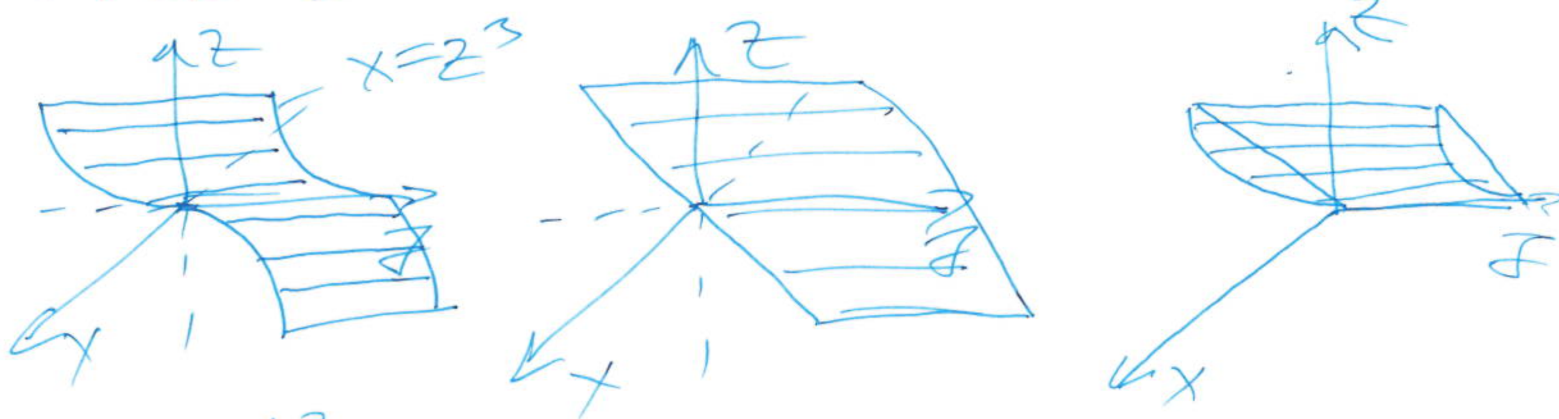
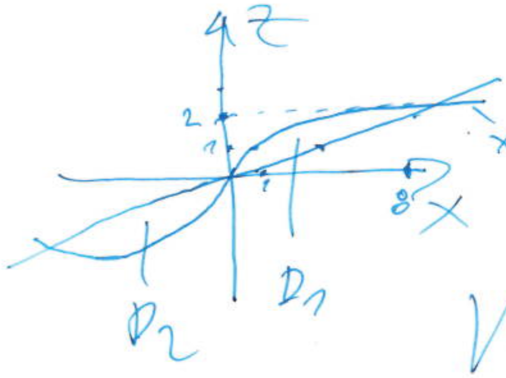
NAĆI ZAPREMINU TIJELA OGRANIČENOG POVRŠIMA $S_1: y=2xz$; $S_2: y=0$; $S_3: x=4z$; $S_4: x=z^3$

RJEŠENJE:





$\left.\begin{array}{l} x=4z \\ x=z^3 \end{array}\right\}$ SIJEKU SE U TAČKAMA $(8;2);(0;0);(-8;-2)$

$$V=\iint_{D_1} dx\,dz\int_0^{2xz} dy+\iint_{D_2} dx\,dz\int_0^{2xz} dy$$

IZNAD OBLASTI D_1 I D_2 JE $y=2xz>0$ JER SU x I z ISTOG ZNAKA.

$$V=2\iint_D 2xz\,dx\,dz=4\cdot\int_0^8 x\,dx\int_{\frac{x}{4}}^{\sqrt[3]{x}} z\,dz$$

$$=4\int_0^8 x\left.\frac{z^2}{2}\right|_{\frac{x}{4}}^{\sqrt[3]{x}} dx=2\int_0^8 x\cdot\left(x^{\frac{2}{3}}-\frac{x^2}{16}\right)dx$$

$$=2\cdot\left.\left(\frac{x^{\frac{8}{3}}}{\frac{8}{3}}-\frac{1}{16}\,\frac{x^4}{4}\right)\right|_0^8=2\cdot\left(\frac{3}{8}\cdot2^8-\frac{1}{64}2^{12}\right)$$

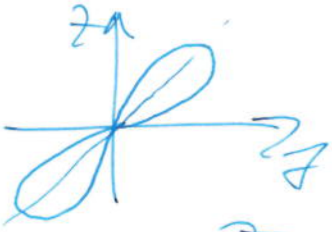
$$=2(3\cdot2^5-2^6)=2\cdot2^5(3-2)=2^6=64$$

P.S. MOŽE I $V=4\int_0^2 z\,dz\int_{4z}^{z^3} x\,dx$

NAĆI ZAPREMINU TIJELA OGRANIČENOG POVRŠIMA $S_1: x^2=2yz$; $S_2: (y^2+z^2)^2=2yz$

REŠENJE:

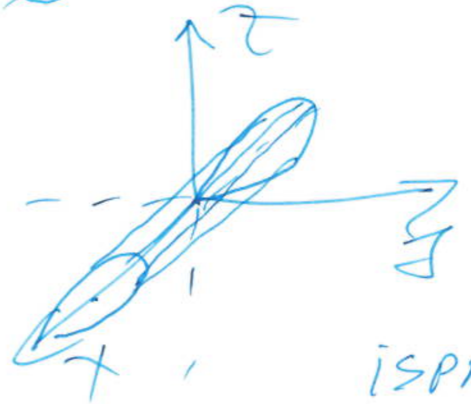
POVRŠ S_2 JE CILINDRIČNA POVRŠ NORMALNA NA RAVAN Oyz KOJA U PRESJEKU SA Oyz IMA LEMNISKATU



$y=\rho\cos\varphi$
$z=\rho\sin\varphi$

$(y^2+z^2)^2=2yz$

$\rho=\sqrt{\sin 2\varphi}$, $\varphi\in(0;\frac{\pi}{2})\cup(\pi;\frac{3\pi}{2})$



POVRŠ S_1: $x=\pm\sqrt{2yz}$ SA POVRŠI S_2 FORMIRA TIJELO KOJE SE NALAZI ISPRED I IZA PRVOG I TREĆEG KVADRANTA RAVNI Oyz. $V=4V_1$ GDJE JE V_1 DIO U PRVOM OKTANTU.

$$V_1=\iiint_V dx\,dy\,dz=\iint_{D_{yz}} dy\,dz\int_0^{\sqrt{2yz}} dx=\iint_{D_{yz}}\sqrt{2yz}\,dy\,dz$$

$$=\sqrt{2}\int_0^{\pi/2} d\varphi\int_0^{\sqrt{\sin 2\varphi}}\sqrt{\rho\cos\varphi\,\rho\sin\varphi}\cdot\rho\,d\rho=\int_0^{\pi/2}\sqrt{\sin 2\varphi}\,d\varphi\int_0^{\sqrt{\sin 2\varphi}}\rho^2\,d\rho$$

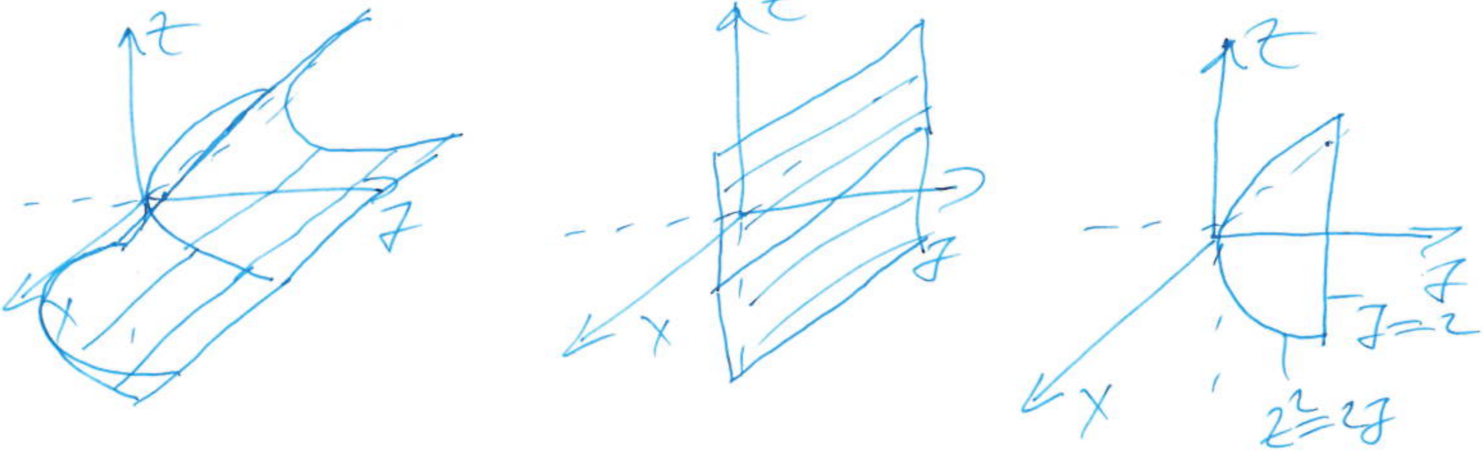
$$=\int_0^{\pi/2}\sqrt{\sin 2\varphi}\cdot\frac{\rho^3}{3}\Big|_0^{\sqrt{\sin 2\varphi}}d\varphi=\frac{1}{3}\int_0^{\pi/2}\sin^2 2\varphi\,d\varphi$$

$$=\frac{1}{3}\int_0^{\pi/2}\frac{1-\cos 4\varphi}{2}d\varphi=\frac{1}{6}\left(\varphi-\frac{\sin 4\varphi}{4}\right)\Big|_0^{\pi/2}=\frac{\pi}{12}$$

$$V=4V_1=\frac{\pi}{3}$$

NAĆI POVRŠINU DIJELA RAVNI ~~2x+y~~ $2x+y+2z=1$ OGRANIČENOG POVRŠIMA $z^2=2y$ i $y=2$.

RJEŠENJE:



DIO RAVNI OGRANIČEN POVRŠIMA $z^2=2y$ i $y=2$ SE PROJEKTUJE U RAVNI Oyz U OBLAST OGRANIČENU PRAVOM $y=2$ i PARABOLOM $z^2=2y$

$$P=\iint_{D_{yz}}\sqrt{1+(x'_y)^2+(x'_z)^2}\,dy\,dz=\iint_{D_{yz}}\sqrt{1+\left(\tfrac{1}{2}\right)^2+1^2}\,dy\,dz$$

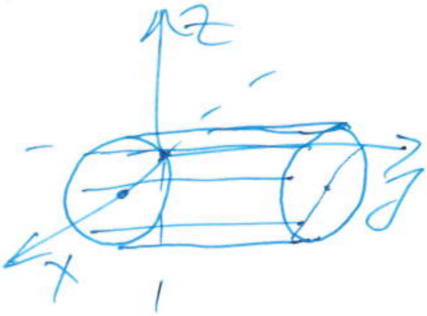
$$=\cancel{\ldots}=\iint_{D_{yz}}\sqrt{\tfrac{9}{4}}\,dy\,dz=\frac{3}{2}\cdot 2\int_0^2 dy\int_0^{\sqrt{2y}}dz$$

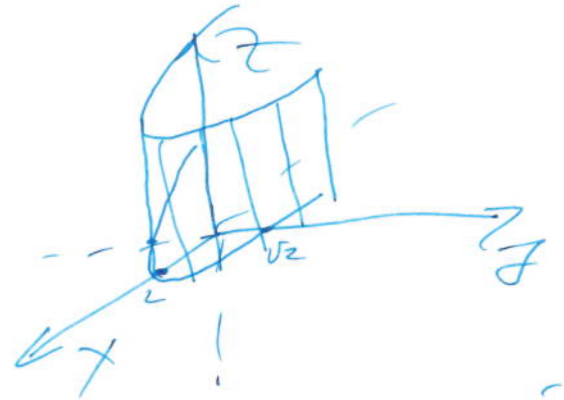
$$=3\cdot\int_0^2\sqrt{2y}\,dy=3\cdot\sqrt{2}\,\frac{y^{\frac{3}{2}}}{\frac{3}{2}}\Big|_0^2$$

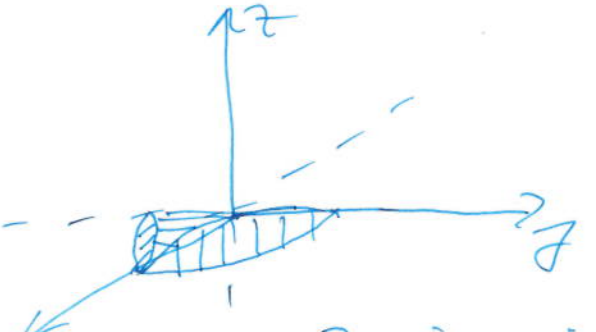
$$=3\cdot\sqrt{2}\cdot\frac{2}{3}\,2^{\frac{3}{2}}=2\sqrt{2}\,2\sqrt{2}=8$$

NAĆI POVRŠINU TIJELA OGRANIČENOG POVRŠIMA $S_1: x^2+z^2=2x$; $S_2: y^2=2-x$.

REŠENJE:







OBIJE POVRŠI SU SIMETRIČNE U ODNOSU NA RAVNI $z=0$ i $y=0$ tj. Oyz i Oxz, PA JE I TIJELO SIMETRIČNO U ODNOSU NA TE DVIJE RAVNI. $P=4P_1$; P_1 – U PRVOM OKTANTU

$$P_1=P(S_1)+P(S_2)$$

$$P(S_1)=\iint_{D_{xy}}\sqrt{1+z_x'^2+z_y'^2}\,dxdy \qquad z=\sqrt{2x-x^2};\ z_x'=\frac{1-x}{\sqrt{2x-x^2}}$$

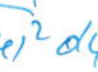
I NAČIN

$$z_y'=0$$

$$=\iint_{D_{xy}}\sqrt{1+\frac{1-2x+x^2}{2x-x^2}}\,dxdy=\iint_{D_{xy}}\frac{1}{\sqrt{2x-x^2}}\,dxdy=\int_0^2\frac{dx}{\sqrt{x}\sqrt{2-x}}\int_0^{\sqrt{2-x}}dy=2\sqrt{x}\Big|_0^2=2\sqrt{2}$$

II NAČIN $P(S_1)=\int_C\sqrt{2-x}\,dl$

$C:\ x^2+z^2=2x$, $x=\rho\cos\varphi$, $z=\rho\sin\varphi$, $\rho=2\cos\varphi$: $x=2\cos^2\varphi$, $z=2\cos\varphi\sin\varphi$, $\varphi\in\left(0,\frac{\pi}{2}\right)$



$$P(S_1)=\int_0^{\pi/2}\sqrt{2-2\cos^2\varphi}\cdot\sqrt{(-4\cos\varphi\sin\varphi)^2+(-2\sin^2\varphi+2\cos^2\varphi)^2}\,d\varphi=$$

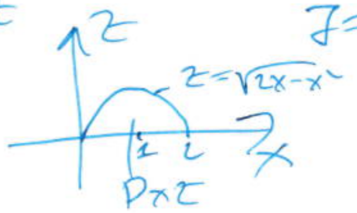
$$=\int_0^{\pi/2}\sqrt{2}\sin\varphi\cdot\sqrt{4\sin^2 2\varphi+4\cos^2 2\varphi}\,d\varphi=\sqrt{2}\cdot 2\cdot(-\cos\varphi)\Big|_0^{\pi/2}$$

$$=2\sqrt{2}(0+1)=2\sqrt{2}$$

$$P(S_L) = \iint_{D_{xz}} \sqrt{1+y_x'^2+y_z'^2}\,dx\,dz$$

I NAČIN

$y=\sqrt{2-x}$; $y_x' = \frac{-1}{2\sqrt{2-x}}$; $y_z'=0$



$$P(S_L)=\iint_{D_{xz}}\sqrt{1+\frac{1}{4(2-x)}}\,dx\,dz = \iint_{D_{xz}} \frac{\sqrt{9-4x}}{2\sqrt{2-x}}\,dx\,dz =$$

$$= \int_0^2 \frac{\sqrt{9-4x}}{2\sqrt{2-x}}\,dx \int_0^{\sqrt{2x-x^2}} dz = \frac12\int_0^2 \frac{\sqrt{9-4x}}{\cancel{\sqrt{2-x}}}\cdot\sqrt{x}\,\cancel{\sqrt{2-x}}\,dx$$

$$=\frac12\int_0^2\sqrt{9x-4x^2}\,dx = \dots$$

II NAČIN

$$P(S_L)=\int_C z(x,y)\,dl = \int_C\sqrt{2x-x^2}\,dl$$

$C:\ y=\sqrt{2-x}$

$x\in(0;2)$

$$dl=\sqrt{1+y_x'^2}\,dx$$

$$= \sqrt{1+\left(\frac{-1}{2\sqrt{2-x}}\right)^2}\,dx = \sqrt{\frac{4(2-x)+1}{4(2-x)}}\,dx$$

$$= \frac{\sqrt{9-4x}}{2\sqrt{2-x}}\,dx$$

$$P(S_L)=\int_0^2\sqrt{x(2-x)}\cdot\frac{\sqrt{9-4x}}{2\sqrt{2-x}}\,dx = \frac12\int_0^2\sqrt{9x-4x^2}\,dx$$

$$=\frac12\int_0^2\sqrt{\left(\frac92\right)^2-\left(\frac92-2x\right)^2}\,dx$$

$$= \left(\text{SMJENA } \frac92-2x=\frac92 t\ ;\ \begin{array}{l} x=0\to t=1 \\ x=2\to t=\frac12 \end{array}\right)$$

$$= \frac12\int_1^{1/2}\sqrt{\left(\frac92\right)^2-\left(\frac92 t\right)^2}\,dt = \dots$$