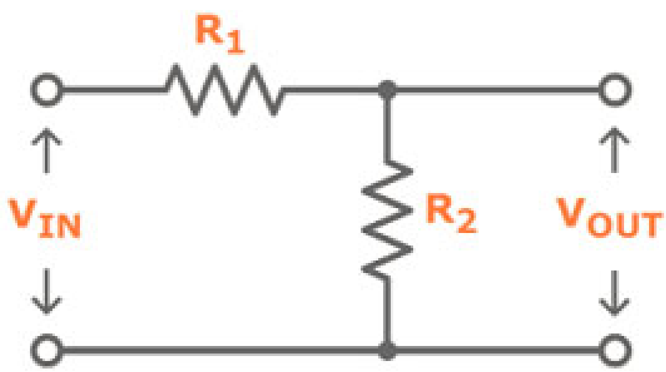
Frekvencijske karakteristike

- Prenosne funkcije (nule i polovi)
- Amplitudno-frekvencijeski dijagrami
- Fazno-frekvencijski dijagrami

Prenosne funkcije

- Frekvencijski nezavisno kolo.
- Pojačanje (*A*) ne zavisi od učestanosti (*f*)

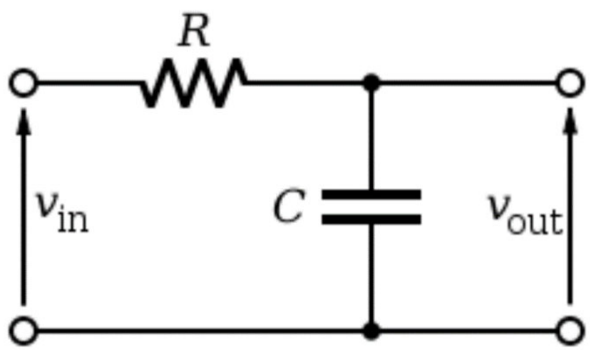


$$V_{out} = \frac{R_2}{R_1 + R_2} V_{ul}$$

$$A = \frac{V_{out}}{V_{in}} = \frac{R_2}{R_1 + R_2}$$

Prenosne funkcije

- **Frekvencijski zavisno kolo.**



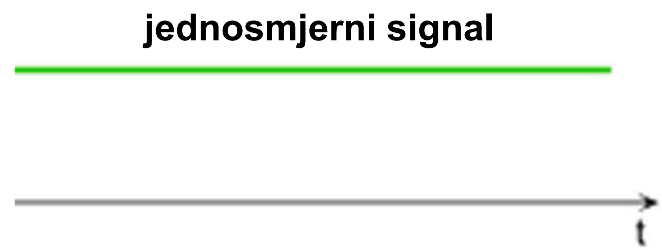
$$A(s) = \frac{V_{out}(s)}{V_{in}(s)} = \frac{\frac{1}{sC}}{R + \frac{1}{sC}}$$

$$A(s) = \frac{1}{1 + sCR}$$

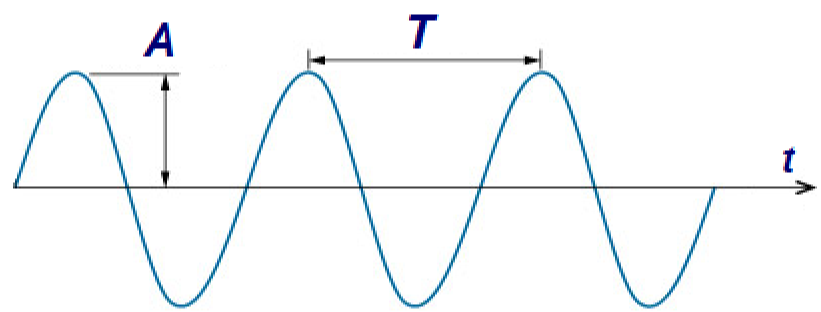
$$s_p = -\frac{1}{CR}$$

Frekvencija (učestanost) *s*

➢ **Nulta učestanost s=0.**



➢ **Realna učestanost s=jω**



harmonijski signal

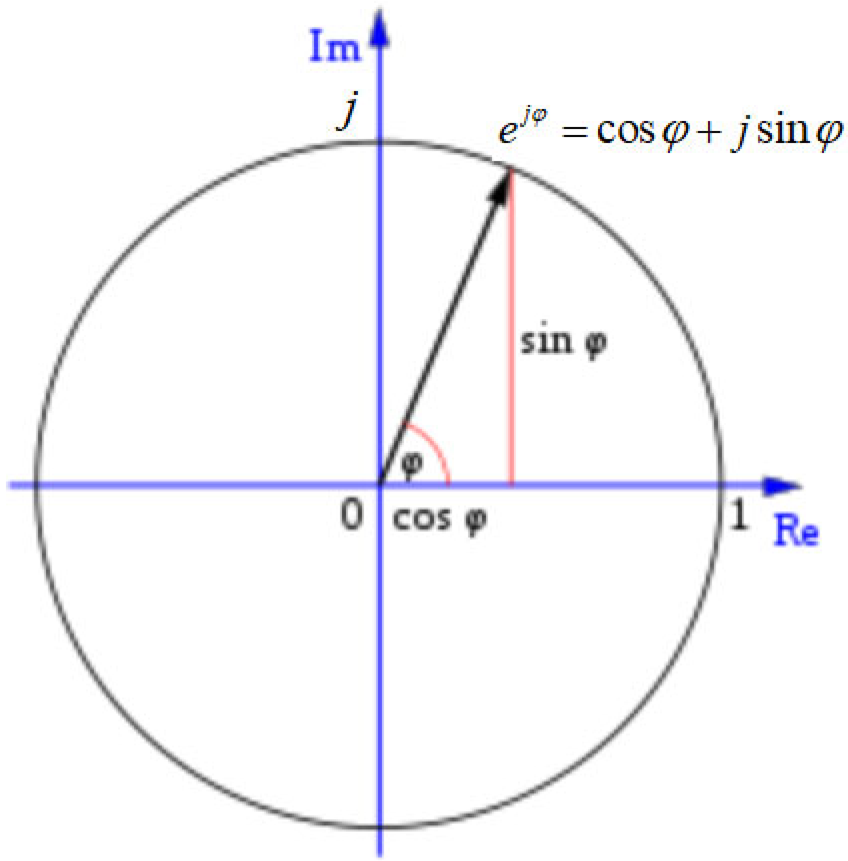
$$\omega - kružna \quad frekvencija$$

$$\omega = 2\pi f$$

$$f = \frac{1}{T} \qquad T - perioda$$

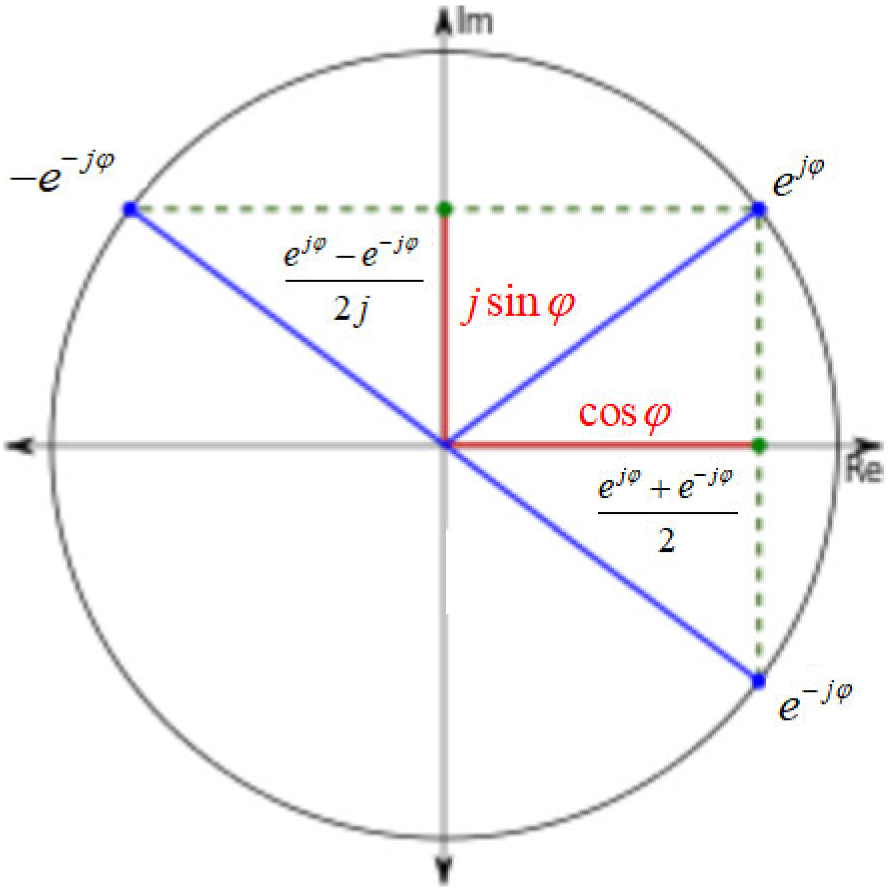
Harmonijski signali

Matematička pozadina je kompleksna



Harmonijski signali

Matematička pozadina je kompleksna



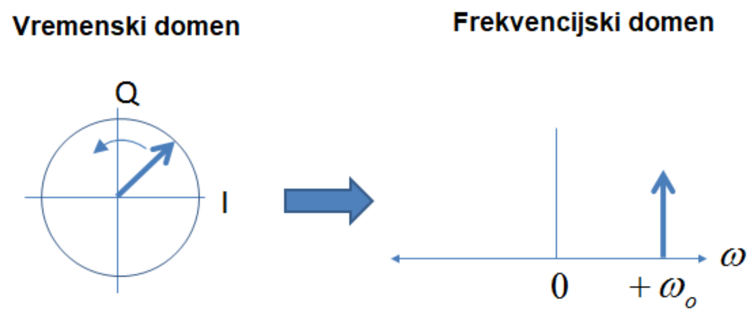
Sinusni (i kosinusni) talas je konstruisan iz dva rotirajuća vektora, koji rotiraju u suprotnim smjerovima (jedan smjer se smatra pozitivnom frekvencijom, a drugi negativnom)

$$\sin\varphi = \frac{e^{j\varphi}-e^{-j\varphi}}{2j}$$

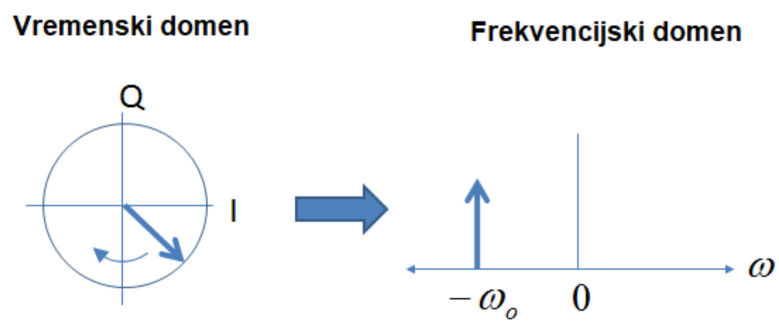
$$\cos\varphi = \frac{e^{j\varphi}+e^{-j\varphi}}{2}$$

Fourier-ova transformacija

Fourier-ove transformacija je korelacija (množenje i akumuliranje) signala u vremenskom domenu sa svakom mogućom frekvencijom.



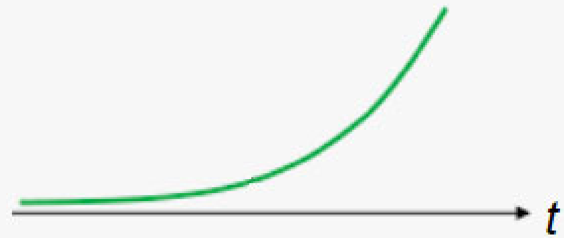
$$e^{j\omega_o t} \qquad \delta(\omega - \omega_o)$$



$$e^{-j\omega_o t} \qquad \delta(\omega + \omega_o)$$

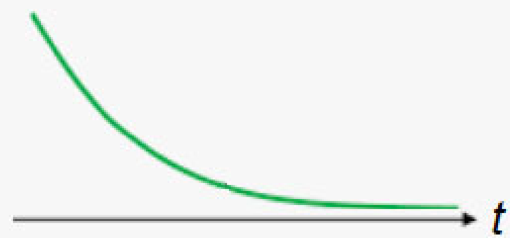
Imaginarna učestanost

$\sigma > 0$



Eksponencijalno rastući signal

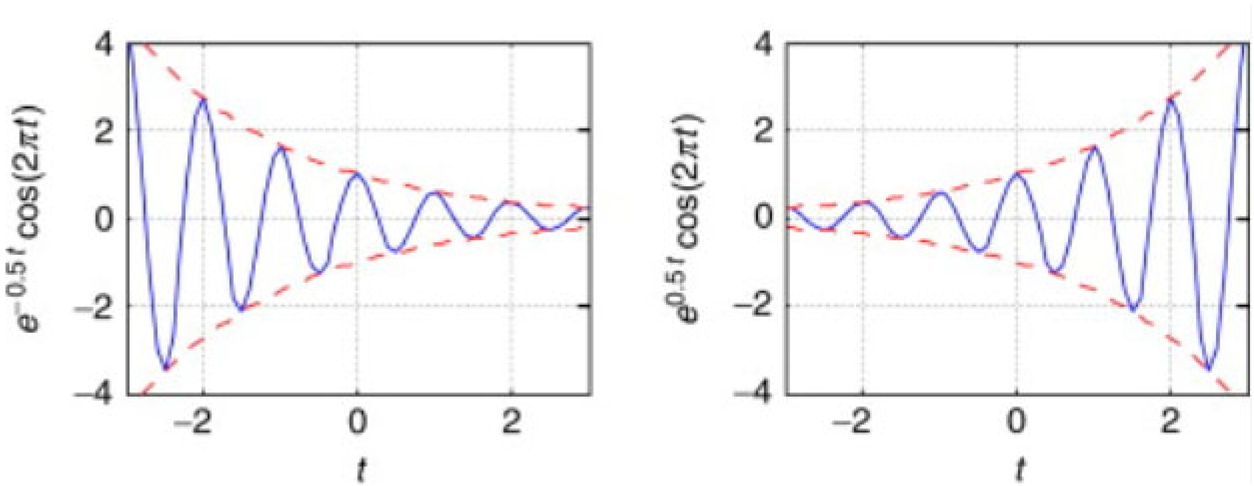
$\sigma < 0$



Eksponencijalno opadajući signal

Kompleksna učestanost

$$s = \sigma + j\omega$$



Eksponencijalno opadajući (rastući) harmonijski signal

ω **- realna učestanost**

σ **- imaginarna učestanost**

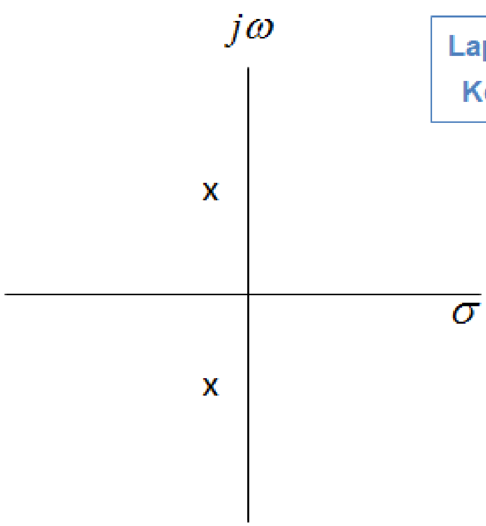
Laplace-ova transformacija

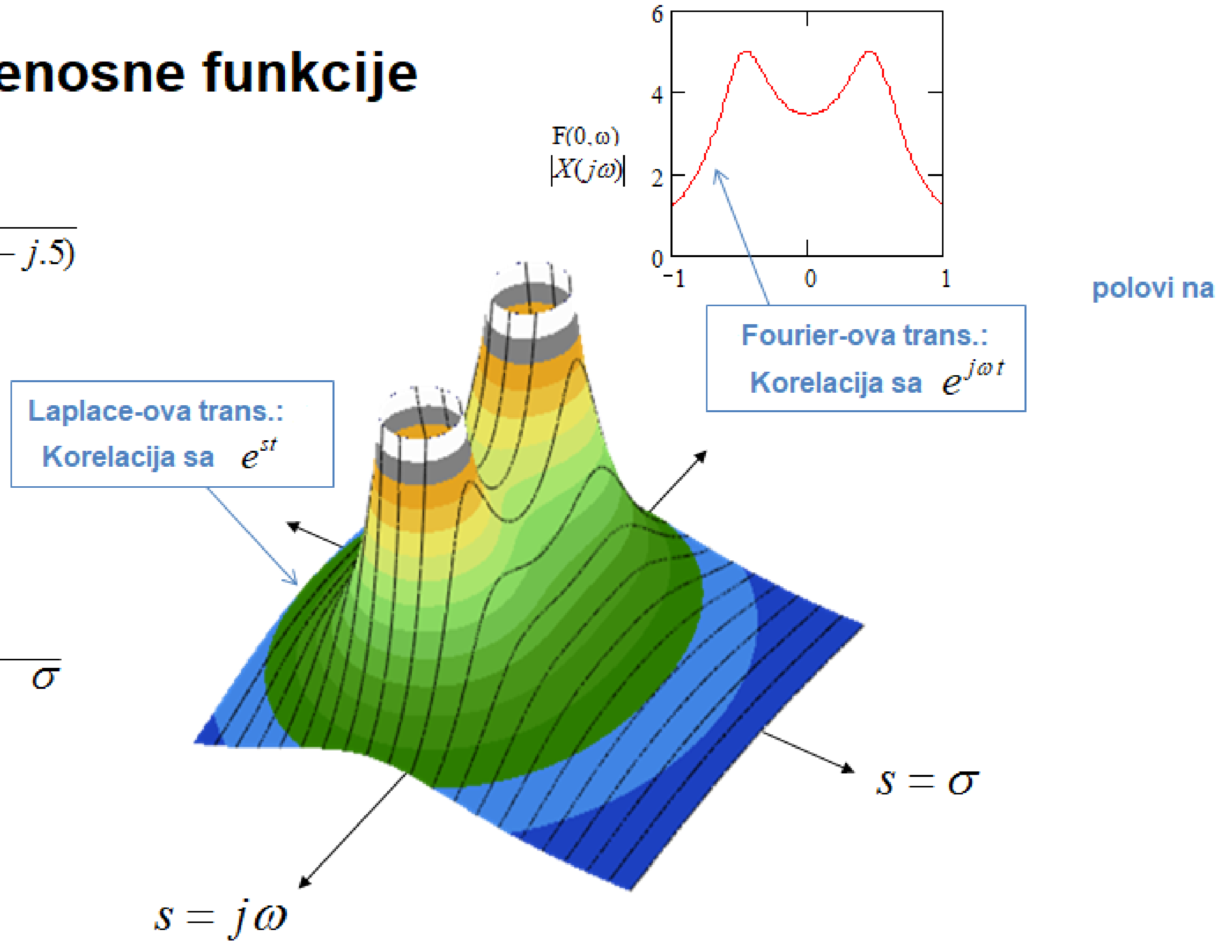
Laplace-ova transforamcija se može predstaviti kao površ u kompleksnoj ravni koja pokazuje korelaciju ulazne funkcije sa svim mogućim rotirajućim fazorima (kao kod Fourier-a), koji rastu i opadaju u vremenu.

Primjer prenosne funkcije

$$X(s) = \frac{1}{(s+.2+j.5)(s+.2-j.5)}$$

(polovi na $s = -.2 \pm j.5$)

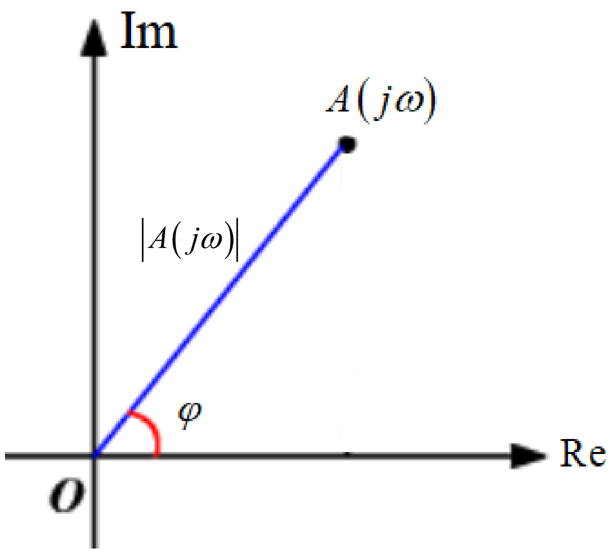




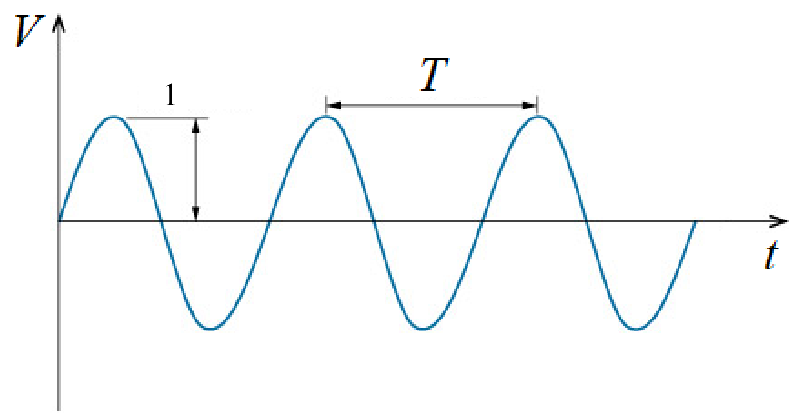
Prenosna funkcija

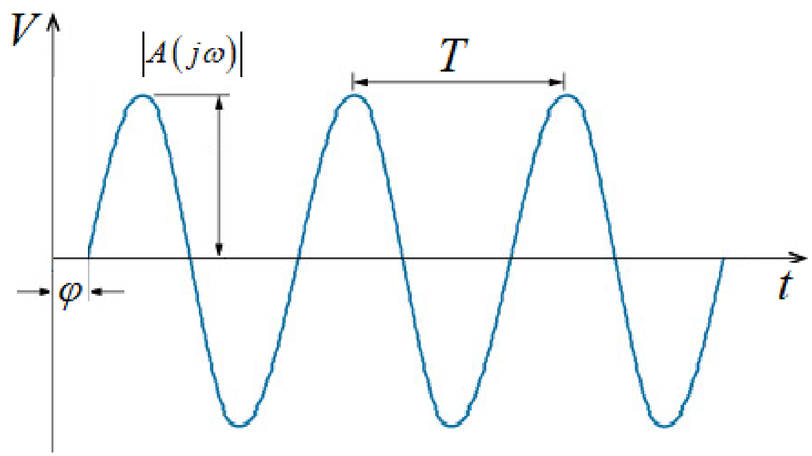
Kada u prenosnu funkciju $A(s)$, stavimo $s=j\omega$ dobijamo prenosnu funkciju za realne učestanosti

Kompleksno pojačanje



$$\varphi = Arg\{A(j\omega)\}$$





Moduo pojačanja - primjer

$$A(s) = \frac{1}{1+sRC} \qquad s_p = -\frac{1}{RC}$$

$$\left|A(j\omega)\right| = \left|\frac{1}{1+j\omega RC}\right| = \frac{1}{\sqrt{1+\omega^2C^2R^2}}$$

$$A_{dB} = 20\log\left|A(j\omega)\right| = 20\log\frac{1}{\sqrt{1+\omega^2C^2R^2}}$$

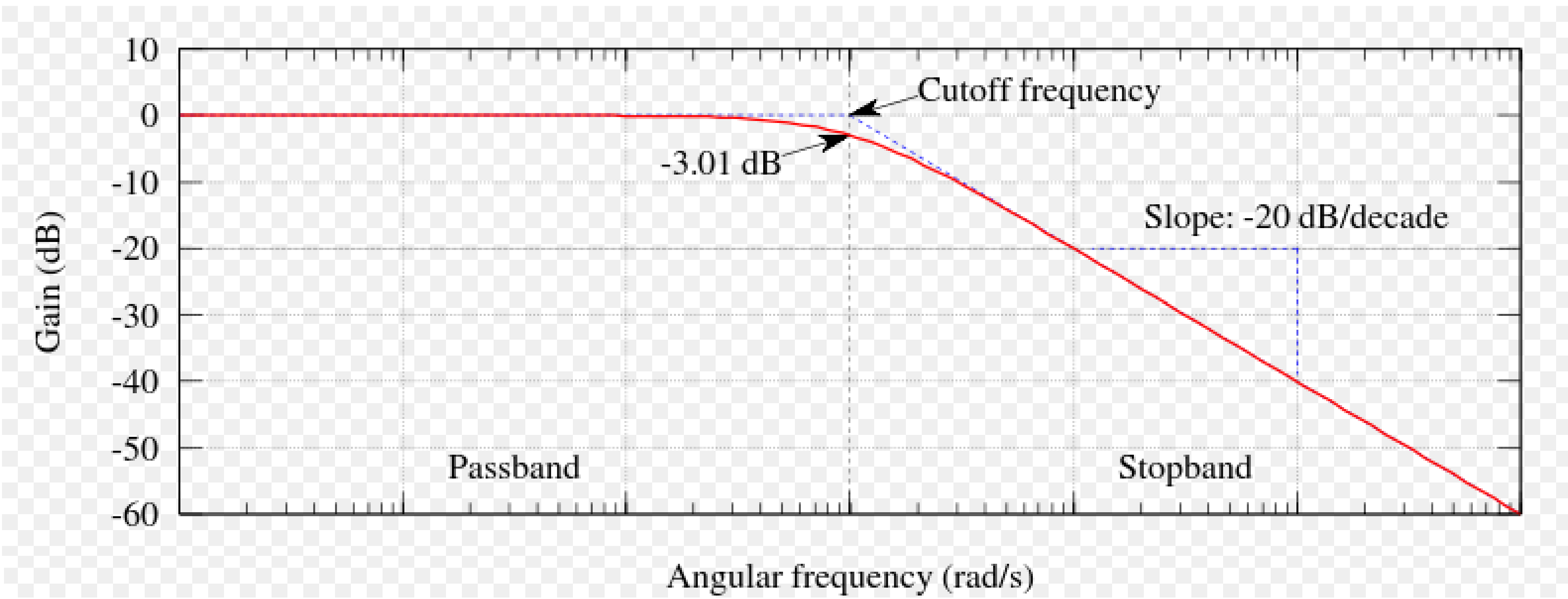
$$A_{dB} = 20(-1)\frac{1}{2}\log\left(1+\omega^2C^2R^2\right)$$

Asimptotski amplitudno frekvencijski dijagram

$$A_{dB} = -10\log\left(1+\omega^2 C^2 R^2\right)$$

$$\omega \to 0 \quad A_{dB} \to -10\log 1 = 0$$

$$\omega CR >> 1 \quad A_{dB} \approx -10\log\left(\omega^2 C^2 R^2\right) = -20\log \omega CR$$



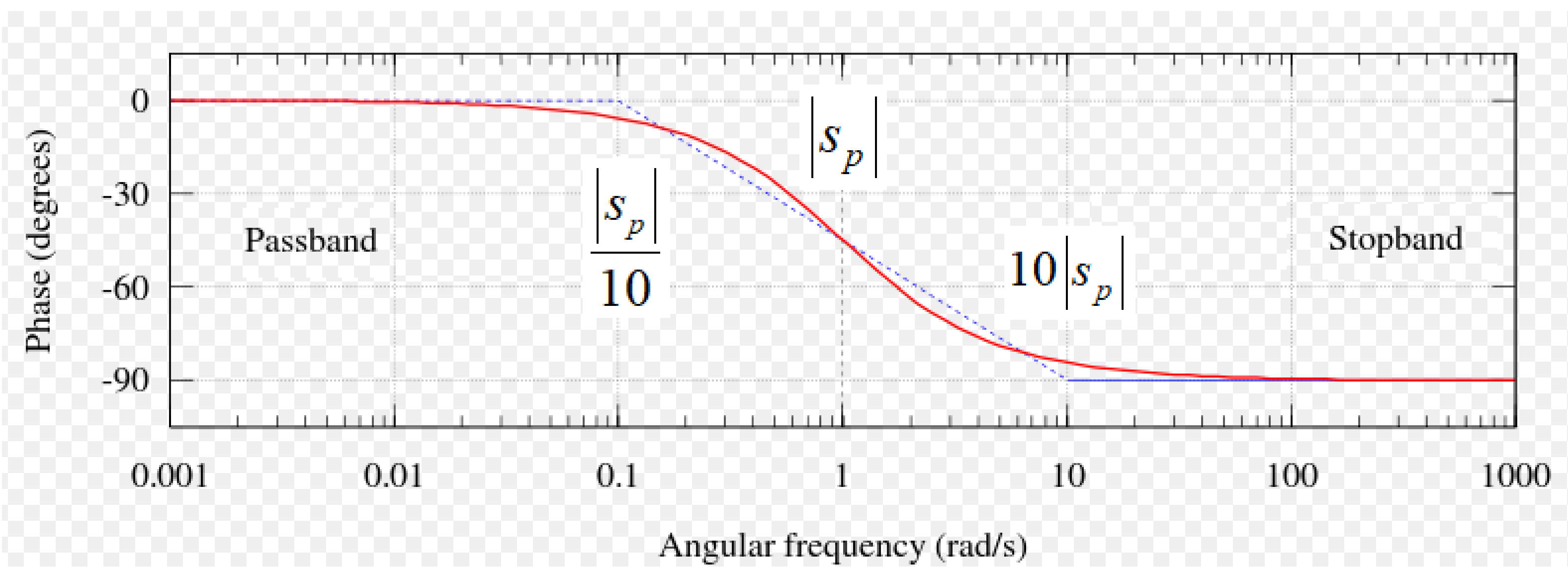
Logaritamsko pojačanje A_{dB}

• A= 0.001	AdB= -60dB
• A=0.01	AdB= -40dB
• A=0.1	AdB= -20dB
• A=1/2	AdB= -6dB
• A=1	AdB= 0dB
• A=2	AdB= +6dB
• A=10	AdB= +20dB
• A=100	AdB= +40dB

Faza prenosne funkcije

$$A(s) = \frac{1}{1 + sCR} \qquad A(j\omega) = \frac{1}{1 + j\omega CR}$$

$$\varphi(\omega) = Arg\{A(j\omega)\} = arctg\frac{\mathrm{Im}\{A(j\omega)\}}{\mathrm{Re}\{A(j\omega)\}} = -arctg\frac{\omega RC}{1} = -arctg(\omega RC)$$



Prenosna funkcija sa nulom

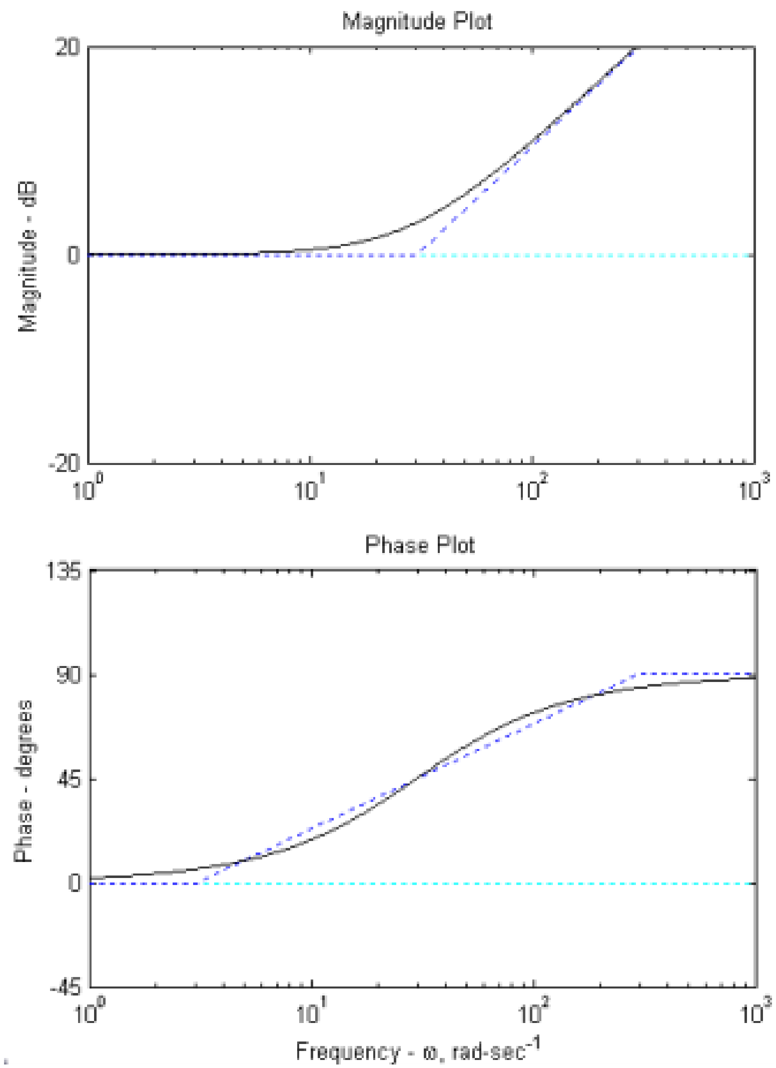
$$A(s) = 1 + sCR \qquad s_n = -\frac{1}{RC}$$

$$\left|A(j\omega)\right| = \sqrt{1 + \omega^2 C^2 R^2}$$

$$A_{dB} = 20\log\left|A(j\omega)\right| = +10\log\left(1 + \omega^2 C^2 R^2\right)$$

$$\varphi(\omega) = Arg\left\{A(j\omega)\right\} = arctg\frac{\mathrm{Im}\left\{A(j\omega)\right\}}{\mathrm{Re}\left\{A(j\omega)\right\}} = arctg\frac{\omega RC}{1} = arctg(\omega RC)$$

Amplitudni i fazni dijagram prenosne funkcije sa nulom



Nula prenosne funkcije podiže nagib amplitudsko frekvencijeske karakteristike za 20dB/dec

Nula podiže faznu karakteristiku za $\pi/2$

Opšti slučaj prenosne funkcije

$$A(s) = K\frac{(s-s_{n1})(s-s_{n2})\ldots}{(s-s_{p1})(s-s_{p2})\ldots}$$

$$A_{dB} = 20\log K + 20\log\left|j\omega - s_{n1}\right| + 20\log\left|j\omega - s_{n2}\right| + \ldots$$
$$-20\log\left|j\omega - s_{p1}\right| - 20\log\left|j\omega - s_{p2}\right| - \ldots$$

$$\varphi = Arg(j\omega - s_{n1}) + Arg(j\omega - s_{n2}) + \ldots$$
$$-Arg(j\omega - s_{p1}) - Arg(j\omega - s_{p2}) - \ldots$$

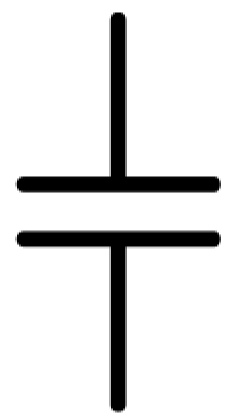
Analiza frekvencijskih karakteristika

- **Nalaženje prenosne funkcije kola.**
- **Nalaženje nula i polova prenosne funkcije**
- **Crtanje AF i FF dijagrama**

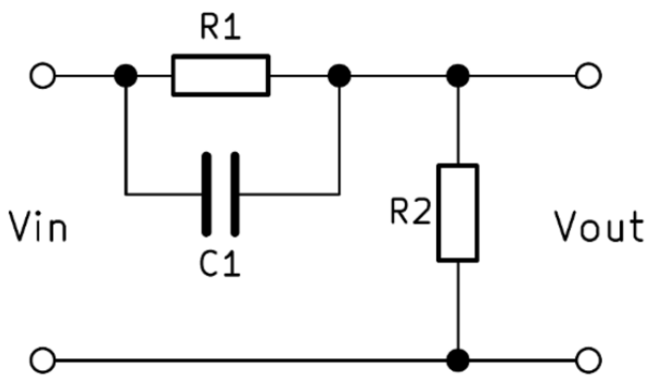
Nalaženje prenosne funkcije kola

- **Elektronske komponente (tranzistore, diode, itd) zamjenimo modelima za male signale i svodimo problem na linearno kolo sa koncentrisanim parametrima.**

- **Reaktanse uvijek donose polove.**



Kondezator u direktnoj grani donodi nulu

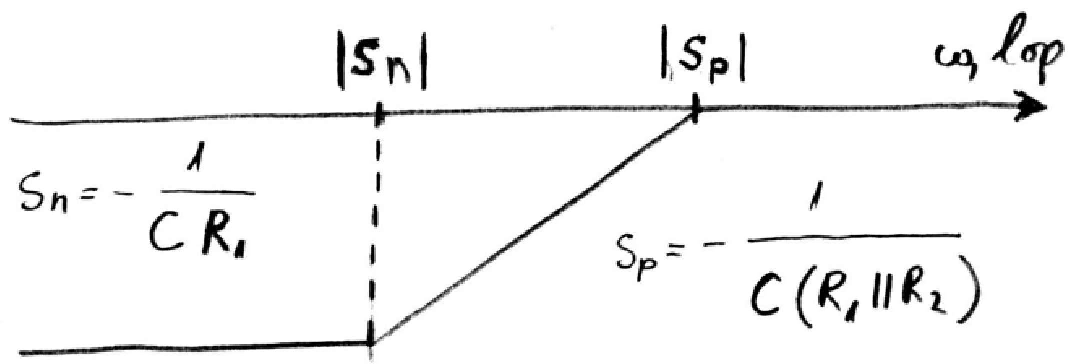


$$A(s)=\frac{R_2}{R_1+R_2}\frac{1+sC_1R_1}{1+sC_1\left(R_1\|R_2\right)}$$

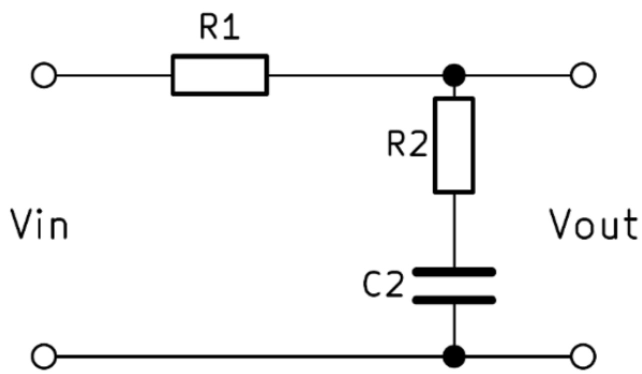
$$s_n=-\frac{1}{CR_1} \qquad s_p=-\frac{1}{C\left(R_1\|R_2\right)}$$

$$A(j\omega)=\frac{R_2}{R_1+R_2}\frac{1+j\omega C_1R_1}{1+j\omega C_1\left(R_1\|R_2\right)}$$

$$\omega=0 \quad\Rightarrow\quad A=\frac{R_2}{R_1+R_2} \qquad\qquad \omega\to\infty \quad\Rightarrow\quad A=1$$



Kondezator u otočnoj grani donosi nulu kada je vezan ne red sa otpornikom

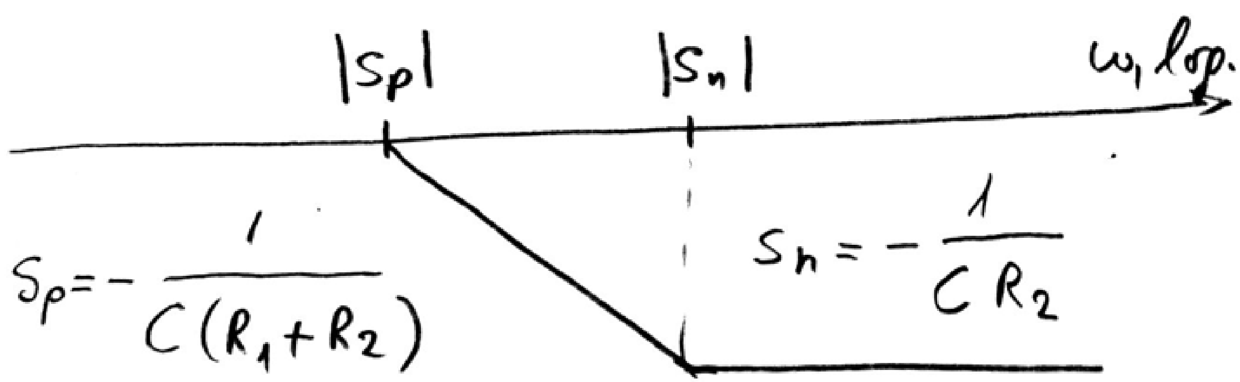


$$A(s) = \frac{1 + sC_2R_2}{1 + sC_2\left(R_1 \| R_2\right)}$$

$$s_n = -\frac{1}{CR_2} \qquad s_p = -\frac{1}{C_2\left(R_1 + R_2\right)}$$

$$A(j\omega) = \frac{1 + j\omega C_2 R_1}{1 + j\omega C_2\left(R_1 + R_2\right)}$$

$$\omega = 0 \quad \Rightarrow \quad A = 1 \qquad\qquad \omega \to \infty \quad \Rightarrow \quad A = \frac{R_2}{R_1 + R_2}$$

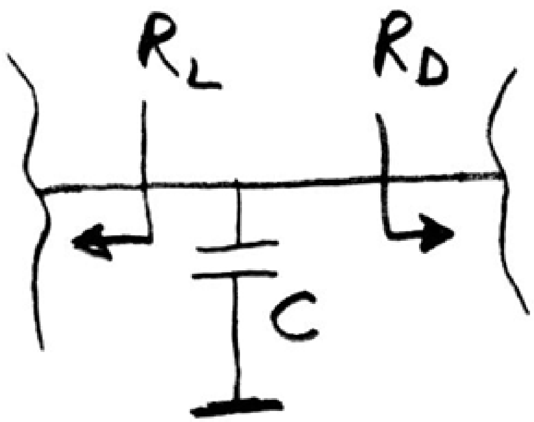


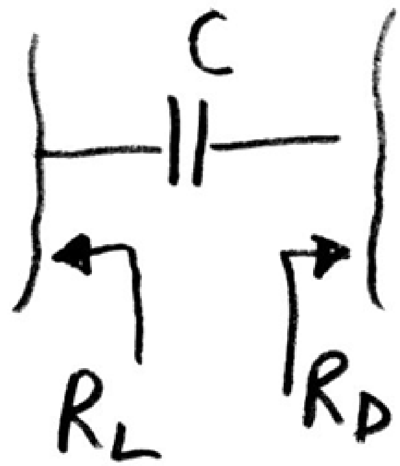
Još po nešto

- Induktivnost u otočnoj grani donosi nulu.
- Induktivnost u direktnoj grani donosi nulu kada je na red vezana sa otpornikom.
- Učestanost pola Sp=-1/τ, τ=C*Re, gdje je Re ekvivalentna otpornost koju “vidi” kondenzator.
- Analogno τ=L/Re, gdje je Re otpornost koju “vidi” induktivitet.

Nalaženje pola od kondezatora

$$s_p = -\frac{1}{\tau} \qquad \tau = CR_{ek}$$





$$\tau = C\left(R_L \| R_D\right) \qquad \tau = C\left(R_L + R_D\right)$$