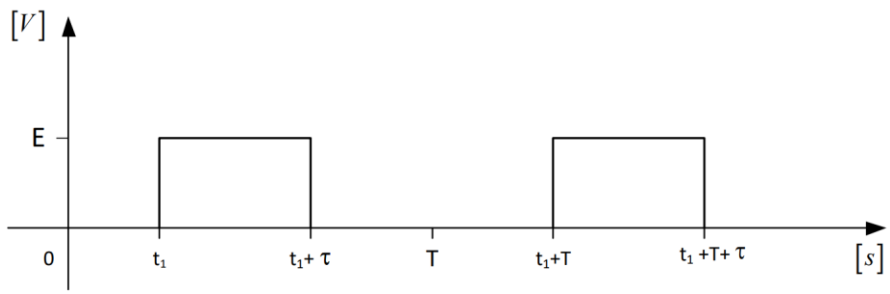
1. Pronaći amplitudski i fazni spektar periodičnog signala *f(t)*, koji je u intervalu jedne periode T definisan na sledeći način:

$$f(t)=\begin{cases}0, & 0<t<t_1\\ E, & t_1<t<t_1+\tau\\ 0, & t_1+\tau<t<T\end{cases}$$

Grafički predstaviti amplitudski i fazni spektar ovog signala za slučaj da je $\tau = T/7$, a trenutak početka impulsa t_1=0.

Rešenje:

Funkciju f(t) možemo grafički predstaviti:



Slika 1. Pravougaona povorka impulsa f(t).

Periodiči signal f(t), periode $T = 2\pi / \omega_o$, može se predstaviti Furijeovim redom:

$$f(t)=\sum_{n=-\infty}^{\infty} F_n e^{jn\omega_o t} = \sum_{n=-\infty}^{\infty} |F_n| e^{j(n\omega_o t+\theta_n)} = F_o + \sum_{n=1}^{\infty} 2|F_n|\cos(n\omega_o t+\theta_n)$$

$$\mathrm{F_n} = |\mathrm{F_n}|\mathrm{e}^{\mathrm{j}\theta_\mathrm{n}} = \frac{1}{\mathrm{T}}\int_{-\frac{\mathrm{T}}{2}}^{\frac{\mathrm{T}}{2}} \mathrm{f(t)e}^{-\mathrm{j}n\omega_0\mathrm{t}}\mathrm{dt} \qquad \mathrm{n} = 0, \pm 1, \pm 2, \ldots$$

$$\mathrm{F_n} = \frac{1}{\mathrm{T}}\int_{\mathrm{t_1}}^{\mathrm{t_1}+\tau} \mathrm{Ee}^{-\mathrm{jn}\omega_0\mathrm{t}}\mathrm{dt} = \frac{\mathrm{E}}{-\mathrm{jn}\omega_0 T}\left(\mathrm{e}^{-\mathrm{jn}\omega_0\mathrm{t_1}} - \mathrm{e}^{-\mathrm{jn}\omega_0(\mathrm{t_1}+\tau)}\right) =$$

$$= \frac{\mathrm{Ee}^{-\mathrm{jn}\omega_0\mathrm{t_1}}}{\mathrm{jn}\omega_0\mathrm{T}}\left(1 - \mathrm{e}^{-\mathrm{jn}\omega_0\tau}\right) = \frac{\mathrm{Ee}^{-\mathrm{jn}\omega_0(\mathrm{t_1}+\frac{\tau}{2})}}{\mathrm{jn}\omega_0\mathrm{T}}\left(\mathrm{e}^{\mathrm{jn}\omega_0\frac{\tau}{2}} - \mathrm{e}^{-\mathrm{jn}\omega_0\frac{\tau}{2}}\right)$$

$$= \frac{2\mathrm{Esin}\left(\mathrm{n}\omega_0\frac{\tau}{2}\right)}{\mathrm{n}\omega_0\mathrm{T}}\mathrm{e}^{-\mathrm{jn}\omega_0\left(\mathrm{t_1}+\frac{\tau}{2}\right)} =$$

$$= \frac{\mathrm{E}\tau}{\mathrm{T}}\frac{\sin\left(\mathrm{n}\omega_0\frac{\tau}{2}\right)}{\mathrm{n}\omega_0\frac{\tau}{2}}\mathrm{e}^{-\mathrm{jn}\omega_0\left(\mathrm{t_1}+\frac{\tau}{2}\right)}$$

Amplitudski spektar:

$$|\mathrm{F_n}| = \frac{\mathrm{E}\tau}{\mathrm{T}}\left|\frac{\sin\left(\mathrm{n}\omega_0\frac{\tau}{2}\right)}{\mathrm{n}\omega_0\frac{\tau}{2}}\right| \mathrm{n} = 0, \pm 1, \pm 2, \ldots$$

Anvelopa amplitudskog spektra:

$$\alpha(\omega) = \frac{\mathrm{E}\tau}{\mathrm{T}}\left|\frac{\sin\left(\omega\frac{\tau}{2}\right)}{\omega\frac{\tau}{2}}\right|$$

Anvelopa spektra ima nule na učestanostima gdje je $\sin\left(\omega\frac{\tau}{2}\right) = 0$, tj.:

$$\frac{\omega\tau}{2} = \mathrm{k}\pi \Rightarrow \omega = \frac{2\mathrm{k}\pi}{\tau}, \mathrm{k} = \pm 1, \pm 2, \ldots$$

Rastojanja nula anvelope od koordinatnog početka obrnuto su proporcionalna trajanju impulsa τ.

Fazni spektar:

$$\theta_{\mathrm{n}} = -\mathrm{n}\omega_0\left(\mathrm{t_1} + \frac{\tau}{2}\right) + \Delta\theta_{\mathrm{n}}$$

$$\Delta\theta_n = \begin{cases} 0, & \dfrac{\sin\left(n\omega_0\frac{\tau}{2}\right)}{n\omega_0\frac{\tau}{2}} > 0 \\ \pm\pi, & \dfrac{\sin\left(n\omega_0\frac{\tau}{2}\right)}{n\omega_0\frac{\tau}{2}} < 0 \end{cases}$$

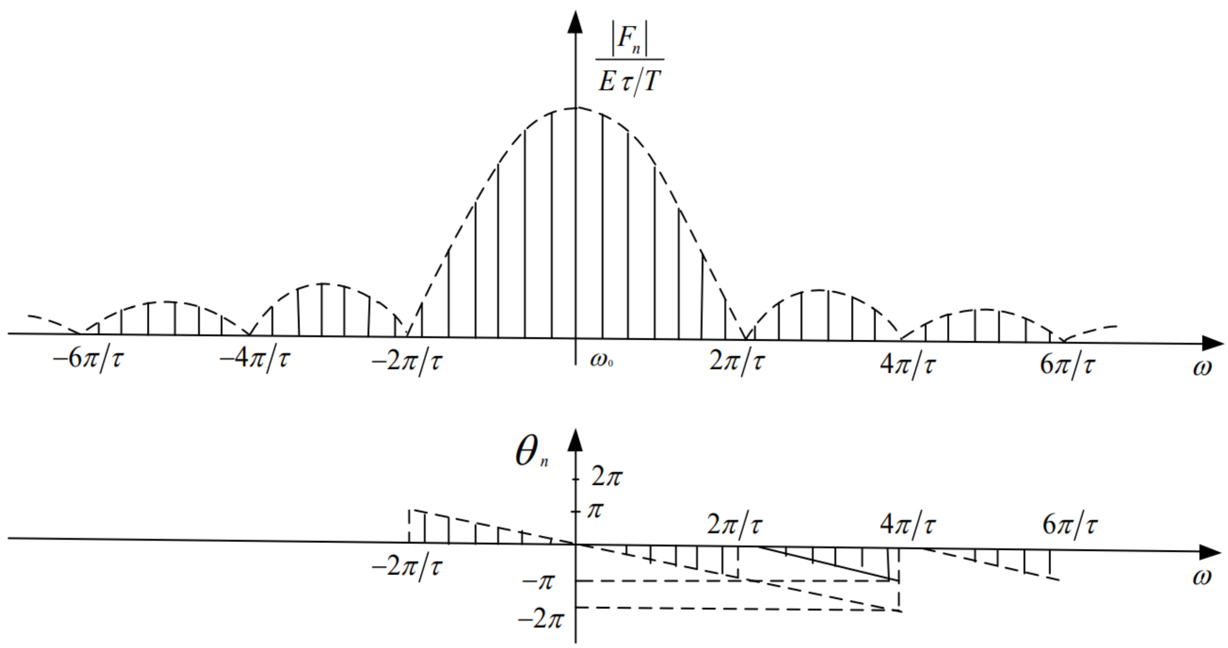
Anvelopa faznog spektra:

$$\beta(\omega) = -\omega\left(t_1 + \frac{\tau}{2}\right) + \Delta\theta_n$$

Zaključujemo da trenutak uspostavljanja impulsa t_1 utiče samo na fazni spektar.

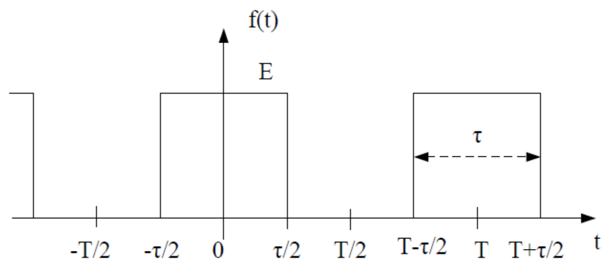
Za zadatu vrijednost periode i t_1 imamo:

$$\tau = \mathrm{T}/7 \Rightarrow |\mathrm{F_n}| = \frac{\mathrm{E}}{7}\left|\frac{\sin\left(\frac{\mathrm{n}\pi}{7}\right)}{\frac{\mathrm{n}\pi}{7}}\right|, \mathrm{F_0} = \frac{\mathrm{E}}{7}, \text{nule} : \omega = 7\omega_0 \mathrm{k}$$

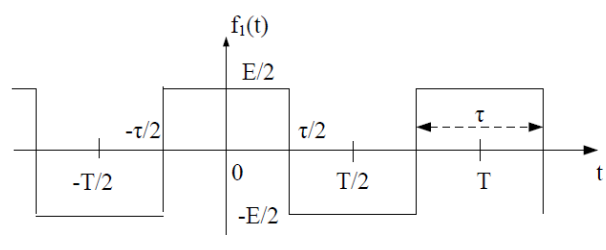


Slika 2. Amplitudski (gore) i fazni spektar (dolje) pravougaone povorke impulsa f(t) za $\tau = T/7$ i t_1=0.

2. Pronaći amplitudski i fazni spektar periodičnog signala *f(t)* sa slike 3, a zatim odrediti amplitudski i fazni spektar signala $f_1(t)$ sa slike 4.



Slika 3.



Slika 4.

<u>Rešenje:</u>

U prethodnom zadatku dokazali smo da za opšti oblik pravougaone povorke važi:

$$|F_n| = \frac{E\tau}{T}\left|\frac{\sin\left(n\omega_0\frac{\tau}{2}\right)}{n\omega_0\frac{\tau}{2}}\right| n = 0, \pm 1, \pm 2, \dots$$

$$\alpha(\omega) = \frac{E\tau}{T}\left|\frac{\sin\left(\omega\frac{\tau}{2}\right)}{\omega\frac{\tau}{2}}\right|$$

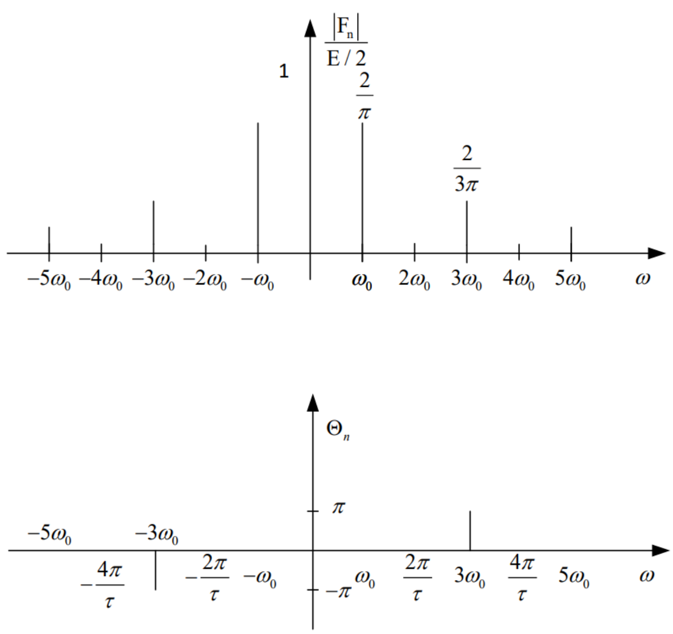
$$\theta_n = -jn\omega_0\left(t_1 + \frac{\tau}{2}\right) + \Delta\theta_n$$

$$\Delta\theta_n = \begin{cases} 0, & \dfrac{\sin\left(n\omega_0\frac{\tau}{2}\right)}{n\omega_0\frac{\tau}{2}} > 0 \\ \pm\pi, & \dfrac{\sin\left(n\omega_0\frac{\tau}{2}\right)}{n\omega_0\frac{\tau}{2}} < 0 \end{cases}$$

Kako je u ovom slučaju, $\tau = T/2$ i $t_1 = -\frac{\tau}{2}$, to su amplitudski i fazni spektar opisani izrazima:

$$|F_n| = \frac{E}{2}\left|\frac{\sin(n\pi/2)}{n\pi/2}\right|, \quad n = 0, \pm 1, \pm 2\dots$$
$$\theta_n = \Delta\theta_n, \quad n = 0, \pm 1, \pm 2\dots$$

Nule anvelope amplitudskog spektra nalaze se na učestanostima $\omega = 2\omega_0 k$



Slika 2. Amplitudski (gore) i fazni spektar (dolje) pravougaone povorke impulsa f(t).

Signal f(t) može da se predstavi Furijeovim redom kao:

$$f(t)=\sum_{n=-\infty}^{\infty}F_n e^{jn\omega_o t}=F_o+\sum_{n=0}^{\infty}2\left|F_n\right|\cos(n\omega_o t+\theta_n)$$

Kako je:

$$F_0=\frac{E}{2}\quad i\quad |F_n|=\begin{cases}\left|\frac{E}{np}\right|, n=\pm1,\pm3,\ldots\\ 0, n=\pm2,\pm4,\ldots\end{cases}$$

To je:

$$f(t)=\frac{E}{2}+\sum_{n=1}^{\infty}(-1)^n\frac{2E}{(2n+1)\pi}\cos\left[(2n+1)\omega_o t\right]$$

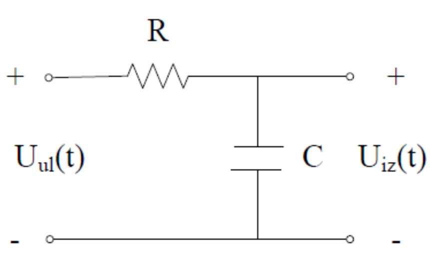
Spektar signala $f_1(t)$ lako se određuje pošto se uspostavi relacija između signala $f(t)$ i $f_1(t)$:

$$f_1(t)=f(t)-E/2$$

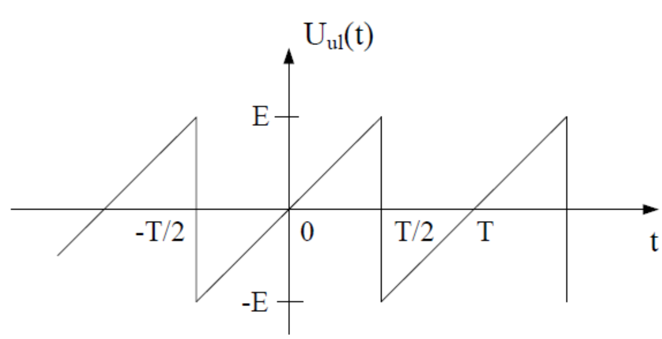
$$f_1(t)=\sum_{n=0}^{\infty}(-1)^n\frac{2E}{(2n+1)\pi}\cos\left[(2n+1)\omega_0 t\right]$$

signal $f_1(t)$ ne sadrži jednosmjernu komponentu tako da njegov spektar, izuzev za $\omega = 0$ jednak spektru signala $f(t)$.

3. Na ulaz kola prikazanog na sl. 4 dovodi se signal prikazan na sl. 5.
   a) Pronaći amplitudski spektar i spektar snage ulaznog signala,
   b) Ako je $1/(RC)=\omega_0$, gdje je ω_0 osnovna kružna učestanost ulaznog signala, odrediti amplitudski spektar i spektar snage izlaznog signala,
   c) Kako treba odrediti elemente kola R i C, pa da snaga trećeg harmonika izlaznog signala ne prelazi 1% srednje snage ulaznog signala?



Slika 4.



Slika 5.