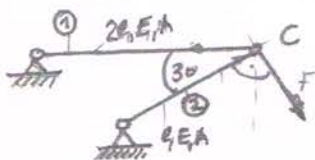
$$\Delta l_1 = \frac{S_1 \cdot l}{EA} = \frac{2F\sqrt{3}l}{EA}$$

$$\Delta l_2 = \frac{S_2 \cdot l}{EA} = \frac{2Fl}{EA}$$

$$f_H = \Delta l_2$$

$$f_V = \frac{\Delta l_1}{\sin 30^\circ} + \frac{\Delta l_2}{\tan 30^\circ}$$

2°



$$\sum F_V = 0 \quad S_2 \cdot \frac{1}{2} - F \cdot \frac{\sqrt{3}}{2} = 0 \quad S_2 = F\sqrt{3}$$

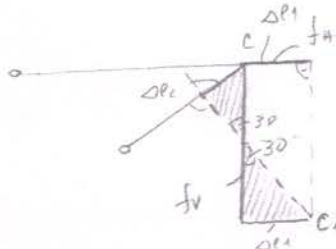
$$\sum F_H = 0 \quad -S_1 + S_2 \cdot \frac{\sqrt{3}}{2} + \frac{F}{2} = 0 \quad S_1 = S_2 \cdot \frac{\sqrt{3}}{2} + \frac{F}{2} = 2F = S_1$$

$$\Delta l_1 = \frac{2F \cdot 2l}{EA} = \frac{4Fl}{EA}$$

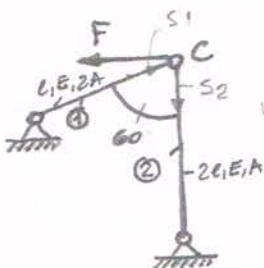
$$\Delta l_2 = \frac{F\sqrt{3} \cdot l}{EA}$$

$$f_H = \Delta l_1$$

$$f_V = 2\Delta l_2 + \frac{\Delta l_1}{\tan 30^\circ}$$



3°



$$\sum F_H = 0 \quad S_1 \cdot \frac{\sqrt{3}}{2} - F = 0 \quad S_1 = \frac{2F}{\sqrt{3}} = \frac{2F\sqrt{3}}{3}$$

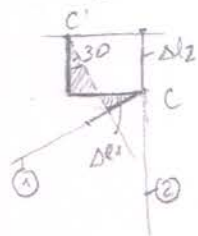
$$\sum F_V = 0 \quad S_1 \cdot \frac{1}{2} - S_2 = 0 \quad S_2 = \frac{S_1}{2} = \frac{2F\sqrt{3}}{3} \cdot \frac{1}{2} = \frac{F\sqrt{3}}{3}$$

$$\Delta l_1 = \frac{2F\sqrt{3} \cdot l}{3EA}$$

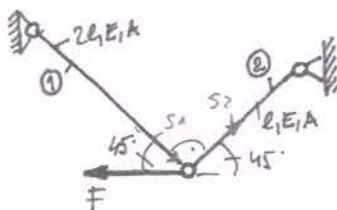
$$\Delta l_2 = \frac{F\sqrt{3} \cdot 2l}{3EA} \quad \Delta l_2 = \frac{2F\sqrt{3}l}{3EA}$$

$$f_V = \Delta l_2$$

$$f_H = \Delta l_2 \cdot \tan 30^\circ + \frac{\Delta l_1}{\cos 30^\circ}$$



4°

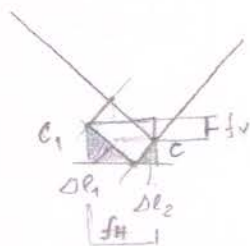


$$\sum F_V = 0 \quad S_1 \cdot \frac{\sqrt{2}}{2} + S_2 \cdot \frac{\sqrt{2}}{2} = 0 \quad S_1 = S_2$$

$$\sum F_H = 0 \quad -F + S_1 \cdot \frac{\sqrt{2}}{2} + S_2 \cdot \frac{\sqrt{2}}{2} = 0 \quad S_1 = S_2 = \frac{F}{\sqrt{2}} = \frac{F\sqrt{2}}{2}$$

$$\Delta l_1 = \frac{F\sqrt{2} \cdot 2l}{2EA} = \frac{F\sqrt{2}l}{EA}$$

$$\Delta l_2 = \frac{F\sqrt{2} \cdot l}{2EA}$$



$$f_H = \frac{\Delta l_1}{\sqrt{2}} + \frac{\Delta l_2}{\sqrt{2}}$$

$$f_V = \frac{\Delta l_1}{\sqrt{2}} - \frac{\Delta l_2}{\sqrt{2}}$$

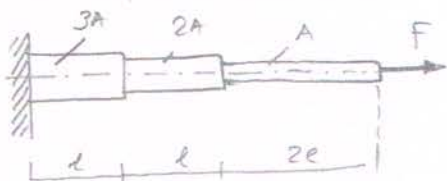
2A GREDNI NOSAČ STEPENASTOG POPREČNOG PRESJERA, KOJI JE OPTEREĆEN KAO NA SLICI ODREDITI:

a) NAPONE U POJEDINIM DJELOVIMA NOSAČA I MAKSIMALNI NAPON;

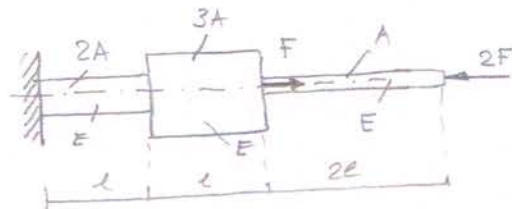
b) UKUPNO IZDUŽENJE NOSAČA

DATO JE: F, l, A, E

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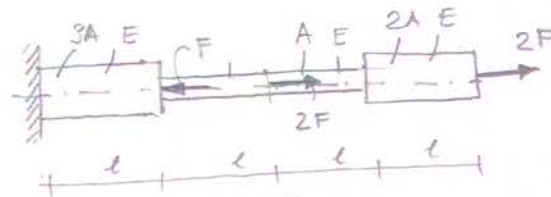
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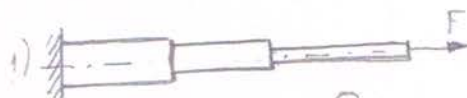
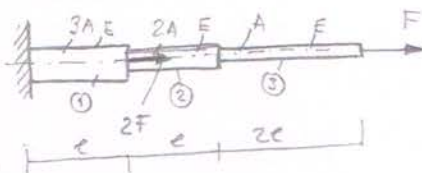
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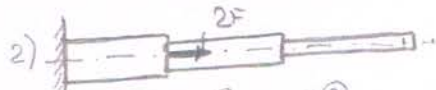
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$$\sigma_2^{(1)} = \frac{F}{3A}, \sigma_2^{(2)} = \frac{F}{2A}, \sigma_2^{(3)} = \frac{F}{A}$$

$$\Delta l^{(1)} = \frac{Fl}{E3A}, \Delta l^{(2)} = \frac{Fl}{E2A}, \Delta l^{(3)} = \frac{F \cdot 2l}{EA}$$

$$\Delta l_{\text{uk}} = \frac{Fl}{E3A} + \frac{Fl}{E2A} + \frac{2Fl}{EA} = \frac{17Fl}{6EA}$$



$$\sigma_2^{(1)} = \frac{2F}{3A}, \sigma_2^{(2)} = 0, \sigma_2^{(3)} = 0$$

$$\Delta l^{(1)} = \frac{2Fl}{E3A}, \Delta l^{(2)} = 0, \Delta l^{(3)} = 0$$

$$\Delta l_{\text{uk}} = \frac{2Fl}{E3A}$$

$$a) \left. \begin{aligned} \sigma_2^{(1)} &= \frac{F}{3A} + \frac{2F}{3A} = \frac{F}{A} \\ \sigma_2^{(2)} &= \frac{F}{2A} + 0 = \frac{F}{2A} \\ \sigma_2^{(3)} &= \frac{F}{A} + 0 = \frac{F}{A} \end{aligned} \right\} \Rightarrow \sigma_{\text{max}} = \frac{F}{A}$$

$$b) \Delta l_{\text{uk}} = \Delta l_{\text{uk}}^{(1)} + \Delta l_{\text{uk}}^{(2)}$$

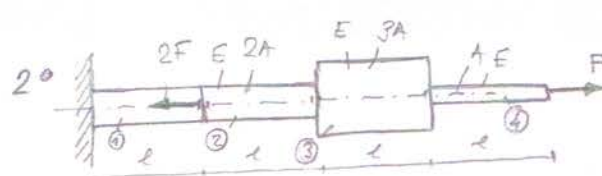
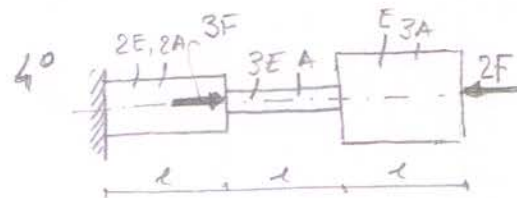
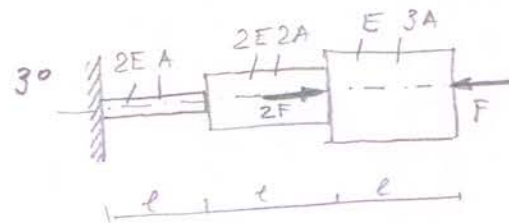
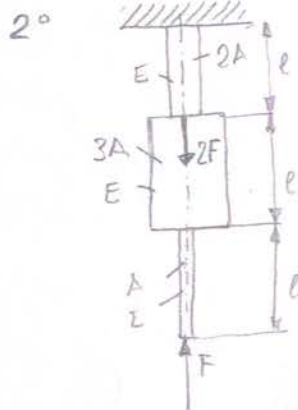
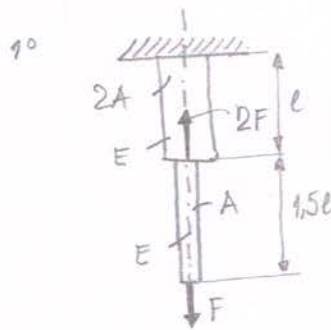
$$\Delta l_{\text{uk}} = \frac{17Fl}{6EA} + \frac{2Fl}{3EA} = \frac{21Fl}{6EA}$$

ZADATAK

ZA GREDNI NOSAČ STEPENASTOG POPREČNOG PRESJERA, KOJI JE OPTEREĆEN KAO NA SLICI, ODREDITI:

a) NAPONE U POJEDINIM DJELOVIMA NOSAČA I MAKSIMALNI NAPON

b) UKUPNO KUDUĆENJE NOSAČA



$$a) \sigma_2^{(1)} = \frac{F}{2A} - \frac{2F}{2A} = -\frac{F}{2A}; \sigma_2^{(2)} = \frac{F}{2A}; \sigma_2^{(3)} = \frac{F}{3A}; \sigma_2^{(4)} = \frac{F}{A}$$

$$\sigma_{2\text{max}} = \frac{F}{A} \text{ NA DIOU 4}$$

$$b) \Delta l_{\text{uk}} = \Delta l_{\text{uk}}^{(1)} + \Delta l_{\text{uk}}^{(2)}$$

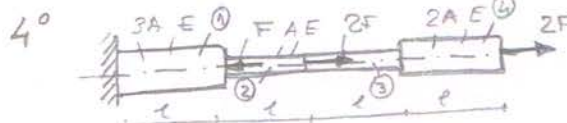
$$\Delta l_{\text{uk}}^{(1)} = \frac{F \cdot 2l}{E \cdot 2A} + \frac{F \cdot l}{E \cdot 3A} + \frac{F \cdot l}{E \cdot A} = \frac{Fl}{EA} \left(2 + \frac{1}{3} \right) = \frac{5Fl}{3EA}$$

$$\Delta l_{\text{uk}}^{(2)} = -\frac{2F \cdot l}{E \cdot 2A} = -\frac{Fl}{EA}$$

$$\Delta l_{\text{uk}} = \frac{5Fl}{3EA} - \frac{Fl}{EA} = \frac{Fl}{EA} \left(\frac{5}{3} - 1 \right) = \frac{2Fl}{3EA}$$

$$30 \text{ E: } \sigma_2^{(1)} = -\frac{F}{A}; \sigma_2^{(2)} = -\frac{F}{3A}; \sigma_2^{(3)} = -\frac{2F}{A}; \sigma_{2\text{max}} = -\frac{2F}{A}$$

$$\Delta l_{\text{uk}}^{(1)} = -\frac{17Fl}{3EA}; \Delta l_{\text{uk}}^{(2)} = \frac{5Fl}{6EA}; \Delta l_{\text{uk}} = -\frac{29Fl}{6EA}$$



$$a) \sigma_2^{(1)} = \frac{F}{3A} (2 + 2 - F) = \frac{F}{A}; \sigma_2^{(2)} = \frac{F}{A} (2 + 2) = \frac{4F}{A}$$

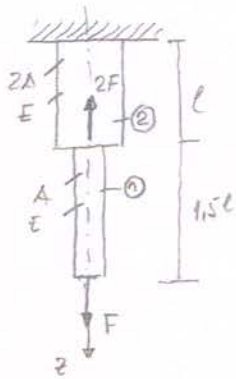
$$\sigma_2^{(3)} = \frac{2F}{A}; \sigma_2^{(4)} = \frac{2F}{2A} = \frac{F}{A} \quad \sigma_{2\text{max}} = \sigma_2^{(2)} = \frac{4F}{A}$$

$$b) \Delta l_{\text{uk}} = \Delta l_{\text{uk}}^{(1)} + \Delta l_{\text{uk}}^{(2)} + \Delta l_{\text{uk}}^{(3)}$$

$$\Delta l_{\text{uk}} = \frac{2F \cdot l}{E \cdot 3A} + \frac{2F \cdot 2l}{E \cdot A} + \frac{2F \cdot l}{E \cdot 2A} + \frac{2F \cdot l}{E \cdot 3A} + \frac{2F \cdot l}{EA} - \frac{Fl}{E \cdot 3A}$$

$$\Delta l_{\text{uk}} = \frac{Fl}{EA} \left(\frac{2}{3} + 4 + 1 + \frac{2}{3} + 2 - \frac{1}{3} \right) = \frac{8Fl}{EA}$$

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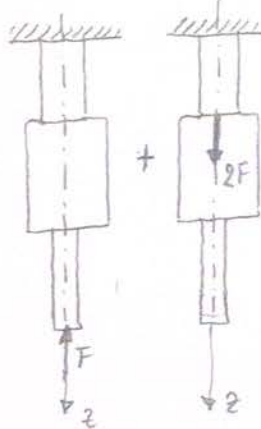
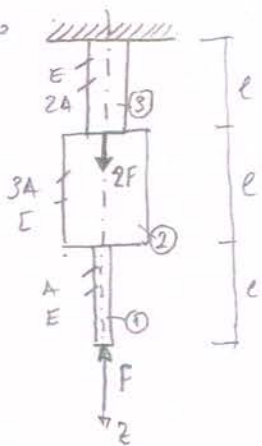


$$a) \left. \begin{aligned} \delta_2^{(1)} &= \delta_2^F + \delta_2^{2F} = \frac{F}{A} + 0 = \frac{F}{A} \\ \delta_2^{(2)} &= \delta_2^F + \delta_2^{2F} = \frac{F}{2A} - \frac{2F}{2A} = -\frac{F}{A} \end{aligned} \right\} \delta_{\max} = \delta_2^{(1)} = \delta_2^{(2)} = \frac{F}{A}$$

$$b) \Delta l_{\text{tot}} = \Delta l_{\text{tot}}^{(1)} + \Delta l_{\text{tot}}^{(2)} = \Delta l_{\text{tot}}^{(1)F} + \Delta l_{\text{tot}}^{(1)2F} + \Delta l_{\text{tot}}^{(2)F} + \Delta l_{\text{tot}}^{(2)2F}$$

$$\Delta l_{\text{tot}} = \frac{1.3l}{2EA} + 0 + \frac{Fl}{EA} - \frac{2Fl}{EA} = \frac{Fl}{EA} \left(\frac{3}{2} + \frac{1}{2} - 1 \right) = \frac{Fl}{EA}$$

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$$a) \left. \begin{aligned} \delta_2^{(1)F} &= \frac{F}{A} ; \delta_2^{(2)F} = \frac{2F}{3A} ; \delta_2^{(3)F} = \frac{F}{2A} \\ \delta_2^{(1)2F} &= 0 ; \delta_2^{(2)2F} = 0 ; \delta_2^{(3)2F} = \frac{2F}{2A} = \frac{F}{A} \end{aligned} \right\}$$

$$\delta_2^{(1)} = \frac{F}{A} + 0 = \frac{F}{A} ; \delta_2^{(2)} = \frac{2F}{3A} + 0 = \frac{2F}{3A}$$

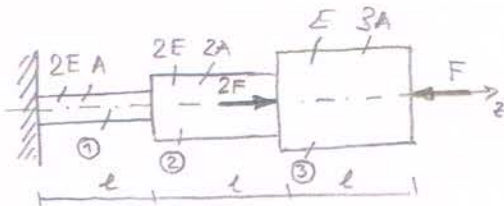
$$\delta_2^{(3)} = \frac{F}{2A} + \frac{2F}{2A} = \frac{3F}{2A} \quad \delta_{\max} = \delta_2^{(3)} = \frac{3F}{2A}$$

$$b) \Delta l^{(1)F} = \frac{Fl}{EA} \quad \Delta l^{(2)F} = \frac{2Fl}{E3A} \quad \Delta l^{(3)F} = \frac{Fl}{E2A}$$

$$\Delta l^{(1)2F} = 0 \quad \Delta l^{(2)2F} = 0 \quad \Delta l^{(3)2F} = \frac{2Fl}{E2A}$$

$$\Delta l_{\text{tot}} = -\frac{Fl}{EA} - \frac{Fl}{3EA} - \frac{Fl}{2EA} + 0 + 0 + \frac{2Fl}{2EA} = \frac{Fl}{EA} \left(-1 - \frac{1}{3} - \frac{1}{2} + 1 \right) = -\frac{5Fl}{6EA}$$

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$$a) \delta_2^{(1)} = \frac{-F}{A} + \frac{2F}{2A} = 0$$

$$\delta_2^{(2)} = \frac{-F}{2A} + \frac{2F}{2A} = \frac{F}{2A}$$

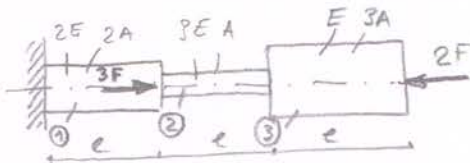
$$\delta_2^{(3)} = \frac{-F}{3A}$$

$$\delta_{\max} = \delta_2^{(2)} = \frac{F}{2A}$$

$$b) \Delta l^{(1)} = \frac{-F \cdot l}{2E \cdot A} + \frac{2F \cdot l}{2EA} = \frac{Fl}{2EA} ; \Delta l^{(2)} = \frac{-Fl}{2E \cdot 2A} + \frac{2Fl}{2E \cdot 2A} = \frac{Fl}{4EA}$$

$$\Delta l^{(3)} = \frac{-Fl}{E3A} \Rightarrow \Delta l_{\text{tot}} = \Delta l^{(1)} + \Delta l^{(2)} + \Delta l^{(3)} = \frac{Fl}{EA} \left(\frac{1}{2} + \frac{1}{4} - \frac{1}{3} \right) = \frac{7Fl}{12EA}$$

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$$a) \delta_2^{(1)} = \frac{-2F}{2A} + \frac{3F}{2A} = \frac{F}{2A}$$

$$\delta_2^{(2)} = \frac{-2F}{A}$$

$$\delta_2^{(3)} = \frac{2F}{3A}$$

$$\Rightarrow \delta_{\max} = \delta_2^{(2)} = -\frac{2F}{A}$$

$$b) \Delta l^{(1)} = \frac{-2Fl}{2E \cdot 2A} + \frac{3Fl}{2E \cdot 2A} = \frac{Fl}{4EA}$$

$$\Delta l^{(2)} = \frac{-2Fl}{3EA}$$

$$\Delta l^{(3)} = \frac{2Fl}{E \cdot 3A}$$

$$\Rightarrow \Delta l_{\text{tot}} = \Delta l^{(1)} + \Delta l^{(2)} + \Delta l^{(3)}$$

$$\Delta l_{\text{tot}} = \frac{Fl}{EA} \left(\frac{1}{4} - \frac{2}{3} + \frac{2}{3} \right) = \frac{Fl}{4EA}$$