1. Ravno stanje napona u tački je zadato pomoću tenzora napona (xy ravan).

[a] Prikazati okolinu tačke

[b] Sračunati glavne napone i glavne pravce i prikazati okolinu tačke u sistemu glavnih osa;

[c] Sračunati max smičući napon i odgovarajući normalni napon (σ_{odg}), prikazati okolinu tačke;

[d] Morov krug napona

[e] odrediti tenzor deformacije ako je $E = 200\,GPa$ i $\nu = 0{,}2$; prikazati deformisanu okolinu tačke

[f] Sračunati glavne dilatacije i prikazati deformisanu okolinu tačke

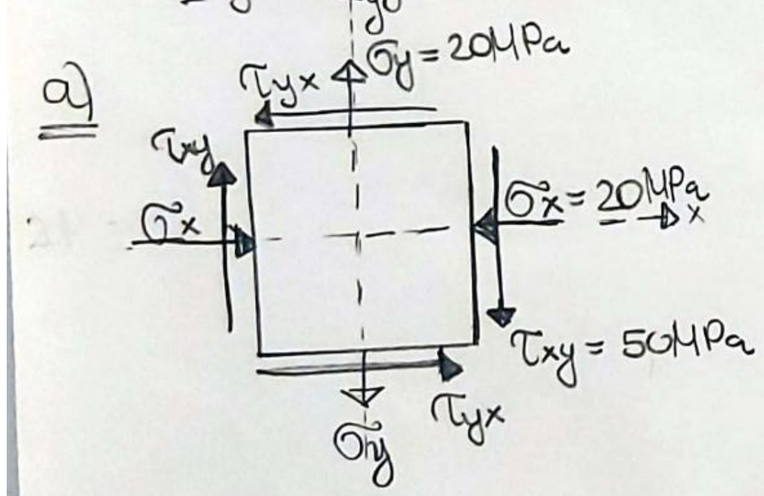
[g] Sračunati max. klizanje i prikazati pravce iz smera kojih se dešava

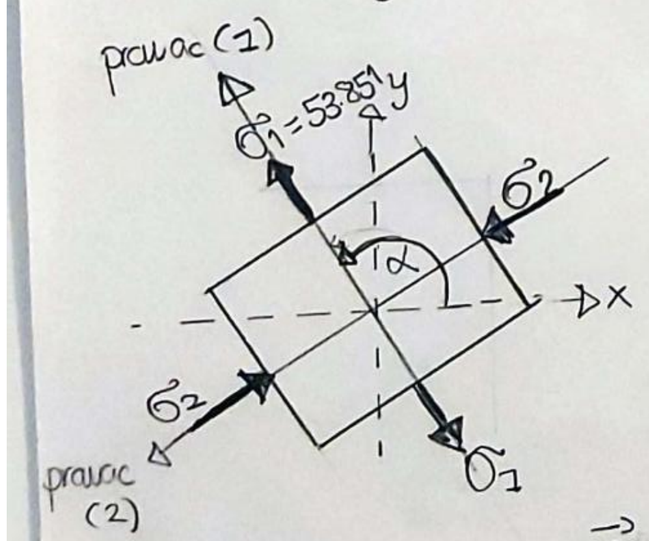
[h] Nacrtati Morov krug deformacije.

$$S = \begin{bmatrix} -20 & -50 \\ -50 & 20 \end{bmatrix} MPa$$

$$T_S = \begin{bmatrix} \sigma_x & \tau_{xy} \\ \tau_{yx} & \sigma_y \end{bmatrix} \rightarrow \text{tenzor napona}$$

a)





b) $\sigma_{1/2} = 0{,}5(\sigma_x + \sigma_y) \pm 0{,}5\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}$

$\sigma_{1/2} = 0{,}5(-20+20) \pm 0{,}5\cdot\sqrt{(-20-20)^2 + 4\cdot(-50)^2}$

$= 0 \pm 53{,}851$

$\begin{cases} \sigma_1 = 53{,}851\,MPa \\ \sigma_2 = -53{,}851\,MPa \end{cases} \quad \tau_{12} = 0$

Pravci glavnih napona:

$$tg2\alpha = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{2\cdot(-50)}{-20-20} = \frac{-100}{-40}$$

$$tg2\alpha = \frac{10}{4}$$

$\sigma_x - \sigma_y < 0 \Rightarrow \alpha = 0.5\,arctg\left(\frac{10}{4}\right) + 90°$

$\boxed{\alpha = 124.1°}$

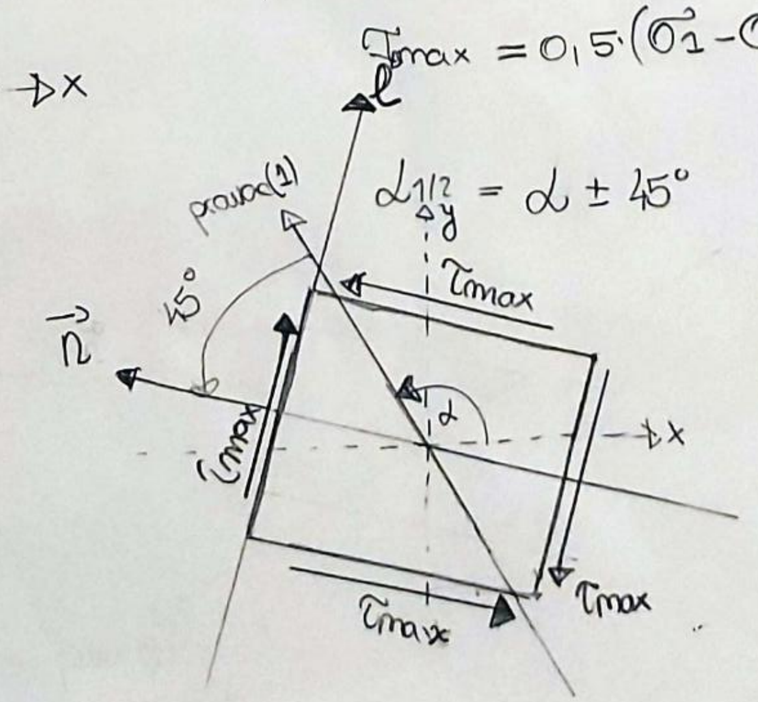
c) maksimalni smičući napon:

$\tau_{max} = 0{,}5\cdot(\sigma_1 - \sigma_2) = 0{,}5\cdot(53{,}851 + 53{,}851)$

$= 53{,}851\,MPa$

$\alpha_{1/2} = \alpha \pm 45°$

$\sigma_{odg} = 0{,}5\cdot(\sigma_1 + \sigma_2) = 0{,}5\cdot(53{,}851 - 53{,}851) = 0$

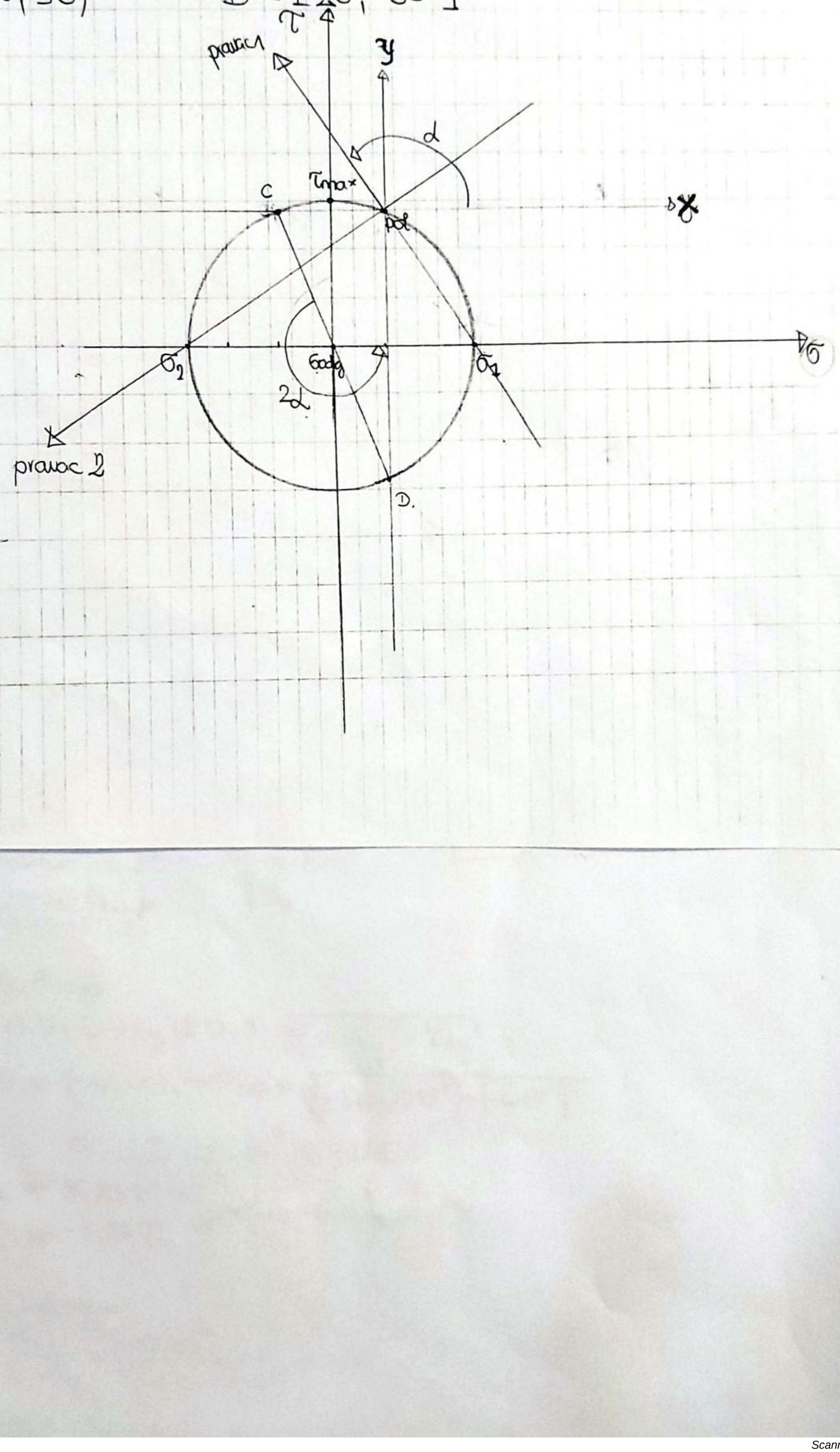


d) Mohrov krug naprezanja:

$C = (\sigma_x, -\tau_{xy})$ $\quad D = [\sigma_y, \tau_{xy}]$

$C = (-20, 50)$ $\quad D = [20, -50]$

1 cm = 20 MPa



e) $E = 200\,GPa = 200 \cdot 10^3\,MPa$

$\nu = 0{,}2$

$G = \frac{200\,GPa}{2\cdot(1+0{,}2)} = 83.3\,GPa$ → modul klizanja

$\varepsilon_x = [\sigma_x - \nu(\sigma_y + \sigma_z)]/E = \frac{1}{200\cdot10^3}\cdot[-20 - 0{,}2\cdot(20+0)] = \frac{-24}{200\cdot10^3} = -1.2\cdot10^{-4}$

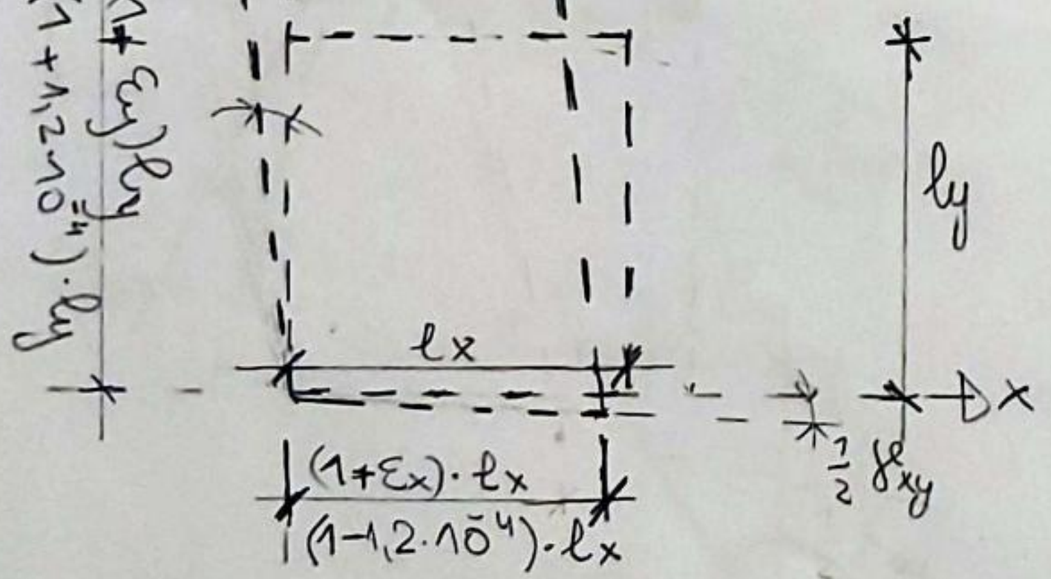
$\varepsilon_y = [\sigma_y - \nu\cdot(\sigma_x + \sigma_z)]/E = \frac{1}{200\cdot10^3}[20 - 0{,}2\cdot(-20+0)] = \frac{24}{200\cdot10^3} = 1{,}2\cdot10^{-4}$

$\varepsilon_z = [\sigma_z - \nu\cdot(\sigma_x + \sigma_y)]/E = \frac{1}{200\cdot10^3}\cdot[0 - 0{,}2\cdot(-20+20)] = 0$

$\gamma_{xy} = \frac{\tau_{xy}}{G} = \frac{-5^0}{83.3} = -6\cdot10^{-4}$

$\gamma_{yz} = \frac{\tau_{yz}}{G} = 0,\ \gamma_{xz} = \frac{\tau_{xz}}{G} = 0$

$$[D] = \begin{bmatrix} \varepsilon_x & \frac{1}{2}\gamma_{xy} & \frac{1}{2}\gamma_{xz} \\ \frac{1}{2}\gamma_{yx} & \varepsilon_y & \frac{1}{2}\gamma_{yz} \\ \frac{1}{2}\gamma_{zx} & \frac{1}{2}\gamma_{zy} & \varepsilon_z \end{bmatrix} = \begin{bmatrix} -1{,}2 & -\frac{1}{2}\cdot6 & 0 \\ -\frac{1}{2}\cdot6 & 1{,}2 & 0 \\ 0 & 0 & 0 \end{bmatrix}\cdot10^{-4} \qquad [D] = \begin{bmatrix} -1{,}2 & -\frac{1}{2}\cdot6 \\ -\frac{1}{2}\cdot6 & 1{,}2 \end{bmatrix}\cdot10^{-4}$$



$\varepsilon_x = \frac{\Delta l_x}{l_x}$, $\Delta l_x = \varepsilon_x\cdot l_x$

$l_x' = l_x + \Delta l_x = l_x + \varepsilon_x\cdot l_x \Rightarrow$

$\boxed{l_x' = l_x\cdot(1+\varepsilon_x)}$ *

f) Glavne dilatacije:

$\varepsilon_{1/2} = 0{,}5(\varepsilon_x + \varepsilon_y) \pm 0{,}5\cdot\sqrt{(\varepsilon_x - \varepsilon_y)^2 + \gamma_{xy}^2}$

$= 0{,}5\cdot(-1{,}2+1{,}2)\cdot10^{-4} \pm 0{,}5\cdot\sqrt{(-1{,}2-1{,}2)^2\cdot10^{-4} + (-6\cdot10^{-4})^2}$

$= 0 \pm 0{,}5\cdot10^{-4}\cdot6{,}4622$

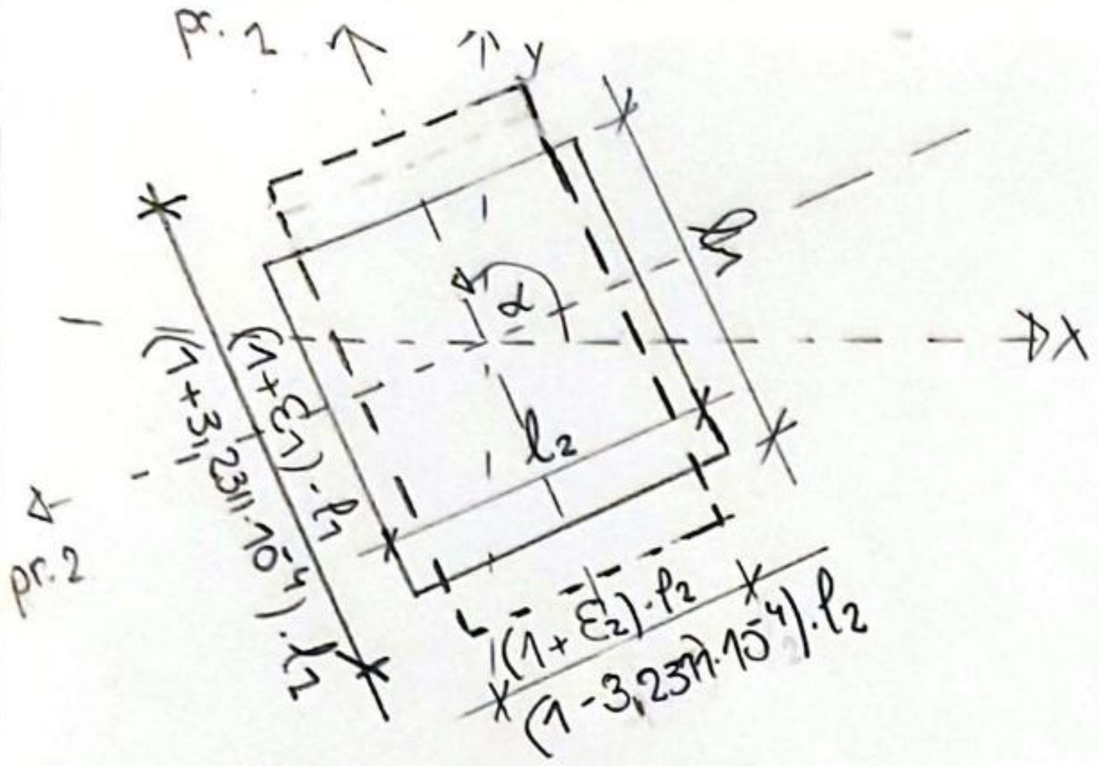
$\varepsilon_1 = 3{,}2311\cdot10^{-4}$

$\varepsilon_2 = -3{,}2311\cdot10^{-4}$ $\qquad \gamma_{12} = 0$

Pravci gl. dilatacija:

$\operatorname{tg}2\alpha = \frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y} = \frac{-6\cdot10^{-4}}{(-1{,}2-1{,}2)\cdot10^{-4}} = 2{,}5$

za $(\varepsilon_x - \varepsilon_y) < 0$, $\alpha = 0{,}5\operatorname{arctg}(2{,}5) + 90^0$, $\boxed{\alpha = 124{,}1^0}$

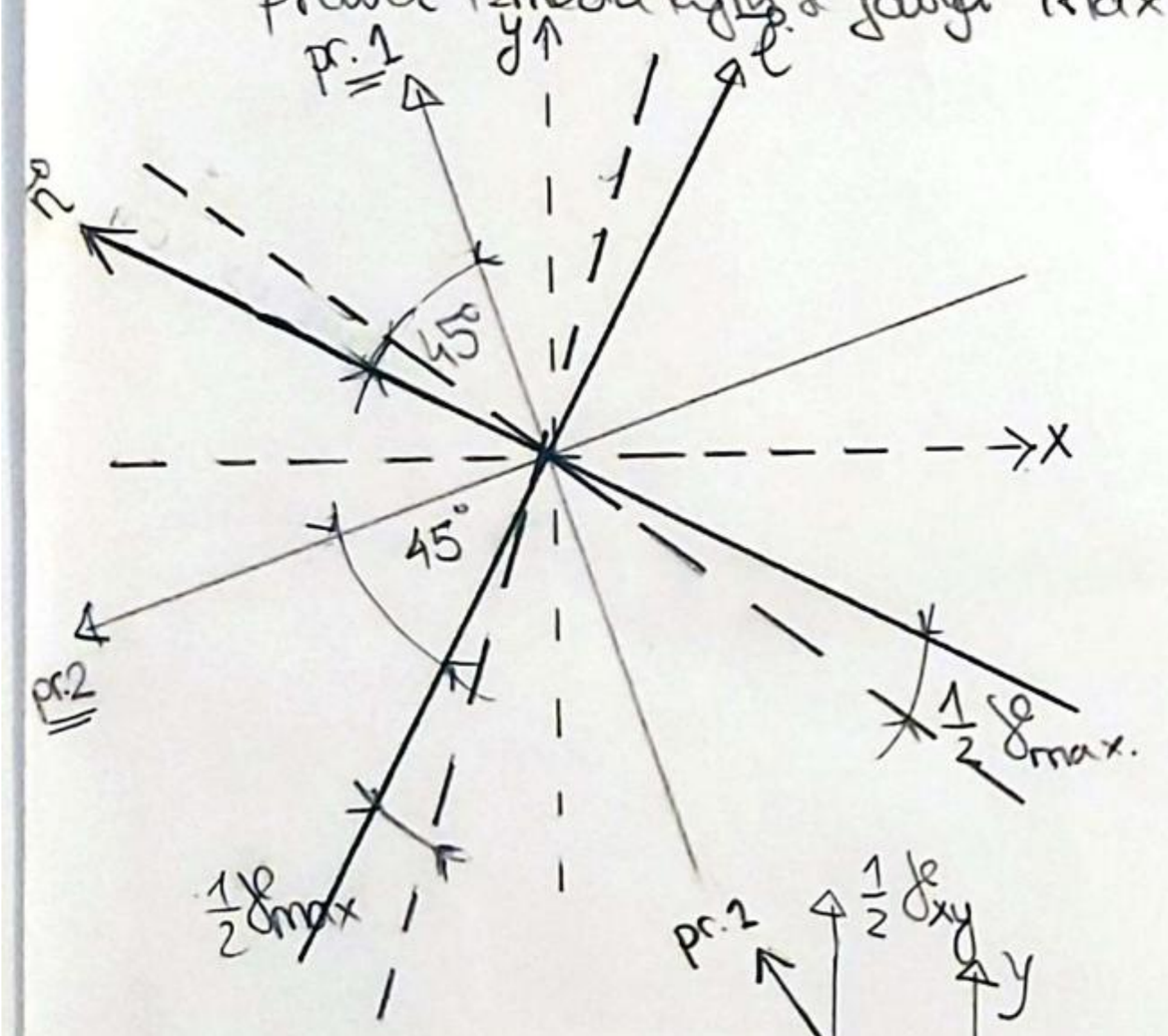


g) Max klizanje:

$$\frac{1}{2}\gamma_{max} = \frac{1}{2}(\varepsilon_1 - \varepsilon_2) = 0{,}5\cdot(3{,}2311\cdot10^{-4} + 3{,}2311\cdot10^{-4}) = 3{,}2311\cdot10^{-4}$$

$$\varepsilon_{odg} = \frac{1}{2}(\varepsilon_1 + \varepsilon_2) = \frac{1}{2}\cdot 0 = 0$$

– pravci izmedu kojih djeluju max. klizanje su pod uglom od 45° u odnosu na prave 1 i 2;



h) Morov krug deformacije:

$$[C] = [\varepsilon_{x_i} - \tfrac{1}{2}\gamma_{xy}]$$

$$[C] = [-1{,}2_i + 3]\cdot10^{-4}$$

$$[D] = [\varepsilon_y, \tfrac{1}{2}\gamma_{xy}]$$

$$[D] = [1{,}2_i - 3]\cdot10^{-4}$$

$$1\,cm = 1\cdot10^{-4}$$