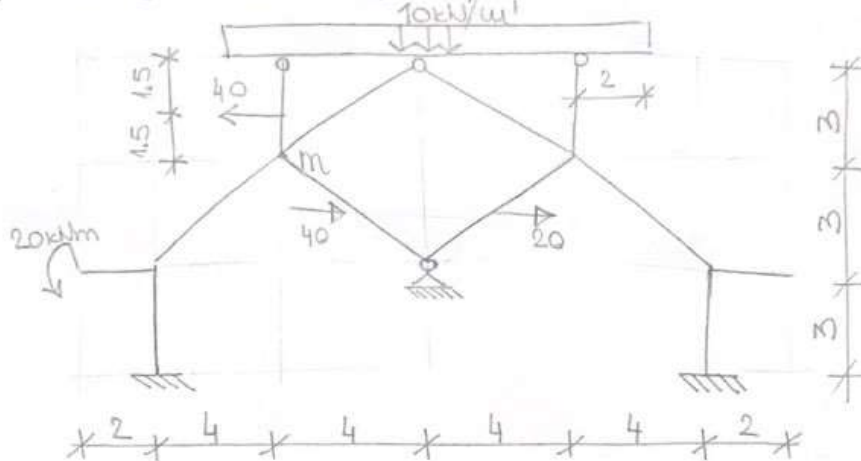


VJEŽBE VI

①

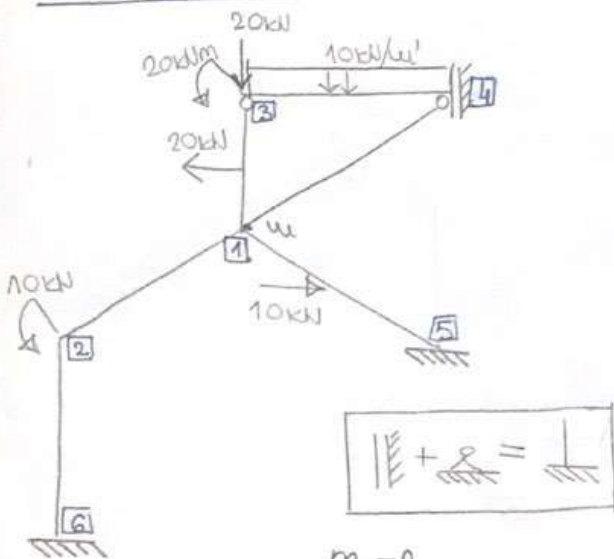
Rok 2015.

PRIMJENOM Približne metode deformacija odrediti pomjeranje tačke m uslijed zadatog opterećenja.
 $EI = \text{const.}$ $E = 3 \cdot 10^7 \text{ kN/m}^2$



Potrebno je utvrditi da li se pomjeranje tačke javlja uslijed simetrične ili antisimetrične, ili uslijed oboje. U ovom slučaju, pomjeranje se javlja uslijed oboje stanja. Da je čvor u osi simetrije vertikalna komponenta pomjeranja se javlja samo uslijed simetrične deformacije, a horizontalno pomjeranje uslijed antisimetrične deformacije.

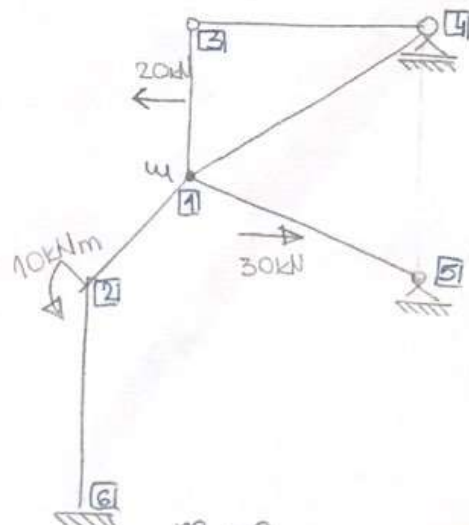
SIMETRIJA



$$m = 2$$

$$n = 1$$

ANTISIMETRIJA

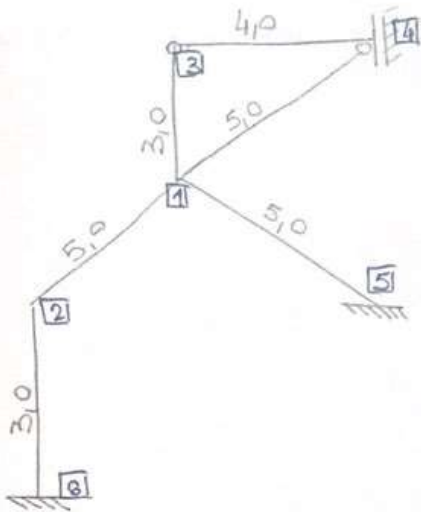


$$m = 2$$

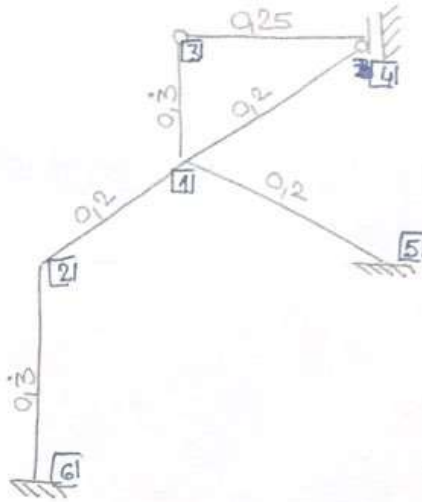
$$n = 1$$

SIMETRIJA

ŠEMA lik!



ŠEMA Kik!



USLOVNE JEDNAČINE

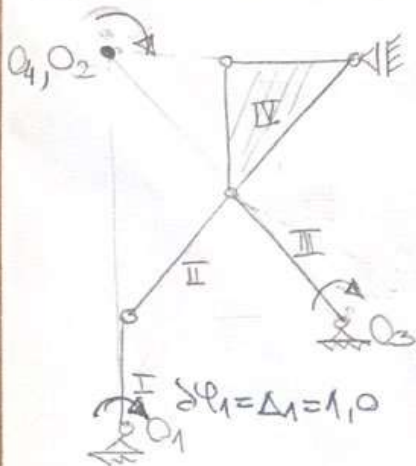
$$A_{11}\varphi_1 + A_{12}\varphi_2 + B_{11}\Delta_1 + A_{10} = 0$$

$$A_{21}\varphi_1 + A_{22}\varphi_2 + B_{21}\Delta_1 + A_{20} = 0$$

$$B_{11}\varphi_1 + B_{21}\varphi_2 + C_{11}\Delta_1 + C_{10} = 0$$

~~XXXXXXXXXX~~

Rešetka sistena

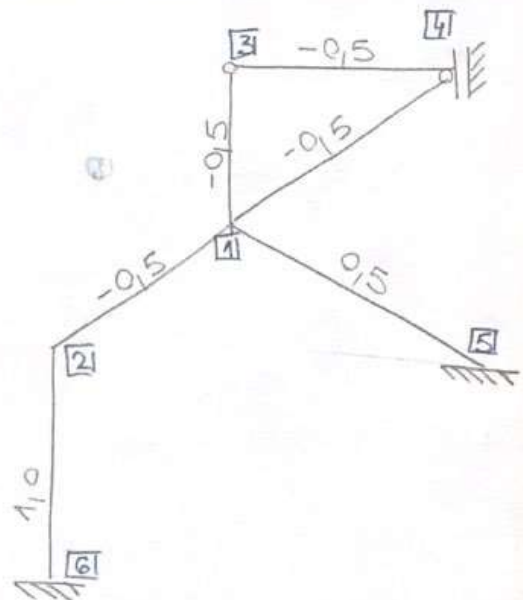


$$\delta\varphi_1 \cdot 3 = -\delta\varphi_2 \cdot 6 \Rightarrow \delta\varphi_2 = -0,5$$

$$\delta\varphi_2 = -\delta\varphi_3 \Rightarrow \delta\varphi_3 = 0,5$$

$$\delta\varphi_3 = -\delta\varphi_4 \Rightarrow \delta\varphi_4 = -0,5$$

ŠEMA PsiKj



KONSTANTE

$$A_{11} = a_{12} + a_{15} + d_{13} + d_{14}$$

$$A_{12} = b_{12}$$

$$A_{22} = a_{21} + a_{26}$$

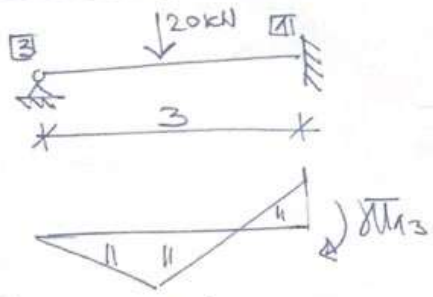
$$B_{11} = -(C_{12}\Psi_{12,1} + C_{15}\Psi_{15,1} + d_{13}\Psi_{13,1} + d_{14}\Psi_{14,1})$$

$$B_{21} = -(C_{21}\Psi_{12,1} + C_{26}\Psi_{26,1})$$

$$C_{11} = (C_{12} + C_{21})\Psi_{12,1}^2 + (C_{15} + C_{51})\Psi_{15,1}^2 + (C_{26} + C_{62})\Psi_{26,1}^2 + d_{13}\Psi_{13,1}^2 + d_{14}\Psi_{14,1}^2 + d_{43}\Psi_{43,1}^2$$

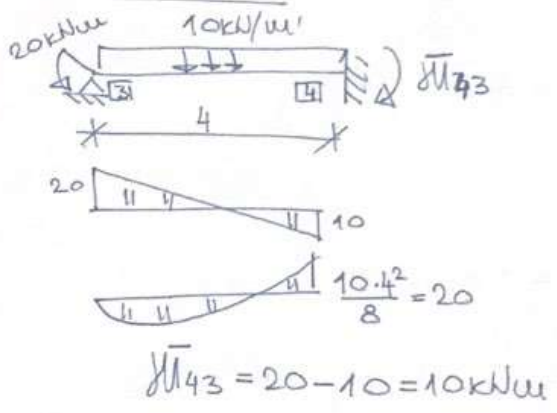
GOTSKI MOMENTI

STAP 1-3

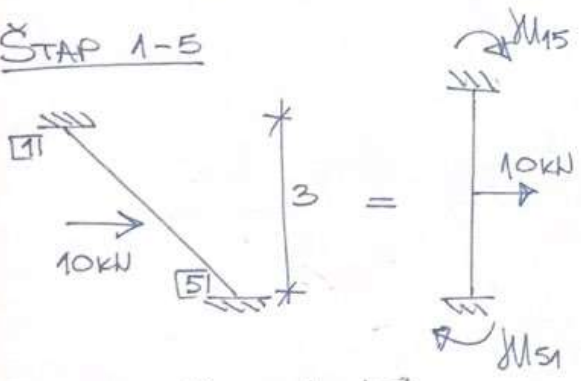


$$\bar{M}_{13} = \frac{20 \cdot 1,5^2}{2 \cdot 3^2} (3 + 1,5) = 11,25$$

STAP 3-4



STAP 1-5



$$M_{15} = -M_{51} = \frac{10 \cdot 1,5^3}{3^2} = 3,75 \text{ kNm}$$

$$A_{10} = 11,25 + 3,75 = 15$$

A₂₀ = 10 - čvorni moment

$$C_{10} = -[(-3,75) \cdot 3,75 \cdot (-0,5) + 10 \cdot (-0,5) + 11,25 \cdot (-0,5) + R_1]$$

$$R_1 = 10 \cdot 1,5 \cdot (+0,5) + 20 \cdot 1,5 \cdot (-0,5) + 20 \cdot 0,5 + 20 \cdot (-0,5) \cdot 4 + 10 \cdot 4 \cdot 6 \cdot (-0,5)$$

RIJEŠI SE SISTEM JEDNAČINA

ANTIMETRIJA

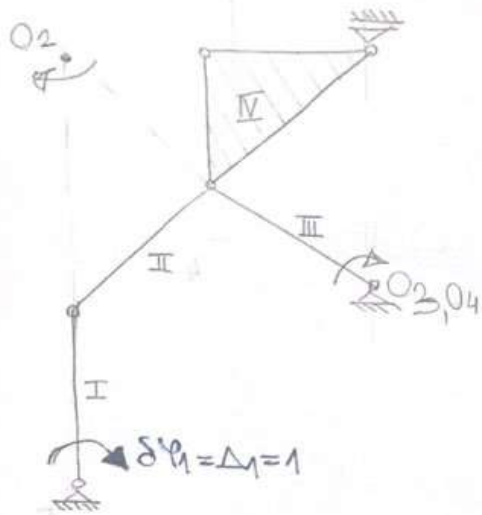
USLOVNE J-NE

$$A_{11}\psi_1 + A_{12}\psi_2 + B_{11}\Delta_1 + A_{10} = 0$$

$$A_{21}\psi_1 + A_{22}\psi_2 + B_{21}\Delta_1 + A_{20} = 0$$

$$B_{11}\psi_1 + B_{21}\psi_2 + C_{11}\Delta_1 + C_{10} = 0$$

REŠETKA SISTEMA



$$\partial\psi_1 \cdot 3 = -\partial\psi_2 \cdot 6 \Rightarrow \partial\psi_2 = -0,5$$

$$\partial\psi_2 \cdot 4 = -\partial\psi_3 \cdot 4 \Rightarrow \partial\psi_4 = 0,5 = \partial\psi_3$$

~~ČLANOV I~~

ČLANOVI

$$A_{11} = a_{12} + d_{13} + d_{14} + d_{15}$$

$$A_{12} = b_{12}$$

$$A_{22} = a_{21} + a_{26}$$

$$B_{11} = -[c_{12}\psi_{12,1} + d_{13}\psi_{13,1} + d_{14}\psi_{14,1} + d_{15}\psi_{15,1}]$$

$$B_{21} = -[c_{21}\psi_{12,1} + c_{26}\psi_{26,1}]$$

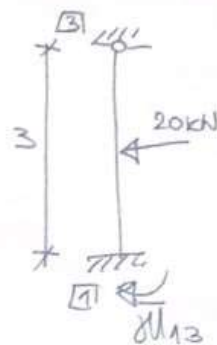
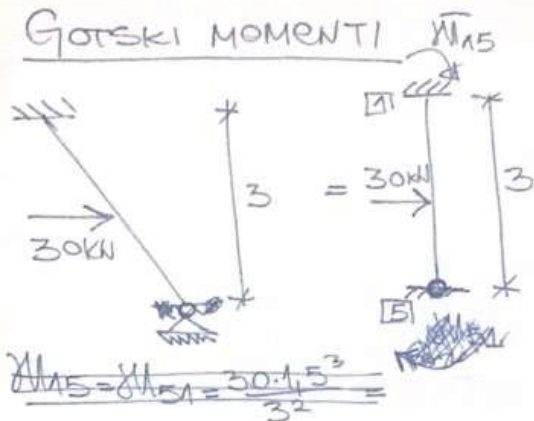
$$C_{11} = (c_{26} + c_{62})\psi_{26,1}^2 + (c_{21} + c_{12})\psi_{12,1}^2 + d_{13}\psi_{13,1}^2 + d_{14}\psi_{14,1}^2 + d_{15}\psi_{15,1}^2$$

ŠEMA OBRATANJA $\psi_{ik,1}$



GOTSKI MOMENTI

(3)



$$\bar{M}_{15} = \frac{30 \cdot 1,5^2}{3^2} = 5$$

$$\bar{M}_{13} = \frac{20 \cdot 1,5^2}{2 \cdot 3^2} (3 + 1,5) = 11,25$$

$$\bar{M}_{15} = \frac{30 \cdot 1,5^2}{2 \cdot 3^2} (3 + 1,5) = 16,875$$

$$A_{10} = 11,25 + 16,875 = 28,125$$

$$A_{20} = 10 - \text{čvorni moment}$$

$$C_{10} = -[16,875 \cdot 0,5 + 11,25 \cdot 0,5 + R_1]$$

$$R_1 = 30 \cdot 1,5 \cdot 0,5 + 20 \cdot 1,5 \cdot 0,5 = 7,5$$

- Riješi se sistem jednačina i dobiju ψ_1, ψ_2, Δ_1

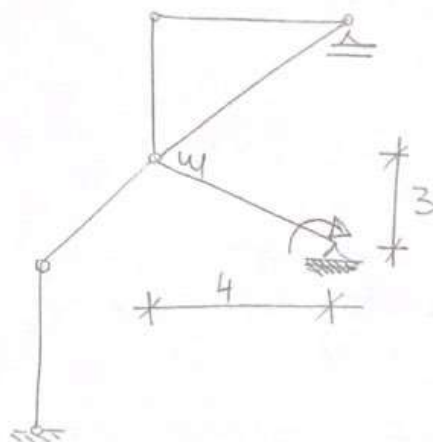
- Potrebno je odrediti pomjeranje tačke M, tj. ~~u~~ VERTIKALNU I HORIZONTALNU PROJEKCIJU POMJERANJA.

~~u = \psi_{15,1} \cdot 3 \cdot \Delta_1 \cdot C~~

PRVO NA ANTIMETRIČNOM DIJELU

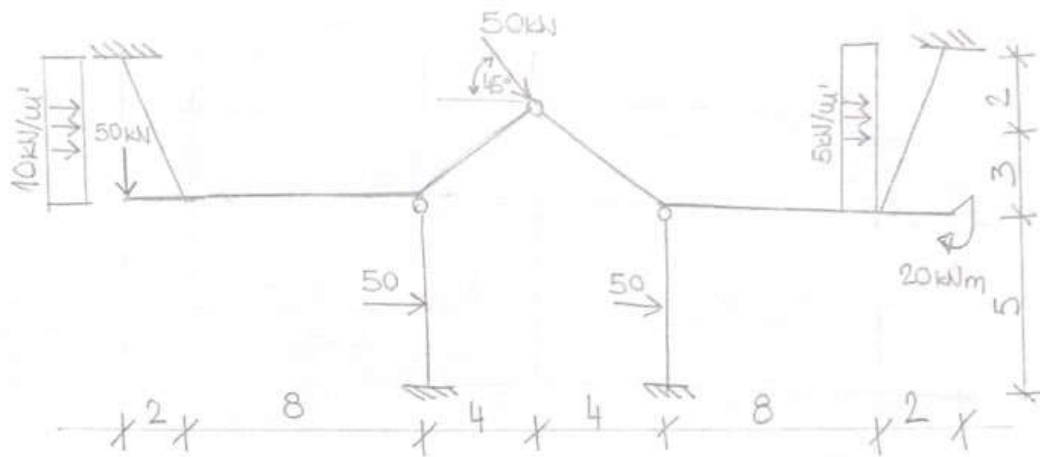
$$u = \psi_{15,1} \cdot 3 \cdot \Delta_1 \cdot C$$

$$v = \psi_{15,1} \cdot 4 \cdot \Delta_1 \cdot C$$

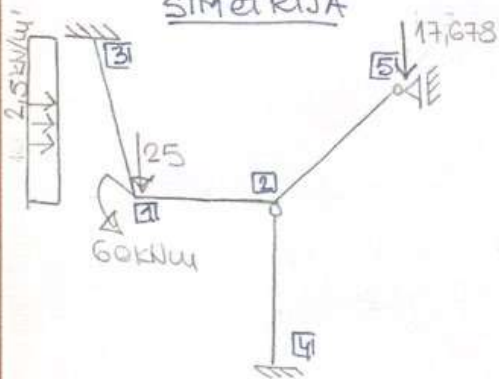


ROK APRIL 2017

ODREDITI PRESJEČNE SILE DATOG NOSAČA. $EI = \text{const.}$



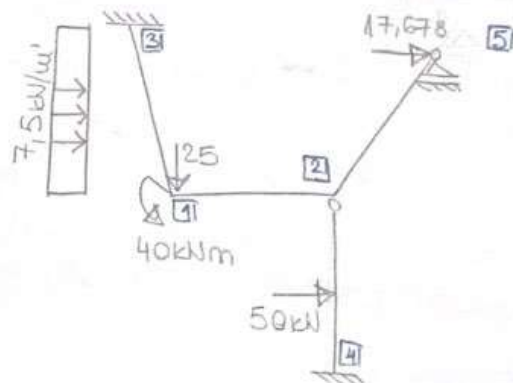
SIMETRIČAN NOSAČ
SIMETRJA



$u = 2$

$n = 2 \cdot 5 - 5 - 4 = 1$

ANTIMETRJA



$u = 2$

$n = 2 \cdot 5 - 5 - 4 = 1$

SIMETRJA

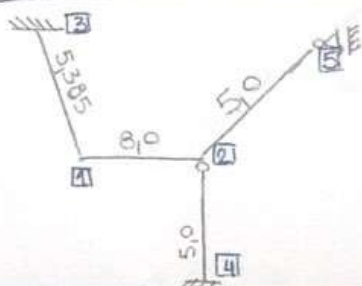
USLOVNE J-NE

$A_{11} \psi_1 + A_{12} \psi_2 + B_{11} \Delta_1 + A_{10} = 0$

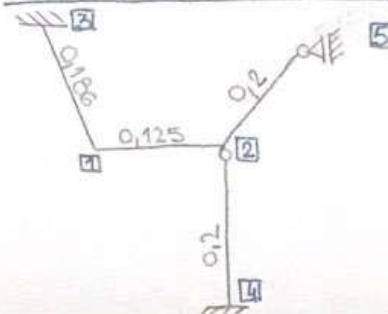
$A_{21} \psi_1 + A_{22} \psi_2 + B_{21} \Delta_1 + A_{20} = 0$

$B_{11} \psi_1 + B_{21} \psi_2 + C_{11} \Delta_1 + C_{10} = 0$

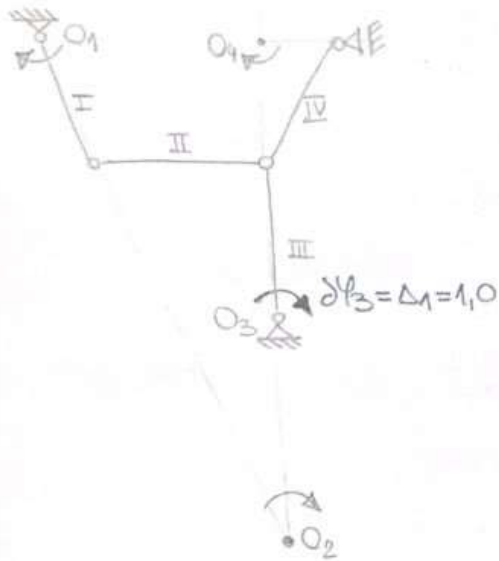
ŠEMA REDUKOVANIH DUŽINA l_{ik}



ŠEMA KRUTOSTI K_{ik}

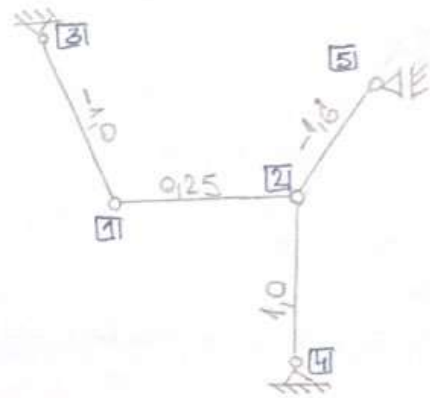


Rešetka sistema



ŠEMA OBRTANJA $\Psi_{ik,1}$

(4)



$$\delta\varphi_3 \cdot 5 = \delta\varphi_2 \cdot 20 \Rightarrow \delta\varphi_2 = \frac{1}{4}$$

$$\delta\varphi_3 \cdot 5 = -\delta\varphi_4 \cdot 3 \Rightarrow \delta\varphi_4 = -1,6$$

$$\delta\varphi_2 \cdot 8 = -\delta\varphi_1 \cdot 2 \Rightarrow \delta\varphi_1 = -1$$

UKOLIKO JE STUDENTIMA LAKŠE MOGU FORMIRATI JOŠ DVIJE ŠEME, ŠEMU $a_{ik}, a_{ki}, b_{ik}, d_{ig}$, I ŠEMU c_{ik}, c_{ki}, d_{ig} .

$$A_{11} = a_{12} + a_{13}$$

$$A_{12} = b_{12}$$

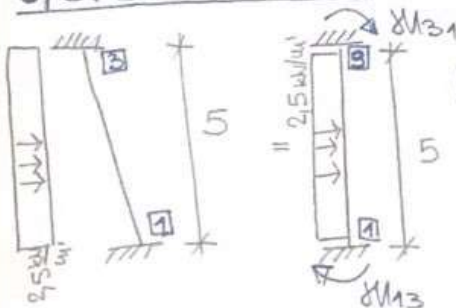
$$A_{22} = a_{21} + d_{25}$$

$$B_{11} = -(C_{13}\Psi_{13,1} + C_{12}\Psi_{12,1})$$

$$B_{21} = -(C_{21}\Psi_{12,1} + d_{25}\Psi_{25,1})$$

$$C_{11} = (C_{13} + C_{31})\Psi_{13,1}^2 + (C_{12} + C_{21})\Psi_{12,1}^2 + d_{42}\Psi_{42,1}^2 + d_{25}\Psi_{25,1}^2$$

GOTSKI MOMENTI



$$\delta M_{13} = -\delta M_{31} = -\frac{2,5 \cdot 5^2}{12} = -5,208$$

$$A_{10} = -5,208 + 60 = 54,792 \text{ kNm}$$

$$A_{20} = 0$$

$$C_{10} = - \left[\frac{5,2 \cdot 8 - 1,2 \cdot 8}{(M_{21} + M_{13})} \Psi_{13,1} + R_1 \right]$$

$$R_1 = -25 \cdot \frac{1}{4} \cdot 8 - 25 \cdot 5 \cdot 25 \cdot (-1) + 17,678 \cdot 4 \cdot (-1,6)$$

RIJEŠIMO SISTEM J-NA I DOBJEMO Ψ_1, Ψ_2, Δ_1 A ZATIM ODREDIMO I DIJAGRAME PRESJEČNIH SILA

$$\underline{M_{ik} = M_{ik}^s + M_{ik}^*}$$

NAKON ŠTO ODREDIMO PRESJEČNE SILE USLIJED SIMETRIČNE DEFORMACIJE, ODREĐUJEMO IH I* ZA ANTIMETRIČNU DEFORMACIJU

ANTIMETRIJA

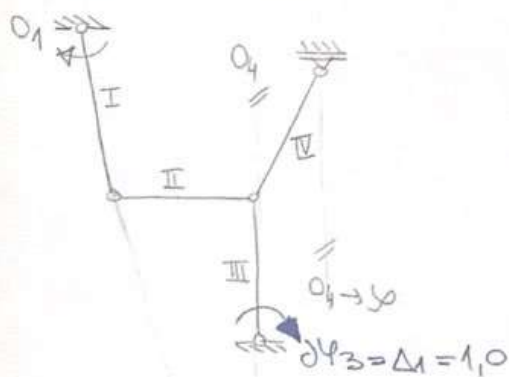
USLOVNE J-NE

$$A_{11} \Psi_1 + A_{12} \Psi_2 + B_{11} \Delta_1 + A_{10} = 0$$

$$A_{21} \Psi_1 + A_{22} \Psi_2 + B_{21} \Delta_1 + A_{20} = 0$$

$$B_{11} \Psi_1 + B_{21} \Psi_2 + C_{11} \Delta_1 + C_{10} = 0$$

Rešetka sistema

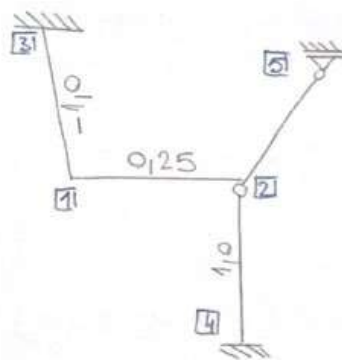


$$\partial \Psi_3 \cdot 5 = \partial \Psi_2 \cdot 20 \Rightarrow \partial \Psi_2 = \frac{1}{4}$$

$$\partial \Psi_2 \cdot 8 = -\partial \Psi_1 \cdot 2 \Rightarrow \partial \Psi_1 = -1$$



ŠEMA OBRATANJA



$$A_{11} = a_{12} + a_{13}$$

$$A_{12} = b_{12}$$

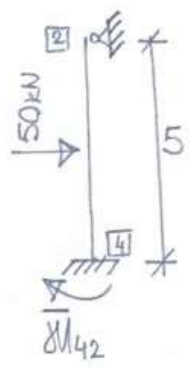
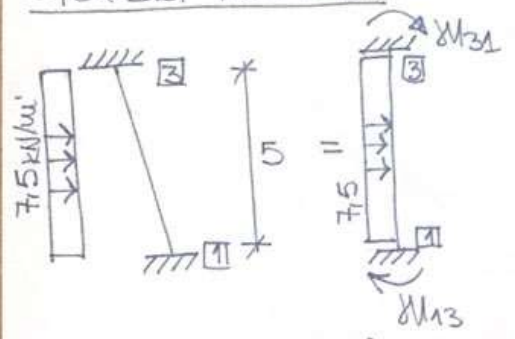
$$A_{22} = a_{21} + d_{25}$$

$$B_{11} = - (C_{13} \Psi_{13,1} + C_{12} \Psi_{12,1})$$

$$B_{21} = - (C_{21} \Psi_{12,1} + d_{25} \Psi_{25,1})$$

$$C_{11} = (C_{13} + C_{31})\Psi_{13,1}^2 + (C_{12} + C_{21})\Psi_{12,1}^2 + d_{25}\Psi_{25,1}^2 + d_{42}\Psi_{24,1}^2$$

GOTSKI MOMENTI



$$\bar{M}_{42} = -\frac{50 \cdot 2.5^2}{2 \cdot 5^2} (2.5 + 5) = -46.875$$

$$M_{13} = -M_{31} = -\frac{7.5 \cdot 5^2}{12} = -15.624$$

$$A_{10} = -15.624 + 40 = 24.376 \text{ kNm}$$

$$A_{20} = -46.875$$

$$C_{10} = -[(M_{13} + M_{31})\Psi_{13,1} + \bar{M}_{42}\Psi_{42,1} + R_1]$$

$$R_1 = 7.5 \cdot 5 \cdot 2.5 \cdot 1 - 25 \cdot 2 \cdot 1 + 50 \cdot 2.5 \cdot 1.0 + \boxed{17.678 \cdot 1.5}$$

NEMA OBRATANJA TE PLOŠE, ONA TRANSLIRA
 NEĐUTIM PRAVI RAD JER TRANSLIRA
 U PRAVCU SILE.

Riješi se sistem j-NA i DOBIJU se Ψ_1, Ψ_2, Δ_1 .

ZATIM SE ODREDE PRESJEČNE SILE ZA ANTIMETRIČNU DEFORMACU
 NA KRAJU SUPERPONIRAMO UTICAJE.

$$M_{ik} = M_{ik}^s + M_{ik}^a$$