* Sistemi linearnih jednačina *

$$a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n = b_1$$
$$a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n = b_2$$
$$\vdots$$
$$a_{n1}x_1 + a_{n2}x_2 + \dots + a_{nn}x_n = b_n$$

Kramerovo pravilo

$$D = \begin{vmatrix} a_{11} \dots a_{1n} \\ a_{21} \dots a_{2n} \\ \vdots \\ a_{n1} \dots a_{nn} \end{vmatrix}$$

D_i - det koja se dobija iz D kada se i-ta vrsta zamjeni kolonama slobodnih članova. $\left(x_i = \frac{D_i}{D}\right)$

1) $D \neq 0 \Rightarrow$ sistem saglasan i određen
2) $D = 0 \wedge D_i \neq 0$ za neko $i \in \{1..n\}$ sistem je nesaglasan
3) $D = 0 \wedge D_i = 0 \ \forall i = \overline{1,n}$ dodatno diskutujemo

Primjer: Riješiti sistem:

$x_1+2x_2+x_3=8$

$3x_1+2x_2+x_3=10$

$4x_1+3x_2-2x_3=4$

$D=\begin{vmatrix}1&2&1\\3&2&1\\4&3&-2\end{vmatrix}=14\neq 0 \Rightarrow$ Sistem je sag

$D_1=\begin{vmatrix}8&2&1\\10&2&1\\4&3&-2\end{vmatrix}=14 \quad D_2=\begin{vmatrix}1&8&1\\3&10&1\\4&4&-2\end{vmatrix}=28$

$D_3=\begin{vmatrix}1&2&8\\3&2&10\\4&3&4\end{vmatrix}=42$

$x_1=\frac{D_1}{D}=\frac{14}{14}=1 \quad x_2=\frac{D_2}{D}=\frac{28}{14}=2 \quad x_3=\frac{42}{14}=3$

Jedinstveno rešenje $(1,2,3)$

Primjenom Kramerovog pravila diskutovati i riješiti sist. lin. jedn. u zavisnosti od parametra a

$ax-y+3z=a-1$

$x+ay-z=1$

$4x+3y+z=3$

$D=\begin{vmatrix}a&-1&3\\1&a&-1\\4&3&1\end{vmatrix}=a^2+4+9-(12a-3a-1)=a^2-9a+14$
$=(a-7)(a-2)$

$D_1=\begin{vmatrix}a-1&-1&3\\1&a&-1\\3&3&1\end{vmatrix}=(a-2)(a-5) \quad D_2=\begin{vmatrix}a&a-1&3\\1&1&-1\\4&3&1\end{vmatrix}=-(a-2)$

$D_3=\begin{vmatrix}a&-1&a-1\\1&a&1\\4&3&3\end{vmatrix}=-(a-2)^2$

I $a\neq 7 \wedge a\neq 2 \quad D\neq 0 \Rightarrow$ sistem saglasan i određen

$x=\frac{D_1}{D}=\frac{\cancel{(a-2)}(a-5)}{(a-7)\cancel{(a-2)}}=\frac{a-5}{a-7} \quad y=\frac{D_2}{D}=\frac{-(a-2)}{(a-7)(a-2)}=\frac{1}{7-a}$

$z=\frac{D_3}{D}=\frac{-(a-2)^2}{(a-7)\cancel{(a-2)}}=\frac{2-a}{a-7}$ Jedinstveno rešenje $\left(\frac{a-5}{a-7},\frac{1}{7-a},\frac{2-a}{a-7}\right)$

II $a=7 \quad D=0 \wedge D_1=10\neq 0 \Rightarrow$ nesaglasan

III $a=2 \quad D=0 \wedge D_1=0 \wedge D_2=0 \wedge D_3=0$ - dodatno diskutujemo

Za $a=2$ sistem glasi

$2x-y+3z=1$

$x+2y-z=1$

$4x+3y+z=3$

$\left(\begin{array}{ccc|c}2&-1&3&1\\1&2&-1&1\\4&3&1&3\end{array}\right)\sim\left(\begin{array}{ccc|c}1&2&-1&1\\2&-1&3&1\\4&3&1&3\end{array}\right)\cdot(-2)\cdot(-4)\sim\left(\begin{array}{ccc|c}1&2&-1&1\\0&-5&5&-1\\0&-5&5&-1\end{array}\right)\cdot(-1)$

$\sim\left(\begin{array}{ccc|c}1&2&-1&1\\0&-5&5&-1\\0&0&0&0\end{array}\right)$

$x+2y-z=1$
$-5y+5z=-1$

$y=\frac{1+5z}{5}$

$x=\frac{3-5z}{5}$

Sistem saglasan i neodređen

z - slobodna nepoznat

Vježbe

1. Primjenom Kramerovog pravila diskutovati i riješiti sistem u zavisnosti od parametra a.

$$\begin{aligned} ax+y+z&=1 \\ x+ay+z&=2 \\ x+y+az&=-3 \end{aligned}$$

$$D=\begin{vmatrix} a & 1 & 1 \\ 1 & a & 1 \\ 1 & 1 & a \end{vmatrix} = -\begin{vmatrix} 1 & 1 & a \\ 1 & a & 1 \\ a & 1 & 1 \end{vmatrix} \begin{matrix} \cdot(-1) & \cdot(-a) \\ & \\ & \end{matrix} = -\begin{vmatrix} 1 & 1 & a \\ 0 & a-1 & 1-a \\ 0 & 1-a & 1-a^2 \end{vmatrix} = -\begin{vmatrix} 1 & 1 & a \\ 0 & a-1 & 1-a \\ 0 & 0 & -a^2-a+2 \end{vmatrix} = -\begin{vmatrix} 1 & 1 & a \\ 0 & a-1 & 1-a \\ 0 & 0 & -(a-1)(a+2) \end{vmatrix} = (a-1)^2(a+2)$$

$$D_1=\begin{vmatrix} 1 & 1 & 1 \\ 2 & a & 1 \\ -3 & 1 & a \end{vmatrix} = (a-1)(a+2)$$

$$D_2=\begin{vmatrix} a & 1 & 1 \\ 1 & 2 & 1 \\ 1 & -3 & a \end{vmatrix} = 2(a-1)(a+2)$$

$$D_3=\begin{vmatrix} a & 1 & 1 \\ 1 & a & 2 \\ 1 & 1 & -3 \end{vmatrix} = -3(a-1)(a+2)$$

I $a\neq 1 \land a\neq -2$ $\quad D\neq 0 \Rightarrow$ sistem je saglasan i određen

$$x=\frac{D_1}{D}=\frac{1}{a-1} \qquad y=\frac{D_2}{D}=\frac{2}{a-1} \qquad z=\frac{D_3}{D}=\frac{3}{1-a}$$

Jedinstveno rešenje $\left(\frac{1}{a-1}, \frac{2}{a-1}, \frac{3}{1-a}\right)$

II $a=1$ $\quad D=0 \land D_1=0 \land D_2=0 \land D_3=0 \Rightarrow$ dodatno disk.

Za $a=1$ sistem glasi:

$$\begin{aligned} x+y+z&=1 \\ x+y+z&=2 \\ x+y+z&=3 \end{aligned} \qquad \left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 2 \\ 1 & 1 & 1 & -3 \end{array}\right) \cdot(-1) \sim \left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -4 \end{array}\right) \cdot 4 \sim \left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$0=1$ $\perp$ $\Rightarrow$ sistem je nesaglasan

III $a=-2$ $\quad D=0 \land D_1=0 \land D_2=0 \land D_3=0 \Rightarrow$ dodatno disk.

Za $a=-2$ sistem glasi:

$$\begin{aligned} -2x+y+z&=1\\ x-2y+z&=2\\ x+y-2z&=-3 \end{aligned} \quad \left(\begin{array}{ccc|c} -2 & 1 & 1 & 1\\ 1 & -2 & 1 & 2\\ 1 & 1 & -2 & -3 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & -2 & 1 & 2\\ -2 & 1 & 1 & 1\\ 1 & 1 & -2 & -3 \end{array}\right) \begin{array}{c} \cdot 2 \quad \cdot(-1) \\ \\ \\ \end{array} \sim$$

$$\left(\begin{array}{ccc|c} 1 & -2 & 1 & 2\\ 0 & -3 & 3 & 5\\ 0 & 3 & -3 & -5 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & -2 & 1 & 2\\ 0 & -3 & 3 & 5\\ 0 & 0 & 0 & 0 \end{array}\right) \quad \begin{aligned} x-2y+z&=2\\ -3y+3z&=5 \end{aligned}$$

$$y=\frac{3z-5}{3}$$

Sistem je saglasan i neodređen $x=2y+2-z=2\cdot\frac{3z-5}{3}+2-z$

Opšte rešenje sistema je: $=\frac{3z-4}{3}$

$\left(\frac{3z-4}{3}, \frac{3z-5}{3}, z\right)$ z - slobodna nepoznata

2. Diskutovati i riješiti sistem primjenom Kramerovog pr. a u zavisnosti od parametara a i b

$$\begin{aligned} ax+by+2z&=1\\ ax+(2b-1)y+3z&=1\\ ax+by+(b+3)z&=2b-1 \end{aligned}$$

$$D=\begin{vmatrix} a & b & 2\\ a & 2b-1 & 3\\ a & b & b+3 \end{vmatrix} \begin{array}{c}\cdot(-1)\\ \\ \\ \end{array} = \begin{vmatrix} a & b & 2\\ 0 & b-1 & 1\\ 0 & 0 & b+1 \end{vmatrix} = a(b-1)(b+1)$$

$$D_1=\begin{vmatrix} 1 & b & 2\\ 1 & 2b-1 & 3\\ 2b-1 & b & b+3 \end{vmatrix} \begin{array}{c}\cdot(1-2b) \quad \cdot(-1)\\ \\ \\ \end{array} = \begin{vmatrix} 1 & b & 2\\ 0 & b-1 & 1\\ 0 & 2b(1-b) & 5-3b \end{vmatrix} \begin{array}{c} \\ \cdot 2b \\ \\ \end{array}$$

$$=\begin{vmatrix} 1 & b & 2\\ 0 & b-1 & 1\\ 0 & 0 & 5-b \end{vmatrix} = (b-1)(5-b) \qquad D_2=\begin{vmatrix} a & 1 & 2\\ a & 1 & 3\\ a & 2b-1 & b+3 \end{vmatrix} = 2a(1-b)$$

$$D_3=\begin{vmatrix} a & b & 1\\ a & 2b-1 & 1\\ a & b & 2b-1 \end{vmatrix} = 2a(b-1)^2$$

I $\quad a\neq 0 \wedge b\neq 1 \wedge b\neq -1 \quad D\neq 0 \Rightarrow$ sistem je saglasan i određen

$$x=\frac{D_1}{D}=\frac{5-b}{a(b+1)} \quad y=\frac{D_2}{D}=-\frac{2}{b+1} \quad z=\frac{D_3}{D}=\frac{2(b-1)}{b+1}$$

Jedinstveno rešenje je $\left(\frac{5-b}{a(b+1)}, \frac{-2}{b+1}, \frac{2(b-1)}{b+1}\right)$

II $b=1$ $D=0 \wedge D_1=0 \wedge D_2=0 \wedge D_3=0 \Rightarrow$ dodatno disk.

Za $b=1$ sistem glasi:

$$\begin{aligned} ax+y+2z&=1\\ ax+y+3z&=1\\ ax+y+4z&=1 \end{aligned} \quad \left(\begin{array}{ccc|c} a & 1 & 2 & 1\\ a & 1 & 3 & 1\\ a & 1 & 4 & 1 \end{array}\right) \cdot(-1) \sim \left(\begin{array}{ccc|c} a & 1 & 2 & 1\\ 0 & 0 & 1 & 0\\ 0 & 0 & 2 & 0 \end{array}\right) \cdot(-2)$$

$$\sim \left(\begin{array}{ccc|c} a & 1 & 2 & 1\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 0 \end{array}\right) \quad \begin{aligned} ax+y+2z&=1\\ z&=0\\ \hline y&=1-ax\\ z&=0 \end{aligned}$$

Sistem je saglasan i neodređen. Opšte rešenje je $(x, 1-ax, 0)$ gdje je x slobodna nepozn.

III $b=-1$ $D=0$ $D_1=-12\neq 0$

$\Rightarrow$ sistem je nesaglasan

IV $a=0 \wedge b\neq \pm 1$ $D=0 \wedge D_1=(b-1)(5-b)$ $D_2=0$ $D_3=0$
$\neq 0$

1° $b\neq 5$

$D_1\neq 0 \Rightarrow$ sistem je nesaglasan

2° $b=5$ $D_1=0 \Rightarrow$ dodatno diskutujemo

Za $a=0 \wedge b=5$ sistem glasi:

$$\begin{aligned} 5y+2z&=1\\ 9y+3z&=1\\ 5y+8z&=9 \end{aligned} \quad z=\frac{8}{6}=\frac{4}{3} \quad y=-\frac{1}{3}$$

Sistem je saglasan i neodređen. Opšte rešenje je $(x, -\frac{1}{3}, \frac{4}{3})$ x-slobodna nepoznata

3. Primjenom Kroneker-Kapelijeve teor. diskutovati i rješiti sistem u zavisnosti od parametra m.

$$\begin{aligned} mx+y+z&=1\\ x+(3-m)y+z&=m\\ mx+(m+1)y+z&=1-m \end{aligned} \quad \left(\begin{array}{ccc|c} m & 1 & 1 & 1\\ 1 & 3-m & 1 & m\\ m & m+1 & 1 & 1-m \end{array}\right)$$

- proširena matrica kolona sl. članova se ne može mijenjati kolonama se mogu samo mijenjati mjesta

$$\sim \left(\begin{array}{ccc|c} 1 & 1 & m & 1\\ 1 & 3-m & 1 & m\\ 1 & m+1 & m & 1-m \end{array}\right) \cdot(-1) \sim \left(\begin{array}{ccc|c} 1 & 1 & m & 1\\ 0 & 2-m & 1-m & m-1\\ 0 & m & 0 & -m \end{array}\right)$$

$$\sim \left(\begin{array}{ccc|c} 1 & m & 1 & 1\\ 0 & 1-m & 2-m & m-1\\ 0 & 0 & m & -m \end{array}\right)$$

I $m\neq 1$ $m\neq 0$

rang $A=3$ $M=\begin{vmatrix} 1 & m & 1\\ 0 & 1-m & 2-m\\ 0 & 0 & m \end{vmatrix}=(1-m)m\neq 0$

rang $B=3$

$rang A = rang B = 3$ - broj nepoznatih $\Rightarrow$ na ons. teor. Kron.-Kapeli sistem je saglasan i odredjen:

$$\begin{aligned} z + mx + y &= 1 \\ (1-m)x + (2-m)y &= m-1 \\ my &= -m \end{aligned} \qquad y = -1, \quad x = \frac{m-1-(2-m)\cdot(-1)}{1-m} = \frac{1}{1-m}$$

$$z = 1 - mx - y = 1 - \frac{m}{1-m} + 1 = 2 - \frac{m}{1-m} = \frac{2-3m}{1-m}$$

Jedinstveno rešenje je $\left(\frac{1}{1-m}, -1, \frac{2-3m}{1-m}\right)$

II $m=1$

$$B \Leftrightarrow \left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & -1 \end{array}\right) \cdot(-1) \sim \left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{array}\right)$$

$rang A = 2 \quad M_1 = \begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix} \neq 0$

$rang B = 3 \quad M_2 = \begin{vmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{vmatrix} \neq 0$

$rang A \neq rang B \Rightarrow$ nesaglasan

III $m=0$

$$B \Leftrightarrow \left(\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 0 & 1 & 2 & -1 \\ 0 & 0 & 0 & 0 \end{array}\right) \quad rang A = 2 \quad M_1 = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1 \neq 0$$

$rang B = 2$ - isti minor

$rang A = rang B = 2 < 3$ br. nepoznatih $\Rightarrow$ sagl. i neod.

$$\begin{aligned} z + y &= 1 \\ x + 2y &= -1 \end{aligned} \qquad \begin{aligned} x &= -1-2y \\ z &= 1-y \end{aligned}$$

Opšte rešenje je $(-1-2y, y, 1-y)$ y - slobodna nepoznata

4. U zavisnosti od parametra t diskutovati i riješiti sistem primjenom Kronek.-Kapeli.-teor.

$$\begin{aligned} tx + y + z &= -t \\ x + ty + z &= -1 \\ x + y + tz &= t+1 \end{aligned} \quad \left(\begin{array}{ccc|c} t & 1 & 1 & -t \\ 1 & t & 1 & -1 \\ 1 & 1 & t & t+1 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 1 & t & t+1 \\ 1 & t & 1 & -1 \\ t & 1 & 1 & -t \end{array}\right) \cdot(-1) \quad \cdot(-t)$$

$$\sim \left(\begin{array}{ccc|c} 1 & 1 & t & t+1 \\ 0 & t-1 & 1-t & -t-2 \\ 0 & 1-t & 1-t^2 & -t^2-2t \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 1 & t & t+1 \\ 0 & t-1 & 1-t & -t-2 \\ 0 & 0 & -t^2-t+2 & -t^2-3t-2 \end{array}\right) = \left(\begin{array}{ccc|c} 1 & 1 & t & t+1 \\ 0 & t-1 & 1-t & -t-2 \\ 0 & 0 & -(t+2)(t-1) & -(t+2)(t+1) \end{array}\right)$$

I $t \neq 1 \land t \neq -2$

$$rang A = 3 \quad M = \begin{vmatrix} 1 & 1 & t \\ 0 & t-1 & 1-t \\ 0 & 0 & -(t+2)(t-1) \end{vmatrix} = -(t+2)(t-1)^2 \neq 0$$

$rang A = rang B = 3$ - br. nepoznatih $\Rightarrow$ sagl. i odred.

$$\begin{aligned} x+y+t\cdot z &= t+1 \\ (t-1)y+(1-t)z &= -t-2 \\ -(t+2)(t-1)z &= -(t+2)(t+1) \end{aligned}$$

$$z=\frac{t+1}{t-1}$$

$$y=\frac{-t-2-(1-t)\cdot\frac{t+1}{t-1}}{t-1}=\frac{1}{1-t}$$

$$x=t+1-y-t\cdot z = t+1-\frac{1}{1-t}-t\cdot\frac{t+1}{t-1}=\frac{t}{1-t}$$

Jedinstveno rešenje je

$$\left(\frac{t}{1-t},\ \frac{1}{1-t},\ \frac{t+1}{t-1}\right)$$

II $t=1$

$$B \Leftrightarrow \left(\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & 0 & 0 & -3 \\ 0 & 0 & 0 & -6 \end{array}\right)\cdot(-2) \sim \left(\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & 0 & 0 & -3 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$\mathrm{rang}\,A=1$

$\mathrm{rang}\,B=2$

$\mathrm{rang}\,A \neq \mathrm{rang}\,B \Rightarrow$ nesagl.

III $t=-2$

$$B \Leftrightarrow \left(\begin{array}{ccc|c} 1 & 1 & -2 & -1 \\ 0 & -3 & 3 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$\mathrm{rang}\,A=2 \quad M_1=\begin{pmatrix} 1 & 1 \\ 0 & -3 \end{pmatrix}\neq 0$

$\mathrm{rang}\,B=\mathrm{rang}\,A=2<3 \Rightarrow$ sagl. i neodr.

$$\begin{aligned} x+y-2z &= -1 \\ -3y+3z &= 0 \end{aligned}$$

$x=-1-y+2z=-1+z$ $\quad$ z - slobodna nep

$y=z$ $\quad$ Opšte reš. $(-1+z, z, z)$ znata

5. U zavisnosti od parametara a i b disk. i rj. sist.

$$\begin{aligned} (3a+2)x_1+(b-2)x_2-5x_3+5x_4 &= 8+a \\ 2x_1-2x_2-2x_3+2x_4 &= 2 \\ ax_1-x_3+x_4 &= 2 \end{aligned}$$

$$B=\left(\begin{array}{cccc|c} 3a+2 & b-2 & -5 & 5 & 8+a \\ 2 & -2 & -2 & 2 & 2 \\ a & 0 & -1 & 1 & 2 \end{array}\right) \sim \left(\begin{array}{cccc|c} 3a+2 & b-2 & -5 & 5 & 8+a \\ 1 & -1 & -1 & 1 & 1 \\ a & 0 & -1 & 1 & 2 \end{array}\right)$$

$$\sim \left(\begin{array}{cccc|c} -5 & 5 & 3a+2 & b-2 & 8+a \\ -1 & 1 & 1 & -1 & 1 \\ -1 & 1 & a & 0 & 2 \end{array}\right) \sim \left(\begin{array}{cccc|c} -1 & 1 & 1 & -1 & 1 \\ -5 & 5 & 3a+2 & b-2 & 8+a \\ -1 & 1 & a & 0 & 2 \end{array}\right)$$

$$\sim\left(\begin{array}{cccc|c} -1 & 1 & 1 & -1 & 1 \\ 0 & 0 & 3(a-1) & b+3 & 3+a \\ 0 & 0 & a-1 & 1 & 1 \end{array}\right) \sim \left(\begin{array}{cccc|c} -1 & 1 & 1 & -1 & 1 \\ 0 & 0 & a-1 & 1 & 1 \\ 0 & 0 & 3(a-1) & b+3 & 3+a \end{array}\right) \cdot(-3) \sim$$

$$\begin{array}{c} \begin{array}{cccc} x_3 & x_4 & x_1 & x_2 \end{array} \\ \sim\left(\begin{array}{cccc|c} -1 & 1 & 1 & -1 & 1 \\ 0 & 0 & a-1 & 1 & 1 \\ 0 & 0 & 0 & b & a \end{array}\right) \end{array}$$

I $a \neq 1 \quad b \neq 0$ rang $A=3$

$$M=\begin{vmatrix} 1 & 1 & -1 \\ 0 & a-1 & 1 \\ 0 & 0 & b \end{vmatrix} = (a-1)b \neq 0$$

rang A = rang B = 3 < 4 - br. nepoznatih $\Rightarrow$ saglasan i neodređen

$$-x_3 + x_4 + x_1 - x_2 = 1$$
$$(a-1)x_1 + x_2 = 1$$
$$b x_2 = a$$

$x_2 = \dfrac{a}{b}$

$x_1 = \dfrac{1-x_2}{a-1} = \dfrac{1-a/b}{a-1} = \dfrac{b-a}{b(a-1)}$

$x_4 = 1 - x_1 + x_2 + x_3$

$= 1 - \dfrac{b-a}{b(a-1)} + \dfrac{a}{b} + x_3 = 1 + \dfrac{-b+a+a^2-a}{b(a-1)} + x_3$

Opšte rešenje je:

$\left(\dfrac{b-a}{b(a-1)}, \dfrac{a}{b}, x_3, 1+\dfrac{a^2-b}{b(a-1)}+x_3\right)$ x_3 - slobodna nepoznata

II. $a=1$

$$B\sim\left(\begin{array}{cccc|c} -1 & 1 & 1 & -1 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & b & 1 \end{array}\right) \cdot(-b) \sim \left(\begin{array}{cccc|c} -1 & 1 & 1 & -1 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1-b \end{array}\right)$$

rang $A=2$ $\quad M=\begin{vmatrix} -1 & -1 \\ 0 & 1 \end{vmatrix} \neq 0$

b ne utiče na rang A

1° $b \neq 1$ rang $B=3$

$$M_1 = \begin{vmatrix} -1 & -1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1-b \end{vmatrix} = b-1 \neq 0$$

rang $A \neq$ rang $B \Rightarrow$ nesagl.

2° $b=1$

$$B\sim\left(\begin{array}{cccc|c} -1 & 1 & 1 & -1 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

rang $B=2$ $\quad M_2 = \begin{vmatrix} -1 & -1 \\ 0 & 1 \end{vmatrix} = -1 \neq 0$

rang A = rang B = 2 < 4 $\Rightarrow$ sagl. i neodred.

$$-x_3 + x_4 + x_1 - x_2 = 1$$
$$x_2 = 1$$

$x_3 = -1 + x_1 - x_2 + x_4 = -2 + x_1 + x_4$

Opšte rešenje je:

$(x_1, 1, -2+x_1+x_4, x_4)$

x_1, x_4 - slobodne nepoznate

III $b=0$

$$B \sim \left(\begin{array}{cccc|c} -1 & 1 & 1 & -1 & 1 \\ 0 & 0 & -1 & 1 & 1 \\ 0 & 0 & 0 & 0 & a \end{array}\right)$$

rang $A = 2$

$$M = \begin{vmatrix} -1 & -1 \\ 0 & 1 \end{vmatrix} = -1 \neq 0$$

ne zavisi od a

1° $a \neq 0$ rang $B = 3$

$$M_1 = \begin{vmatrix} -1 & -1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & a \end{vmatrix} = -a \neq 0$$

rang $A \neq$ rang $B \Rightarrow$ nesaglasan

2° $a = 0$

$$B \sim \left(\begin{array}{cccc|c} -1 & 1 & 1 & -1 & 1 \\ 0 & 0 & -1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

rang $B = 2$

$$M = \begin{vmatrix} -1 & -1 \\ 0 & 1 \end{vmatrix} \neq 0$$

rang A = rang $B < 4$ saglasan i neodr.

$-x_3 + x_4 + x_1 - x_2 = 1$

$-x_1 + x_2 = 1$

$x_2 = 1 + x_1$

$x_3 = -1 + x_4 + x_1 - x_2 = -2 + x_4$

Opšte rešenje je: $(x_1, 1+x_1, -2+x_4, x_4)$ x_1, x_4 - slob. nepozn.

6. U zavisnosti od parametra a ispitati kada sistem ima netrivijalna rešenja i rješiti sistem:

$$x + y + z = 0$$
$$ax + 4y + z = 0$$
$$6x + (a+2)y + 2z = 0$$

Homogeni sistem $\Rightarrow$ slobodni čl. $= 0$

$$A = \begin{pmatrix} 1 & 1 & 1 \\ a & 4 & 1 \\ 6 & (a+2) & 2 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 1 \\ 1 & 4 & a \\ 2 & a+2 & 6 \end{pmatrix} \begin{matrix} \cdot(-1) & \cdot(-2) \\ \\ \\ \end{matrix} \sim \begin{pmatrix} 1 & 1 & 1 \\ 0 & 3 & a-1 \\ 0 & a & 4 \end{pmatrix} \cdot\left(-\frac{a}{3}\right)$$

$$\sim \begin{pmatrix} 1 & 1 & 1 \\ 0 & 3 & a-1 \\ 0 & 0 & -\frac{(a-4)(a+3)}{3} \end{pmatrix} \cdot(-3) \sim \begin{pmatrix} 1 & 1 & 1 \\ 0 & 3 & a-1 \\ 0 & 0 & (a-4)(a+3) \end{pmatrix}$$

I $a \neq 4 \wedge a \neq -3$ rang $A = 3$ (br. nepoznatih) $\Rightarrow$ samo trivijalno rešenje

II $a = 4$

$A \sim \begin{pmatrix} 1 & 1 & 1 \\ 0 & 3 & 3 \\ 0 & 0 & 0 \end{pmatrix}$ rang $A = 2 < 3 \Rightarrow$ ima netriv. rešenja

$z + x + y = 0$ $\quad y = -x$

$3y + 3x = 0$ $\uparrow$ $\quad z = 0$

Opšte rešenje je:

$(x, -x, 0)$ x- slobodna nepoznata

III $a = -3$

$A \sim \begin{pmatrix} 1 & 1 & 1 \\ 0 & 3 & -4 \\ 0 & 0 & 0 \end{pmatrix}$ rang $A = 2 < 3 \Rightarrow$ ima netriv. rešenja

$z + y + x = 0$ $\quad y = \frac{4}{3}x$

$3y - 4x = 0$ $\uparrow$ $\quad z = -y - x = -\frac{4}{3}x - x = -\frac{7}{3}x$

Opšte rešenje $(x, \frac{4}{3}x, -\frac{7}{3}x)$ x-slobodna nepoznata