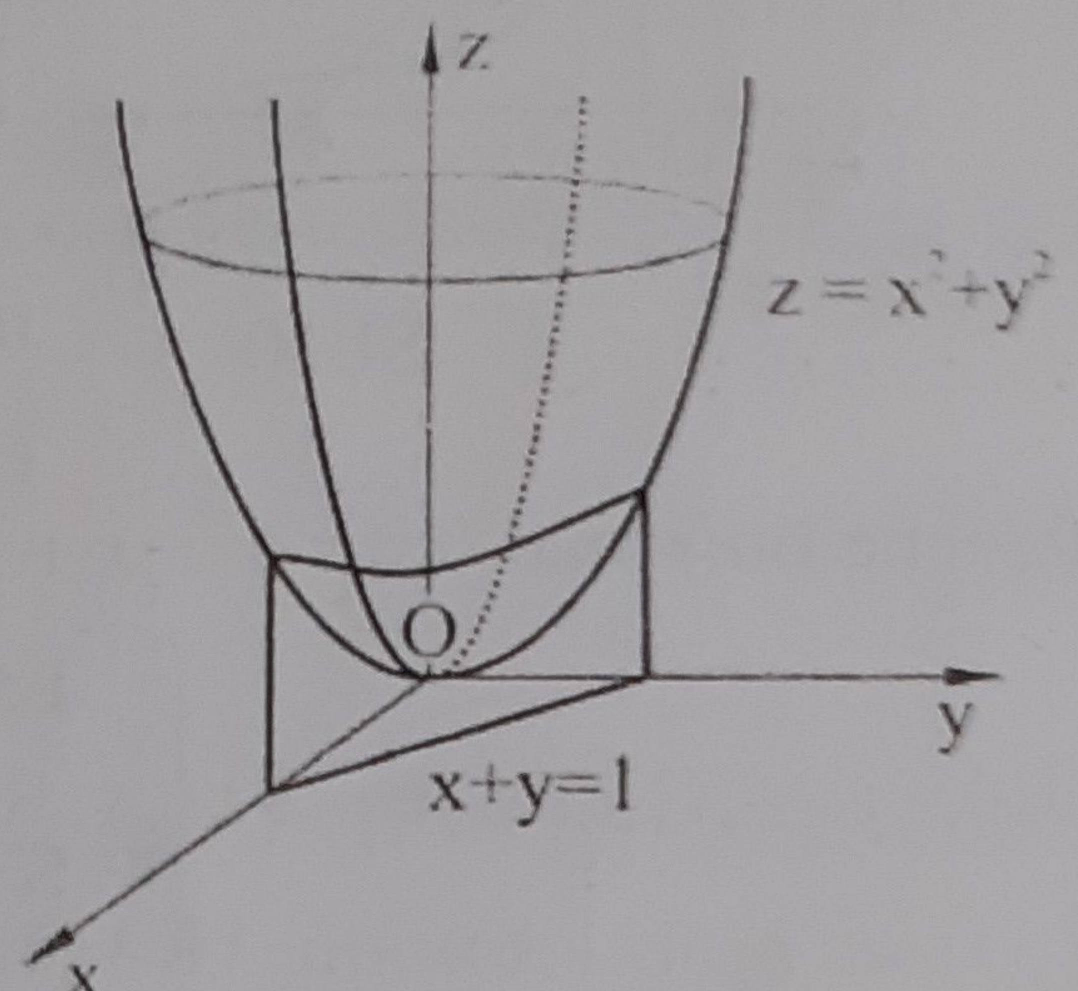
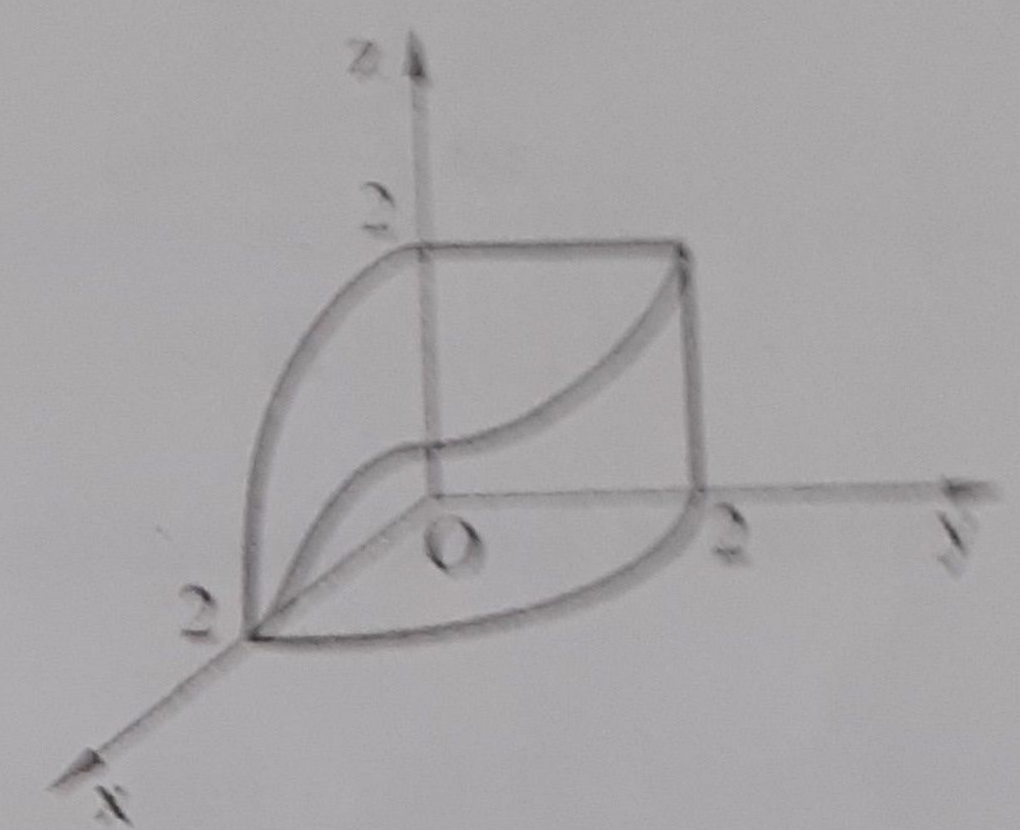
$S:\begin{cases} 0\le x\le 1 \\ 0\le y\le 1-x \end{cases}$ ili $S:\begin{cases} 0\le y\le 1 \\ 0\le x\le 1-y \end{cases}$, to je $V=\int_0^1 dy\int_0^{1-y}(x^2+y^2)dx=$

$\int_0^1\left(\frac{(1-y)^3}{3}+y^2(1-y)\right)dy=\frac{1}{6}$ (kubnih jedinica).



Sl 7



Sl 8

Primjer 5. Naći zapreminu V tijela ograničenog površima $x^2+y^2=4$ i $x^2+z^2=4$.

Uočimo da se osmina datog tijela (Sl 8) nalazi u I oktantu. Tada je

$$\frac{1}{8}V=\iint_S\sqrt{4-x^2}\,dxdy=\int_0^2 dx\int_0^{\sqrt{4-x^2}}\sqrt{4-x^2}\,dy=\int_0^2(4-x^2)dx=\frac{16}{3}\quad\text{(kubnih jed.)},$$

odnosno $V=\frac{128}{3}$ (kubnih jed.).

Primjer 6. Naći površinu figure S ograničene linijama $y^2=4ax+4a^2$ i $x+y=2a$ $(a>0)$.

Zapišimo oblast S u obliku $\begin{cases} -6a\le y\le 2a \\ \frac{y^2-4a^2}{4a}\le x\le 2a-y \end{cases}$ (Sl 9). Kako je $P_S=\iint_S dxdy$, to je

$$P_S=\int_{-6a}^{2a}dy\int_{\frac{y^2-4a^2}{4a}}^{2a-y}dx=\int_{-6a}^{2a}\left(2a-y-\frac{y^2-4a^2}{4a}\right)dy=\ldots=\frac{40a^2}{3}\quad\text{(kvadratnih jedinica).}$$