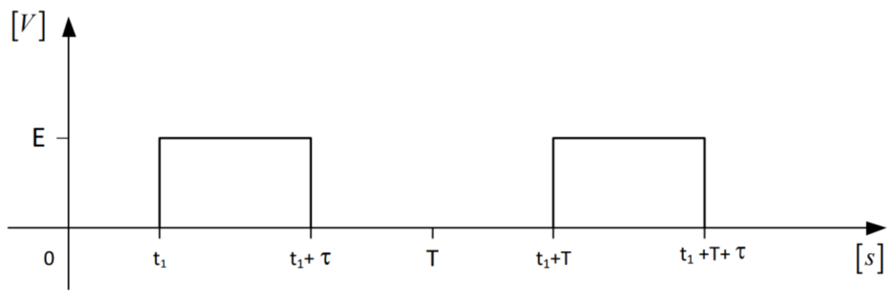
1. Pronaći amplitudski i fazni spektar periodičnog signala *f(t)*, koji je u intervalu jedne periode *T* definisan na sledeći način:

$$f(t) = \begin{cases} 0, & 0 < t < t_1 \\ E, & t_1 < t < t_1 + \tau \\ 0, & t_1 + \tau < t < T \end{cases}$$

Grafički predstaviti amplitudski i fazni spektar ovog signala za slučaj da je $\tau = T/7$, a trenutak početka impulsa t_1=0.

Rešenje:

Funkciju f(t) možemo grafički predstaviti:



Slika 1. Pravougaona povorka impulsa f(t).

Periodiči signal f(t), periode $T = 2\pi / \omega_o$, može se predstaviti Furijeovim redom:

$$f(t) = \sum_{n=-\infty}^{\infty} F_n e^{jn\omega_o t} = \sum_{n=-\infty}^{\infty} |F_n| e^{j(n\omega_o t + \theta_n)} = F_o + \sum_{n=1}^{\infty} 2|F_n| \cos(n\omega_o t + \theta_n)$$

$$F_n = |F_n| e^{j\theta_n} = \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} f(t) e^{-jn\omega_0 t} dt \qquad n = 0, \pm 1, \pm 2, \ldots$$

$$F_n = \frac{1}{T}\int_{t_1}^{t_1+\tau} Ee^{-jn\omega_0 t}dt = \frac{E}{-jn\omega_0 T}\left(e^{-jn\omega_0 t_1} - e^{-jn\omega_0(t_1+\tau)}\right) =$$

$$= \frac{Ee^{-jn\omega_0 t_1}}{jn\omega_0 T}\left(1 - e^{-jn\omega_0 \tau}\right) = \frac{Ee^{-jn\omega_0\left(t_1+\frac{\tau}{2}\right)}}{jn\omega_0 T}\left(e^{jn\omega_0\frac{\tau}{2}} - e^{-jn\omega_0\frac{\tau}{2}}\right)$$

$$= \frac{2E\sin\left(n\omega_0\frac{\tau}{2}\right)}{n\omega_0 T}e^{-jn\omega_0\left(t_1+\frac{\tau}{2}\right)} =$$

$$= \frac{E\tau}{T}\frac{\sin\left(n\omega_0\frac{\tau}{2}\right)}{n\omega_0\frac{\tau}{2}}e^{-jn\omega_0\left(t_1+\frac{\tau}{2}\right)}$$

Amplitudski spektar:

$$|F_n| = \frac{E\tau}{T}\left|\frac{\sin\left(n\omega_0\frac{\tau}{2}\right)}{n\omega_0\frac{\tau}{2}}\right| n = 0, \pm 1, \pm 2, \dots$$

Anvelopa amplitudskog spektra:

$$\alpha(\omega) = \frac{E\tau}{T}\left|\frac{\sin\left(\omega\frac{\tau}{2}\right)}{\omega\frac{\tau}{2}}\right|$$

Anvelopa spektra ima nule na učestanostima gdje je $\sin\left(\omega\frac{\tau}{2}\right) = 0$, tj.:

$$\frac{\omega\tau}{2} = k\pi \Rightarrow \omega = \frac{2k\pi}{\tau}, k = \pm 1, \pm 2, \dots$$

Rastojanja nula anvelope od koordinatnog početka obrnuto su proporcionalna trajanju impulsa τ.

Fazni spektar:

$$\theta_n = -n\omega_0\left(t_1 + \frac{\tau}{2}\right) + \Delta\theta_n$$

$$\Delta\theta_n = \begin{cases} 0, & \dfrac{\sin\left(n\omega_0\frac{\tau}{2}\right)}{n\omega_0\frac{\tau}{2}} > 0 \\ \pm\pi, & \dfrac{\sin\left(n\omega_0\frac{\tau}{2}\right)}{n\omega_0\frac{\tau}{2}} < 0 \end{cases}$$

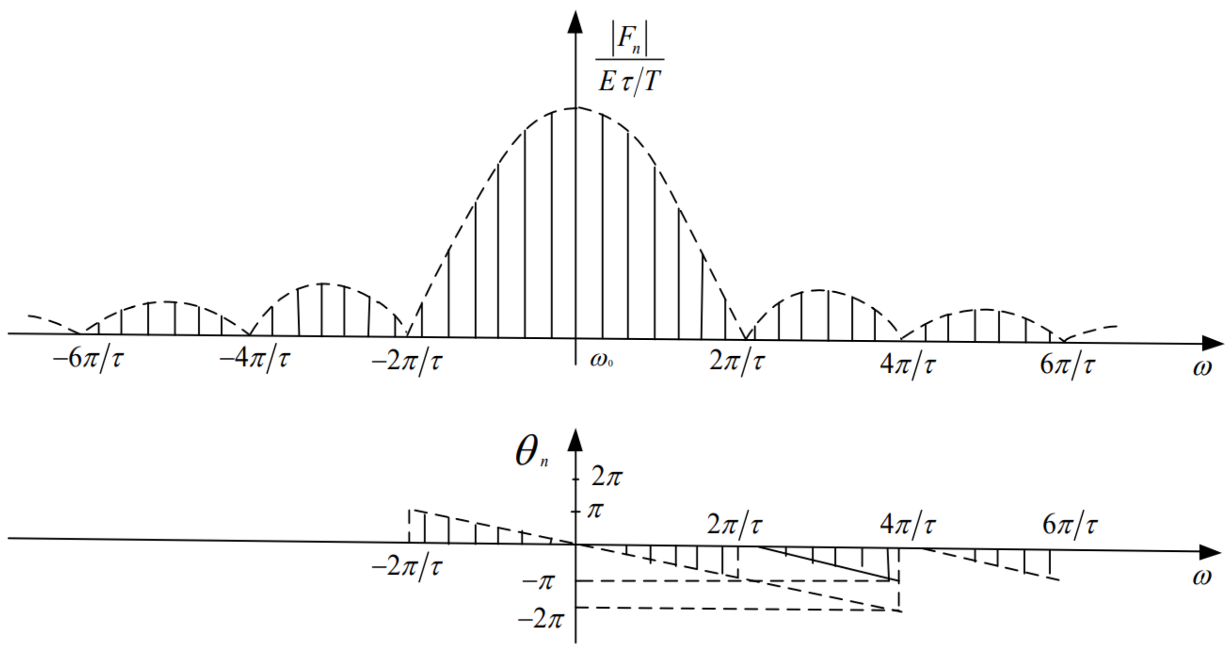
Anvelopa faznog spektra:

$$\beta(\omega) = -\omega\left(t_1 + \frac{\tau}{2}\right) + \Delta\theta_n$$

Zaključujemo da trenutak uspostavljanja impulsa t_1 utiče samo na fazni spektar.

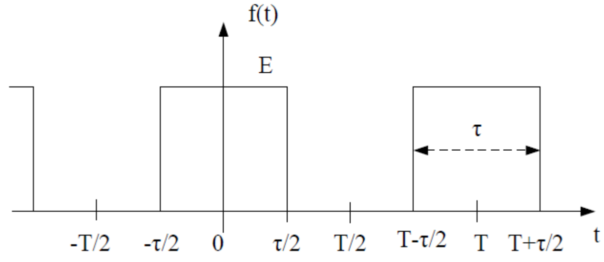
Za zadatu vrijednost periode i t_1 imamo:

$$\tau = \mathrm{T}/7 \Rightarrow |\mathrm{F_n}| = \frac{\mathrm{E}}{7}\left|\frac{\sin\left(\frac{\mathrm{n}\pi}{7}\right)}{\frac{\mathrm{n}\pi}{7}}\right|, \mathrm{F_0} = \frac{\mathrm{E}}{7}, \text{nule} : \omega = 7\omega_0 \mathrm{k}$$

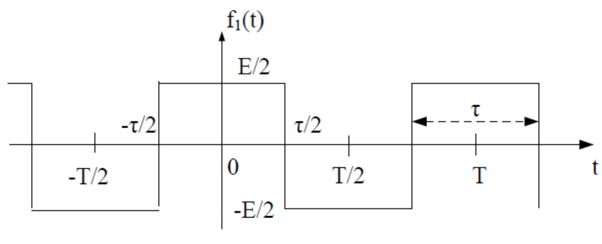


Slika 2. Amplitudski (gore) i fazni spektar (dolje) pravougaone povorke impulsa f(t) za $\tau = T/7$ i t_1=0.

2. Pronaći amplitudski i fazni spektar periodičnog signala *f(t)* sa slike 3, a zatim odrediti amplitudski i fazni spektar signala *$f_1(t)$* sa slike 4.



Slika 3.



Slika 4.

<u>Rešenje:</u>

U prethodnom zadatku dokazali smo da za opšti oblik pravougaone povorke važi:

$$|F_n| = \frac{E\tau}{T}\left|\frac{\sin\left(n\omega_0\frac{\tau}{2}\right)}{n\omega_0\frac{\tau}{2}}\right| n = 0, \pm1, \pm2, \ldots$$

$$\alpha(\omega) = \frac{E\tau}{T}\left|\frac{\sin\left(\omega\frac{\tau}{2}\right)}{\omega\frac{\tau}{2}}\right|$$

$$\theta_n = -jn\omega_0\left(t_1 + \frac{\tau}{2}\right) + \Delta\theta_n$$

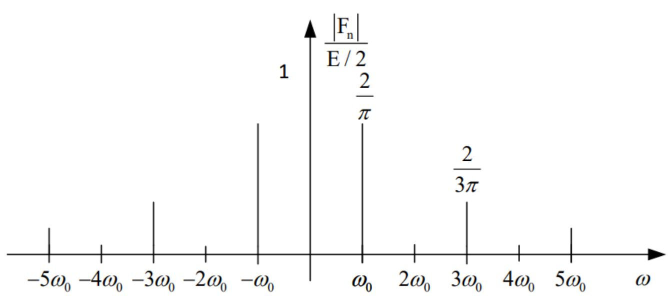
$$\Delta\theta_n = \begin{cases} 0, & \dfrac{\sin\left(n\omega_0\frac{\tau}{2}\right)}{n\omega_0\frac{\tau}{2}} > 0 \\ \pm\pi, & \dfrac{\sin\left(n\omega_0\frac{\tau}{2}\right)}{n\omega_0\frac{\tau}{2}} < 0 \end{cases}$$

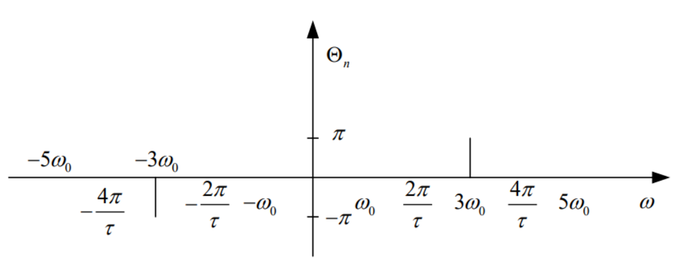
Kako je u ovom slučaju, $\tau = T/2$ i $t_1 = -\frac{\tau}{2}$, to su amplitudski i fazni spektar opisani izrazima:

$$|F_n| = \frac{E}{2}\left|\frac{\sin(n\pi/2)}{n\pi/2}\right|, \quad n = 0, \pm1, \pm2\ldots$$

$$\theta_n = \Delta\theta_n, \quad n = 0, \pm1, \pm2\ldots$$

Nule anvelope amplitudskog spektra nalaze se na učestanostima $\omega = 2\omega_0 k$





Slika 2. Amplitudski (gore) i fazni spektar (dolje) pravougaone povorke impulsa f(t).

Signal f(t) može da se predstavi Furijeovim redom kao:

$$f(t)=\sum_{n=-\infty}^{\infty}F_n e^{jn\omega_o t}=F_o+\sum_{n=1}^{\infty}2\left|F_n\right|\cos(n\omega_o t+\theta_n)$$

Kako je:

$$F_0=\frac{E}{2}\quad i\quad |F_n|=\begin{cases}\left|\frac{E}{np}\right|, & n=\pm1,\pm3,\ldots\\ 0, & n=\pm2,\pm4,\ldots\end{cases}$$

To je:

$$f(t)=\frac{E}{2}+\sum_{n=1}^{\infty}(-1)^n\frac{2E}{(2n+1)\pi}\cos\left[(2n+1)\omega_o t\right]$$

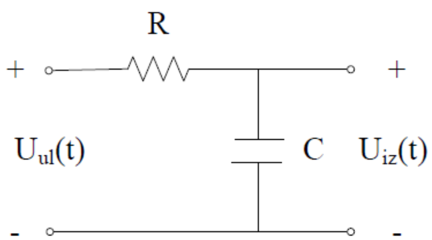
Spektar signala $f_1(t)$ lako se određuje pošto se uspostavi relacija između signala $f(t)$ i $f_1(t)$:

$$f_1(t)=f(t)-E/2$$

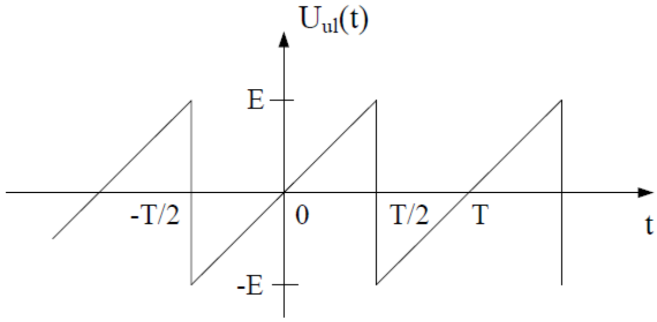
$$f_1(t)=\sum_{n=0}^{\infty}(-1)^n\frac{2E}{(2n+1)\pi}\cos\left[(2n+1)\omega_0 t\right]$$

signal $f_1(t)$ ne sadrži jednosmjernu komponentu tako da njegov spektar, izuzev za $\omega = 0$ jednak spektru signala $f(t)$.

3. Na ulaz kola prikazanog na sl. 4 dovodi se signal prikazan na sl. 5.
   a) Pronaći amplitudski spektar i spektar snage ulaznog signala,
   b) Ako je $1/(RC)=\omega_0$, gdje je ω_0 osnovna kružna učestanost ulaznog signala, odrediti amplitudski spektar i spektar snage izlaznog signala,
   c) Kako treba odrediti elemente kola R i C, pa da snaga trećeg harmonika izlaznog signala ne prelazi 1% srednje snage ulaznog signala?



Slika 4.



Slika 5.

Rešenje:

Funkciju sa Slike 5 možemo zapisati kao:

$$U_{ul}(t)=\frac{2E}{T}t,\qquad -\frac{T}{2}\le t\le\frac{T}{2}$$

Kompleksni spektar $U_{ul}(t)$:

$$U_{uln}=\frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}}U_{ul}(t)e^{-jn\omega_0 t}dt=\frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}}\frac{2E}{T}te^{-jn\omega_0 t}dt$$

$$\text{smjena: }\begin{pmatrix} U=t & dU=dt \\ dV=e^{-jn\omega_0 t}dt & V=-\frac{1}{jn\omega_0}e^{-jn\omega_0 t}\end{pmatrix}$$

$$=\frac{2E}{T^2}\left[\frac{te^{-jn\omega_0}}{jn\omega_0}\Bigg|_{-T/2}^{T/2}+\frac{1}{jn\omega_0}\int_{-T/2}^{T/2}e^{-jn\omega_0 t}dt\right]$$

S obzirom da je $e^{\pm jn\omega_0\frac{T}{2}}=e^{\pm jn\pi}=(-1)^n$, prethodni izraz se može svesti na:

$$U_{uln}=\frac{2E}{T^2}j\left(\frac{T}{2n\frac{2\pi}{T}}e^{-jn\omega_0\frac{T}{2}}+\frac{T/2}{n\frac{2\pi}{T}}e^{jn\,\omega_0\frac{T}{2}}\right)-\frac{1}{(jn\omega_0)^2}\underbrace{\left(e^{-jn\omega_0\frac{T}{2}}-e^{jn\omega_0\frac{T}{2}}\right)}_{0}$$

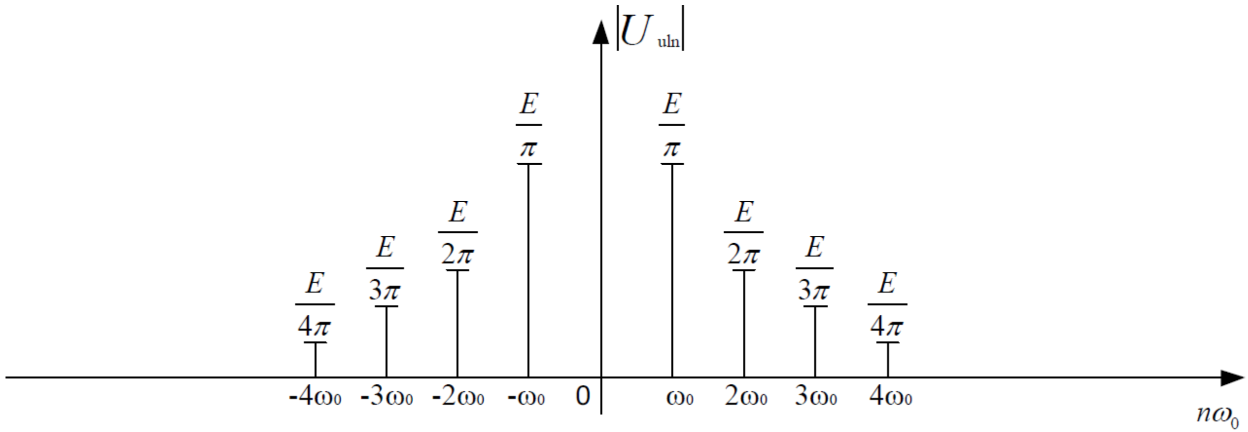
$$U_{uln}=(-1)^n\frac{E}{n\pi}e^{-j\frac{\pi}{2}}$$

Jednosmjernu komponentu nije moguće naći iz prethodnog izraza pa ćemo je naći preko definicije:

$$U_{ul0}=\frac{1}{T}\int_{-T/2}^{T/2}\frac{2E}{T}tdt=0$$

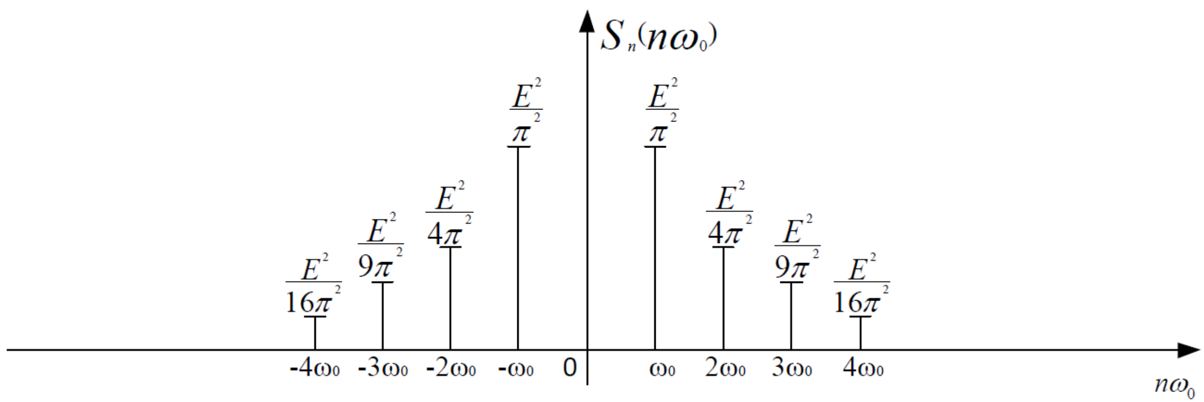
a) Amplitudski spektar ulaznog signala:

$$|U_{uln}|=\begin{cases}0; & n=0\\ \dfrac{E}{|n\pi|}; & n\neq 0\end{cases}$$



Spektar snage ulaznog signala:

$$S_n(n\omega_0) \triangleq |U_{uln}|^2 = \begin{cases} 0, & n = 0 \\ \frac{E^2}{n^2\pi^2}, & n \neq 0 \end{cases}$$



b) Funkcija prenosa kola je:

$$H(j\omega) = \frac{1/j\omega C}{1/j\omega C + R} = \frac{1}{1 + j\omega RC}$$

$$|H(j\omega)| = \frac{1}{\sqrt{1 + (n\omega_0 RC)^2}}$$

$$|\mathrm{U_{izn}}| = \begin{cases} 0; & \mathrm{n} = 0 \\ \dfrac{\mathrm{E}}{|\mathrm{n}\pi|}\dfrac{1}{\sqrt{1+(\mathrm{n}\omega_0\mathrm{RC})^2}}; & \mathrm{n} \neq 0 \end{cases} =$$

$$= \begin{cases} 0; & \mathrm{n} = 0 \\ \dfrac{\mathrm{E}}{|\mathrm{n}\pi|}\dfrac{1}{\sqrt{1+\mathrm{n}^2}}; & \mathrm{n} \neq 0 \end{cases}$$

$$S_{\mathrm{izn}}\,(n\omega_0) = \begin{cases} 0; & \mathrm{n} = 0 \\ \dfrac{E^2}{|n\pi|^2}\dfrac{1}{1+n^2}; & \mathrm{n} \neq 0 \end{cases}$$

d) R,C=?

$$P_{iz}(3\omega_0) \leq 0.01P_{ul}$$

$$P_{iz}(3\omega_0) = 2|U_{ul3}|^2 = \frac{2E^2}{3^2\pi^2}\frac{1}{1+(3\omega_0 RC)^2}$$

$$P_{ul} = \frac{1}{T}\int_0^T U_{ul}(t)^2 dt = \frac{1}{T}\int_{-T/2}^{T/2}\left(\frac{2E}{T}\right)^2 t^2 dt = \frac{4E^2}{T^3}\frac{t^3}{3}\bigg|_{-T/2}^{T/2} = \frac{E^2}{3}$$

$$\frac{2\mathrm{E}^2}{9\pi^2}\frac{1}{1+9(\omega_0\mathrm{RC})^2} \leq 0.01\frac{\mathrm{E}^2}{3}$$

$$\mathrm{RC} \geq \frac{1}{3\omega_0}\sqrt{\frac{2}{0.3\pi^2}-1}$$