11. Koristeći definiciju determinante izračunati:

a) $$D = \begin{vmatrix} 0 & 1 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & & \\ 0 & 0 & 0 & 0 & \cdots & 1 \\ 1 & 0 & 0 & 0 & \cdots & 0 \end{vmatrix} = (-1)^{N(\pi)} \cdot a_{12} \cdot a_{23} \cdot \ldots \cdot a_{n-1\,n} \cdot a_{n1}$$

$\pi: 2\ 3\ 4 \ldots n\ 1$

$N(\pi) = \underbrace{1+1+\ldots+1} = n-1$

$$D = (-1)^{n-1} \cdot 1 \cdot 1 \cdot \ldots \cdot 1 = (-1)^{n-1}$$

Gornja trougaona det.

b) $$\begin{vmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ 0 & a_{22} & a_{23} & \cdots & a_{2n} \\ 0 & 0 & a_{33} & \cdots & a_{3n} \\ \vdots & \vdots & \vdots & & \\ 0 & 0 & 0 & \cdots & a_{nn} \end{vmatrix} = (-1)^{N(\pi)} \cdot a_{11} \cdot a_{22} \cdot \ldots \cdot a_{nn}$$

$\pi: 1\ 2 \ldots n$

$N(\pi) = 0$

Gornja (donja) trougaona det jednaka je proizvodu elemenata sa glavne dijagonale

$$(-1)^0 \cdot a_{11} \cdot a_{22} \cdot \ldots \cdot a_{nn} = a_{11} \cdot a_{22} \cdot \ldots \cdot a_{nn}$$

12. Koristeći osobine determinanti izračunati det

a) $$\begin{vmatrix} a & x & x & \cdots & x \\ x & a & x & \cdots & x \\ x & x & a & \cdots & x \\ \vdots & \vdots & \vdots & & \vdots \\ x & x & x & \cdots & a \end{vmatrix}_n = \begin{vmatrix} (n-1)x+a & (n-1)x+a & \cdots & (n-1)x+a \\ x & a & x \cdots & x \\ x & x & a \cdots & x \\ \vdots & \vdots & & \\ x & x & x \cdots & a \end{vmatrix}$$

$$= ((n-1)x+a) \begin{vmatrix} 1 & 1 & 1 & \cdots & 1 \\ x & a & x & \cdots & x \\ x & x & a & \cdots & x \\ \vdots & \vdots & & & \\ x & x & x & \cdots & a \end{vmatrix} \ (-x) = ((n-1)x+a)$$

$$\begin{vmatrix} 1 & 1 & 1 & \cdots & 1 \\ 0 & a-x & 0 & \cdots & 0 \\ 0 & 0 & a-x & \cdots & 0 \\ \vdots & & \vdots & & \\ 0 & 0 & 0 & \cdots & a-x \end{vmatrix}_n = ((n-1)x+a) \cdot (a-x)^{n-1}$$

b) $\begin{vmatrix} x+a & x & x & \dots & x \\ x & x+a & x & \dots & x \\ x & x & x+a & \dots & x \\ \vdots & \vdots & \vdots & & \vdots \\ x & x & x & \dots & x+a \end{vmatrix} \cdot(-1) = \begin{vmatrix} x+a & x & x & \dots & x \\ -a & a & 0 & \dots & 0 \\ -a & 0 & a & \dots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ -a & 0 & 0 & \dots & a \end{vmatrix}$

$= \begin{vmatrix} nx+a & x & x & x \\ 0 & a & 0 & 0 \\ 0 & 0 & a & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \dots a \end{vmatrix}_n = (nx+a)\cdot a^{n-1}$

13. Ne razvijajući determinantu dokazati:

a) $\begin{vmatrix} a+b & c & 1 \\ b+c & a & 1 \\ a+c & b & 1 \end{vmatrix} = 0 \qquad \begin{vmatrix} a+b & c & 1 \\ b+c & a & 1 \\ a+c & b & 1 \end{vmatrix} = \begin{vmatrix} a+b+c & c & 1 \\ a+b+c & a & 1 \\ a+b+c & b & 1 \end{vmatrix}$

$= (a+b+c) \begin{vmatrix} 1 & c & 1 \\ 1 & a & 1 \\ 1 & b & 1 \end{vmatrix} = (a+b+c)\cdot 0$ Ako imamo dvije iste vrste ili kolone det je 0.

b) $\begin{vmatrix} 1 & a & bc \\ 1 & b & ac \\ 1 & c & ab \end{vmatrix} = \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} \qquad \begin{vmatrix} 1 & a & bc \\ 1 & b & ac \\ 1 & c & ab \end{vmatrix} = \frac{1}{a}\cdot\frac{1}{b}\cdot\frac{1}{c} \begin{vmatrix} a & a^2 & abc \\ b & b^2 & abc \\ c & c^2 & abc \end{vmatrix}$

$= \frac{abc}{abc} \begin{vmatrix} a & a^2 & 1 \\ b & b^2 & 1 \\ c & c^2 & 1 \end{vmatrix} = - \begin{vmatrix} 1 & a^2 & a \\ 1 & b^2 & b \\ 1 & c^2 & c \end{vmatrix} = \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$

c) $\begin{vmatrix} ax & a^2+x^2 & 1 \\ ay & a^2+y^2 & 1 \\ az & a^2+z^2 & 1 \end{vmatrix} = a(x-y)(x-z)(z-y)$

$\begin{vmatrix} ax & a^2+x^2 & 1 \\ ay & a^2+y^2 & 1 \\ az & a^2+z^2 & 1 \end{vmatrix} = a \cdot \begin{vmatrix} x & a^2+x^2 & 1 \\ y & a^2+y^2 & 1 \\ z & a^2+z^2 & 1 \end{vmatrix} = a \cdot \left(\begin{vmatrix} x & a^2 & 1 \\ y & a^2 & 1 \\ z & a^2 & 1 \end{vmatrix} + \begin{vmatrix} x & x^2 & 1 \\ y & y^2 & 1 \\ z & z^2 & 1 \end{vmatrix} \right)$

$$a\cdot\left(a^2\cdot\begin{vmatrix} x & 1 & 1 \\ y & 1 & 1 \\ z & 1 & 1 \end{vmatrix} - \begin{vmatrix} 1 & x^2 & x \\ 1 & y^2 & y \\ 1 & z^2 & z \end{vmatrix}\right) = a\cdot\begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} \cdot(-1)$$

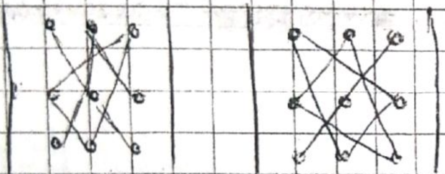
$$= a\cdot\begin{vmatrix} 1 & x & x^2 \\ 0 & y-x & y^2-x^2 \\ 0 & z-x & z^2-x^2 \end{vmatrix} = a\,(y-x)(z-x)\cdot\begin{vmatrix} 1 & x & x^2 \\ 0 & 1 & y+x \\ 0 & 1 & z+x \end{vmatrix} \cdot(-1)$$

$$= a\cdot(y-x)(z-x)\cdot\begin{vmatrix} 1 & x & x^2 \\ 0 & 1 & x+y \\ 0 & 0 & z-y \end{vmatrix} = a(y-x)(z-x)(z-y) = a(x-y)(x-z)(z-y)$$

Primjer 1: $\begin{vmatrix} 5 & 7 \\ -3 & 4 \end{vmatrix} = 5\cdot 4-(-3)\cdot 7 = 20+21 = 41$

Primjer 2: $\left|\begin{matrix} 1 & -1 & 3 \\ 2 & 0 & -4 \\ 5 & -3 & 2 \end{matrix}\right|\begin{matrix} 1 & -1 \\ 2 & 0 \\ 5 & -3 \end{matrix} = 0+20+(-18)-0-12-(-4) = -6$

Sarusovo pravilo

Primjer 3:  $P_1+P_2+P_3 = -(T_1+T_2+T_3)$

Primjer 4: Razvijanjem po drugoj vrsti izračunati det

$\begin{vmatrix} 1 & -1 & 3 \\ 2 & 0 & -4 \\ 5 & -3 & 2 \end{vmatrix} = a_{21}\cdot A_{21} + a_{22}\cdot A_{22} + a_{23}\cdot A_{23}$ $\begin{vmatrix} 1 & -1 & 3 \\ 2 & 0 & -4 \\ 5 & -3 & 2 \end{vmatrix} = 0+20-18-(0+12-4) = -6$

$A_{ij} = (-1)^{i+j} M_{ij}$ M_{ij}-minor koji se dobija kada se iz D izbaci i-ta vrsta j-ta kolona

$\rightarrow \begin{vmatrix} 1 & -1 & 3 \\ 2 & 0 & -4 \\ 5 & -3 & 2 \end{vmatrix} = 2\cdot(-1)^{2+1}\cdot\begin{vmatrix} -1 & 3 \\ -3 & 2 \end{vmatrix} + 0\cdot(-1)^{2+2}\cdot\begin{vmatrix} 1 & 3 \\ 5 & 2 \end{vmatrix} + (-4)\cdot(-1)^{2+3}\begin{vmatrix} 1 & -1 \\ 5 & -3 \end{vmatrix}$

$= -2\cdot(-2-(-9)) + 0 + 4\cdot(-3-(-5)) = -14+8 = -6$ Laplasov razvoj

Primjer 5: Neka je $A = \begin{pmatrix} 1 & 2 & 3 \\ -2 & -4 & -3 \\ 3 & 5 & 6 \end{pmatrix}$. Izračunati A^{-1}.

$A^{-1} = \frac{1}{\det A}\cdot\begin{pmatrix} A_{11} & A_{12} & \dots & A_{1n} \\ A_{21} & A_{22} & \dots & A_{2n} \\ \vdots & & & \\ A_{n1} & A_{n2} & \dots & A_{nn} \end{pmatrix}$

$\det A = \begin{vmatrix} 1 & 2 & 3 \\ -2 & -4 & -5 \\ 3 & 5 & 6 \end{vmatrix} = -24-30-30-(-36-25-24) = -84+85 = 1 \neq 0$ A^{-1} postoji

$A_{11} = \begin{vmatrix} -4 & -5 \\ 5 & 6 \end{vmatrix} = 1$ $A_{12} = -\begin{vmatrix} -2 & -5 \\ 3 & 6 \end{vmatrix} = -3$ $A_{13} = \begin{vmatrix} -2 & -4 \\ 3 & 5 \end{vmatrix} = 2$

$A_{21} = -\begin{vmatrix} 2 & 3 \\ 5 & 6 \end{vmatrix} = 3$, $A_{22} = \begin{vmatrix} 1 & 3 \\ 3 & 6 \end{vmatrix} = 3$ $A_{23} = -\begin{vmatrix} 1 & 2 \\ 3 & 5 \end{vmatrix} = 1$

$A_{31} = \begin{vmatrix} 2 & 3 \\ -4 & -5 \end{vmatrix} = 2$ $A_{32} = -\begin{vmatrix} 1 & 3 \\ -2 & -5 \end{vmatrix} = -1$ $A_{33} = \begin{vmatrix} 1 & 2 \\ -2 & -4 \end{vmatrix} = 0$

$A^{-1} = \frac{1}{1} \cdot \begin{pmatrix} 1 & 3 & 2 \\ -3 & -3 & -1 \\ 2 & 1 & 0 \end{pmatrix}$

Primjenom matrica riješiti sistem:

$x+2y+3z=5$

$4x+5y+6z=8$

$7x+8y=2$

Neka je $A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 0 \end{pmatrix}$, $X = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$, $B = \begin{pmatrix} 5 \\ 8 \\ 2 \end{pmatrix}$.

Tada dati sistem možemo zapisati u obliku matrične jedn.: $A^{-1} / A \cdot X = B$

$X = A^{-1} \cdot B$

$X = \begin{pmatrix} -2 \\ 2 \\ 1 \end{pmatrix} \Rightarrow \begin{matrix} x=-2 \\ y=2 \\ z=1 \end{matrix}$

Riješiti matričnu jednačinu, pa onda naći X ako je $A = \begin{pmatrix} 2 & 3 & 1 \\ 0 & -1 & 2 \\ 3 & 1 & -2 \end{pmatrix}$

$B = \begin{pmatrix} 1 & 3 & 1 \\ -2 & -4 & 2 \\ 2 & -2 & -5 \end{pmatrix}$ $C = \begin{pmatrix} 2 & 1 & 1 \\ 3 & 1 & 1 \\ 4 & 1 & 1 \end{pmatrix}$

$X^T \cdot A + 2C = X^T \cdot B + 3E$

R// $X^T \cdot A + 2C = X^T \cdot B + 3E$

$X^T \cdot A - X^T \cdot B = 3E - 2C$

$X^T (A-B) = 3E - 2C \,/\cdot (A-B)^{-1}$

$X^T = (3E-2C) \cdot (A-B)^{-1}$

$X = ((A-B)^{-1})^T \cdot (3E-2C)^T$

$X = (A^T - B^T)^{-1} \cdot (3E-2C)^T$

$H = A^T - B^T$

$H = \begin{pmatrix} 2 & 0 & 3 \\ 3 & -1 & 1 \\ 1 & 2 & -2 \end{pmatrix} - \begin{pmatrix} 1 & -2 & 2 \\ 3 & -4 & -2 \\ 1 & 2 & -5 \end{pmatrix} = \begin{pmatrix} 0 & 2 & 1 \\ 0 & 3 & 3 \\ 0 & 0 & 3 \end{pmatrix}$ $\det H = \begin{vmatrix} 0 & 2 & 1 \\ 0 & 3 & 3 \\ 0 & 0 & 3 \end{vmatrix} = 9 \neq 0$

$G = 3E - 2C$

$\begin{pmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{pmatrix} - \begin{pmatrix} 4 & 2 & 2 \\ 6 & 2 & 2 \\ 8 & 2 & 2 \end{pmatrix} = \begin{pmatrix} -1 & -2 & -2 \\ -6 & 1 & -2 \\ -8 & -2 & 1 \end{pmatrix}$

$G^T = \begin{pmatrix} -1 & -6 & -8 \\ -2 & 1 & -2 \\ -2 & -2 & 1 \end{pmatrix}$

$H_{11} = +\begin{vmatrix}3&3\\0&3\end{vmatrix} = 9 \quad H_{12} = -\begin{vmatrix}0&3\\0&3\end{vmatrix} = 0 \quad H_{13} = \begin{vmatrix}0&3\\0&0\end{vmatrix} = 0$

$H_{21} = -\begin{vmatrix}2&1\\0&3\end{vmatrix} = -6 \quad H_{22} = \begin{vmatrix}1&1\\0&3\end{vmatrix} = 3 \quad H_{23} = -\begin{vmatrix}1&2\\0&0\end{vmatrix} = 0$

$H_{31} = \begin{vmatrix}2&1\\3&3\end{vmatrix} = 3 \quad H_{32} = -\begin{vmatrix}1&1\\0&3\end{vmatrix} = -3 \quad H_{33} = \begin{vmatrix}1&2\\0&3\end{vmatrix} = 3$

$X = H^{-1} \cdot G^T$

$H^{-1} = \frac{1}{9}\begin{pmatrix}9&-6&3\\0&3&-3\\0&0&3\end{pmatrix} \qquad X = \frac{1}{9}\begin{pmatrix}9&-6&3\\0&3&-3\\0&0&3\end{pmatrix}\begin{pmatrix}-1&-6&-8\\-2&1&-2\\-2&-2&1\end{pmatrix}$

$= \frac{1}{9}\cdot\begin{pmatrix}-3&-66&-57\\0&9&-9\\-6&-6&3\end{pmatrix} = \begin{pmatrix}-1/3&-22/3&-19/3\\0&1&-1\\-2/3&-2/3&1/3\end{pmatrix}$

e

5. $(XA+B)^{-1}\cdot(XD+B) = D$

$A = \begin{pmatrix}-1&2&3\\0&2&1\\0&0&3\end{pmatrix} \quad B = \begin{pmatrix}2&-1&1\\0&-1&1\\0&0&1\end{pmatrix} \quad D = \begin{pmatrix}-1&1&2\\0&-1&2\\0&0&2\end{pmatrix}$

R// $(XA+B) \,/\, (XA+B)^{-1}\cdot(XD+B) = D \qquad XD - XAD = BD - B$

$XD + B = (XA+B)\cdot D \qquad X(D-AD) = BD - B \,/\, (D-AD)^{-1}$

$XD + B = XAD + BD \qquad X = (BD-B)(D-AD)^{-1}$

$G = BD - B = \begin{pmatrix}-4&4&3\\0&2&-1\\0&0&1\end{pmatrix} \quad H = D - AD = \begin{pmatrix}-2&4&-6\\0&1&0\\0&0&-4\end{pmatrix} \quad H^{-1} = \frac{1}{8}\cdot\begin{pmatrix}-4&16&6\\0&8&0\\0&0&-2\end{pmatrix}$

$X = G\cdot H^{-1} = \frac{1}{8}\cdot\begin{pmatrix}-4&4&3\\0&2&-1\\0&0&1\end{pmatrix}\begin{pmatrix}-4&16&6\\0&8&0\\0&0&-2\end{pmatrix} = \frac{1}{8}\begin{pmatrix}16&-32&-30\\0&16&2\\0&0&-2\end{pmatrix} = \begin{pmatrix}2&-4&-15/4\\0&2&1/4\\0&0&-1/4\end{pmatrix}$

$(AB)^{-1} = B^{-1}\cdot A^{-1}$

6. $[(AX)^T - X^T B]^{-1} = A - B, \; A = \begin{pmatrix}1&2&1\\3&0&0\\4&-1&2\end{pmatrix}, B = \begin{pmatrix}1&0&1\\0&0&1\\1&1&0\end{pmatrix}$

$[(AX)^T - X^T B]^{-1} = A - B \,|^{-1}$

$(AX)^T - X^T B = (A-B)^{-1} \qquad X = \left(\left((A^T-B)(A-B)\right)^{-1}\right)^T$

$X^T A^T - X^T B = (A-B)^{-1} \qquad X = \left(\left((A^T-B)(A-B)\right)^T\right)^{-1}$

$X^T(A^T - B) = (A-B)^{-1} \,/\cdot (A^T - B)^{-1}$

$X^T = (A-B)^{-1}\cdot(A^T-B)^{-1} \qquad X = \left((A^T - B^T)(A - B^T)\right)^{-1}$

$X^T = \left((A^T-B)\cdot(A-B)\right)^{-1} \,/^T$

Def. Neka je $A_{n\times n}$ kompleksna matrica, $A=(a_{ij})_{n\times n}$
1. $\overline{A}=(\overline{a}_{ij})_{n\times n}$ - konjugovana matrica matrice A
2. $A^* = (\overline{A})^T = (\overline{A^T})$ konjugovano-transponovana matrica

Svojstva: 1) $\overline{A}=A \Rightarrow A$ je realna 2) $(\overline{A+B}) = \overline{A}+\overline{B}$
3) $(\overline{\alpha A}) = \overline{\alpha}\cdot\overline{A}, \alpha\in C$ 4) $(\overline{AB}) = \overline{A}\cdot\overline{B}$ 5) Ako je A realna matrica tada je $(\overline{A})^{-1} = (\overline{A^{-1}})$ 6) $(A^*)^* = A$ 7) $(A+B)^* = A^*+B^*$
8) $(A\cdot B)^* = B^*\cdot A^*$ 9) $(\alpha A)^* = \overline{\alpha}\cdot A^*, \alpha\in C$ 10) Ako je A regularna matrica $(A^*)^{-1} = (A^{-1})^*$

Def. $A\in C^{n\times n}$ 1) Ako je $A^*=A$ A-hermitska matrica
2) Ako je $A^*=-A$ antihermitska

Def. $A\in R^{n\times n}$ Ako je $A\cdot A^T = A^T\cdot A = E$ A-ortogonalna matrica $(A^{-1}=A^T)$

Def. $A\in C^{n\times n}$

Ako je $A^*\cdot A = A\cdot A^* = E$ A je unitarna $(A^{-1}=A^*)$

7. Neka je A-ortogonalna matrica. Dokazati da je:
a) $\sum_{k=1}^{n} a_{ik}^2 = 1$ b) $\sum_{k=1}^{n} a_{ik}\cdot a_{jk} = 0,\ i\neq j$

a) A je ortogonalna $A\cdot A^T = A^T\cdot A = E$ $A=(a_{ij})_{n\times n}$ $A\cdot A^T = E$
$E=(e_{ij})_{n\times n}$ $e_{ij} = \begin{cases} 1, & i=j \\ 0, & i\neq j\end{cases}$ $e_{ii}=1$ (i-ta vrsta matrice A i-ta kolona A^T)
$\sum_{k=1}^{n} a_{ik}\cdot a_{ik} = 1$ $\sum_{k=1}^{n} a_{ik}^2 = 1$

b) $A\cdot A^T = E$ $e_{ij}=0, i\neq j$ (i-ta vrsta matrice A j-ta kolona A^T)
$\sum_{k=1}^{n} a_{ik}\cdot a_{jk} = 0,\ i\neq j$ j-ta vrsta A

$$A = \begin{pmatrix} 5 & -1 & 3 & 0 & 4 \\ 7 & -2 & 6 & 1 & 5 \\ 3 & 0 & 2 & 9 & 11 \\ 8 & -4 & 6 & 7 & 1 \end{pmatrix} \quad B = \begin{pmatrix} 5 & 0 \\ 3 & 9 \end{pmatrix} \quad \det B = M_1 = \begin{vmatrix} 5 & 0 \\ 3 & 9 \end{vmatrix}$$

$$C = \begin{pmatrix} -1 & 3 & 4 \\ 0 & 2 & 11 \\ -4 & 6 & 1 \end{pmatrix} \quad \det C = M_2 = \begin{vmatrix} -1 & 3 & 4 \\ 0 & 2 & 11 \\ -4 & 6 & 1 \end{vmatrix}$$

Glavni minor - na presjeku prvih k vrsta i k kolona
Rang matrice jednak je redu fundamentalnog minora
Fundamentalni minor, minor različit od nule, a svi minori većeg reda su jednaki nuli.

Primjer Odrediti rang matrice

$$A=\begin{pmatrix}-2&1&3&-1\\2&2&-1&3\\1&3&-2&4\\1&6&0&6\end{pmatrix}\sim\begin{pmatrix}1&3&-2&4\\2&2&-1&3\\-2&1&3&-1\\1&6&0&6\end{pmatrix}\begin{matrix}\cdot(-2)&\cdot 2&\cdot(-1)\end{matrix}\sim\begin{pmatrix}1&3&-2&4\\0&-4&3&-5\\0&7&-1&7\\0&3&2&2\end{pmatrix}\sim$$

$$\begin{pmatrix}1&-2&3&4\\0&3&-4&-5\\0&-1&7&7\\0&2&3&2\end{pmatrix}\sim\begin{pmatrix}1&-2&3&4\\0&-1&7&7\\0&3&-4&-5\\0&2&3&2\end{pmatrix}\begin{matrix}\cdot 3&\cdot 2\end{matrix}\sim\begin{pmatrix}1&-2&3&4\\0&-1&7&7\\0&0&17&16\\0&0&17&16\end{pmatrix}\cdot(-1)\sim$$

$$\sim\begin{pmatrix}1&-2&3&4\\0&-1&7&7\\0&0&17&16\\0&0&0&0\end{pmatrix}$$

rang $A=3$ $\quad M=\begin{vmatrix}1&-2&3\\0&-1&7\\0&0&17\end{vmatrix}=-17\neq 0$

fund. minor jer je $\neq 0$, a svi minori većeg reda su $=0$

$\left(\begin{vmatrix}1&-2&4\\0&-1&7\\0&0&16\end{vmatrix}=-16\neq 0\right)$

8. U zavisnosti od parametra a, odrediti rang matrice A

$$A=\begin{pmatrix}a&0&0&-1\\2&-2&-1&-2\\-1&3&1&1\end{pmatrix}\sim\begin{pmatrix}-1&3&1&1\\2&-2&-1&-2\\a&1&0&-1\end{pmatrix}\sim\begin{pmatrix}1&3&1&-1\\-2&-2&-1&2\\-1&1&0&a\end{pmatrix}\begin{matrix}\cdot 2\\+\\+\end{matrix}\sim\begin{pmatrix}1&3&1&-1\\0&4&1&0\\0&4&1&a-1\end{pmatrix}$$

$$\sim\begin{pmatrix}1&3&1&-1\\0&4&1&0\\0&0&0&a-1\end{pmatrix}$$

Ako je $a\neq 1$ rank$A=3$ $\quad M=\begin{vmatrix}1&3&1\\0&4&0\\0&0&a-1\end{vmatrix}=4(a-1)\neq 0$

Ako je $a=1$ $\quad A\sim\begin{pmatrix}1&3&1&-1\\0&4&1&0\\0&0&0&0\end{pmatrix}$ rang$A=2$,

$$M=\begin{vmatrix}1&3\\0&4\end{vmatrix}=4\neq 0$$