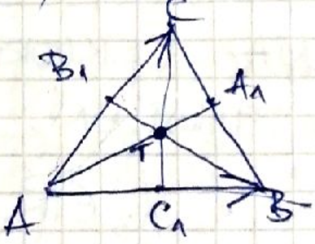
VEKTORI

1. NEKA SU A_1, B_1 I C_1 REDOM SREDINE STRANICA Δ BC, CA I AB TROUGLA ABC I NEKA JE $\vec{AB} = \vec{a}$ I $\vec{AC} = \vec{b}$

1°) POMOĆU VEKTORA $\vec{a}$ I $\vec{b}$ IZRAZITI VEKTORE $\vec{AA_1}$, $\vec{BB_1}$ I $\vec{CC_1}$

2°) DOKAZATI DA JE $\vec{AA_1} + \vec{BB_1} + \vec{CC_1} = \vec{0}$



$$\vec{AA_1} = \vec{AB} + \vec{BA_1}$$
$$+\ \vec{AA_1} = \vec{AC} + \vec{CA_1}$$
$$2\vec{AA_1} = \vec{AB} + \vec{BA_1} + \vec{AC} + \vec{CA_1}$$
$$\vec{AA_1} = \frac{1}{2}(\vec{a} + \vec{b})$$

$\vec{BB_1} = \frac{1}{2}(\vec{BA} + \vec{BC})$ (SLIČNO KAO $\vec{AA_1}$)

$\vec{BB_1} = \vec{BA} + \vec{AB_1} = -\vec{AB} + \frac{1}{2}\vec{AC} = -\vec{a} + \frac{1}{2}\vec{b_1}$

$\vec{CC_1} = \vec{CA} + \vec{AC_1} = -\vec{AC} + \frac{1}{2}\vec{AB} = \vec{a} - \frac{1}{2}\vec{b}$

b) $\vec{AA_1} + \vec{BB_1} + \vec{CC_1} = \vec{0}$

VEKTORI $\vec{a}$ I $\vec{b}$ SU KOLINEARNI AKO SU NA ISTOJ ILI NA PARALELNIM PRAVAMA.

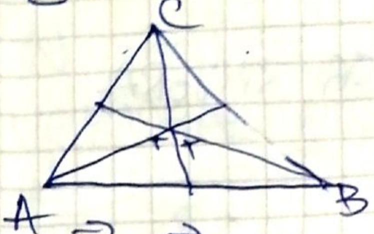
LEMA VEKTORI $\vec{a}$ I $\vec{b}$ SU KOLINEARNI AKKO POSTOJI $k \neq 0$ TAKO DA $\vec{a} = k \cdot \vec{b}$

VEKTORI $\vec{a}, \vec{b}$ I $\vec{c}$ SU KOMPLANARNI AKO PRIPADAJU ISTOJ ILI PARALELNIM RAVNIMA.

LEMA VEKTORI $\vec{a}, \vec{b}$ I $\vec{c}$ SU KOMPLANARNI AKKO $\vec{c} = \alpha\vec{a} + \beta\vec{b}$ $\alpha, \beta \in R$

2. NEKA SU A, B i C TJEMENA A T TEŽIŠTE

∆. DOKAZATI DA JE $\vec{AT}+\vec{BT}+\vec{CT}=\vec{0}$



$\vec{AB}=\vec{a}$

$\vec{AC}=\vec{b}$

$$\vec{AT}=\frac{2}{3}\vec{AA_1}=\frac{2}{3}\cdot\frac{1}{2}\left(\vec{a}+\vec{b}\right)=\frac{1}{3}\left(\vec{a}+\vec{b}\right)$$

$$\vec{BT}=\frac{2}{3}\vec{BB_1}=\frac{2}{3}\left(-\vec{a}+\frac{1}{2}\vec{b}\right)=-\frac{2}{3}\vec{a}+\frac{1}{3}\vec{b}$$

$$\vec{CT}=\frac{2}{3}\vec{CC_1}=\frac{2}{3}\left(\vec{a}-\frac{1}{2}\vec{b}\right)=\frac{2}{3}\vec{a}-\frac{1}{3}\vec{b}$$

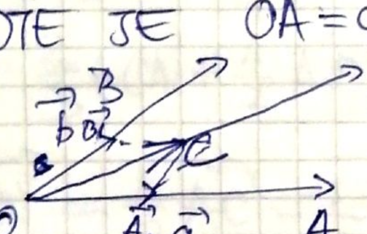
$$\vec{AT}+\vec{BT}+\vec{CT}=\frac{1}{3}\vec{a}+\frac{1}{3}\vec{b}+\cancel{\frac{1}{3}\vec{b}}-\cancel{\frac{2}{3}\vec{a}}+\cancel{\frac{2}{3}\vec{a}}-\cancel{\frac{1}{3}\vec{b}}$$

$$=\vec{0}$$

GREKA!

3. ODREDITI VEKTOR PRAVCA SIMETRALE UGLA $\measuredangle AOB$ GDJE JE $\vec{OA}=\vec{a}$ i $\vec{OB}=\vec{b}$

$\vec{x_0}$ - JEDINIČNI VEKTOR $\vec{x}$



$\vec{OA_1}$ - JEDINIČNI $\vec{OA}$ $\quad \vec{x_0}=\frac{1}{|\vec{x}|}\cdot\vec{x}$

$\vec{OB_1}$ - JEDINIČNI $\vec{OB}$ $\quad \vec{x}=|\vec{x}|\cdot\vec{x_0}$

$$\vec{OA_1}=\frac{1}{|\vec{OA}|}\cdot\vec{OA}=\frac{1}{|\vec{a}|}\cdot\vec{a} \qquad \vec{OB_1}=\frac{1}{|\vec{b}|}\vec{b}$$

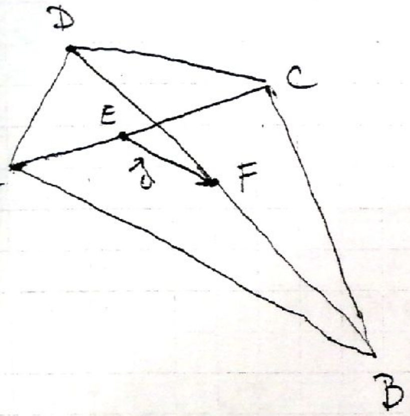
KONSTRUIŠEMO ROMB OA_1CB_1. DIJAGONALA OC POLOVI UGAO $\measuredangle A_1OB_1$, PA TIME I UGAO $\measuredangle AOB$. VEKTOR $\vec{OC}$ JE VEKTOR PRAVCA SIMETRALE $\measuredangle AOB$.

$$\vec{OC}=\vec{OA_1}+\vec{A_1C}$$

$$\vec{OC}=\vec{OA_1}+\vec{OB_1}$$

$$\vec{OC}=\frac{1}{|\vec{a}|}\cdot\vec{a}+\frac{1}{|\vec{b}|}\cdot\vec{b}$$

1) Dokazati da je u četvorouglu ABCD vektor $\vec{d}$ koji spaja središte dijagonala AC i BD dat sa $\vec{d} = \frac{1}{2}(\vec{AD} - \vec{BC})$



E - središte dijagonale AC

F - — " — BD

$\vec{d} = \vec{EF}$

$\vec{EF} = \vec{EA} + \vec{AD} + \vec{DF}$

$\vec{EF} = \vec{EC} + \vec{CB} + \vec{BF}$

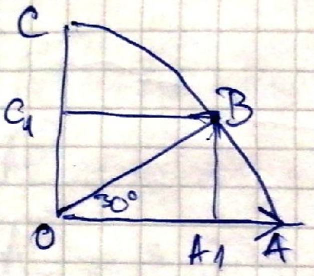
$$2\vec{EF} = \underbrace{\vec{EA} + \vec{EC}}_{0} + \vec{AD} + \vec{CB} + \underbrace{\vec{DF} + \vec{BF}}_{0}$$

$\vec{EF} = \frac{1}{2}(\vec{AD} + \vec{CB})$

$\vec{EF} = \frac{1}{2}(\vec{AD} - \vec{BC})$

$|\vec{a}|$ $|\vec{b}|$

4. TAČKA B DIJELI LUK $\overrightarrow{AC}$ NAD UGLOM 90° U ODNOSU 1:2
RAZLOŽI po VEKTORA $\overrightarrow{OA} = \vec{a}$ i $\overrightarrow{OB} = \vec{b}$ VEKTOR $\overrightarrow{OC}$



A_1 - ORTOGONALNA PROJEKCIJA TAČKE 'B' NA $p(OA)$
C_1 - || - $p(OC)$

$\measuredangle AOB = 30° \Rightarrow \measuredangle A_1OB = 30°$

$\triangle OA_1B$: $\cos 30° = \frac{|\overrightarrow{OA_1}|}{|\overrightarrow{OB}|}$

$\frac{1}{2} = |\overrightarrow{OA_1}|$

$|\vec{OA_1}| = \frac{\sqrt{3}}{2}|\vec{OB}|$ $\qquad |\vec{OA}| = |\vec{OB}| = |\vec{OC}| = r$

$|\vec{OA_1}| = \frac{\sqrt{3}}{2}|\vec{OA}|$

VEKTORI $\vec{OA_1}$ I $\vec{OA}$ SU KOLINEARNI I ISTOG SMJERA

PA SE $\vec{OA_1} = \frac{\sqrt{3}}{2}\vec{OA} = \frac{\sqrt{3}}{2}\vec{a}$

$\sin 30^\circ = \frac{|\vec{A_1B}|}{|\vec{OB}|} = \frac{1}{2}|\vec{OB}|$

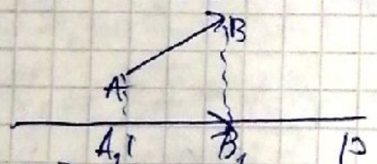
$|\vec{OC_1}| = |\frac{1}{2}| \cdot |\vec{OC}|$

$\vec{OC_1} = \frac{1}{2}\vec{OC}$

$\vec{OB} = \vec{OA_1} + \vec{A_1B} = \vec{OA_1} + \vec{OC_1}$

$\vec{b} = \frac{\sqrt{3}}{2}\vec{a} + \frac{1}{2}\vec{OC}$

$2\vec{b} - \sqrt{3}\vec{a} = \vec{OC}$



$\vec{A_1B_1}$ - VEKTORSKA PROJEKCIJA $\vec{AB}$ NA PRAVU p

$|\vec{A_1B_1}|$ - SKALARNA PROJEKCIJA $\vec{AB}$ NA p

$Pr_p \vec{AB} = \vec{A_1B_1}$ (VEKTORSKA)

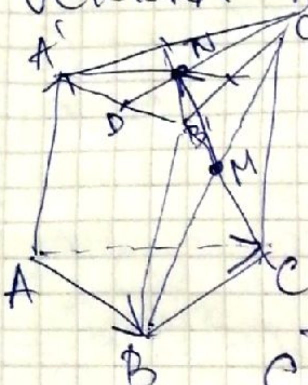
$pr_p \vec{AB} = \vec{A_1B_1}$ (SKALARNA)

$pr_p \vec{AB} = |\vec{AB}| \cdot \cos\varphi \qquad \varphi = \angle(\vec{AB}, p)$

$pr_p(\vec{a} + \vec{b}) = pr_p\vec{a} + pr_p\vec{b}$

$pr_p(\lambda \cdot \vec{a}) = \lambda \cdot pr_p\vec{a}$

5. DATA JE TROSTRANA PRIZMA $ABCA_1B_1C_1$. TAČKA M JE CENTAR PARALELOGRAMA A TAČKA N TEŽIŠTE $\Delta A_1B_1C_1$ IZRAZITI VEKTOR $\overrightarrow{MN}$ POMOĆU VEKTORA $\overrightarrow{AB}, \overrightarrow{AC_1}$ i $\overrightarrow{AA_1}$



D SREDIŠTE IVICE A_1B_1

$$\overrightarrow{MN} = \overrightarrow{MC_1} + \overrightarrow{C_1N}$$

$$\overrightarrow{MC_1} = \frac{1}{2}\overrightarrow{BC_1} = \frac{1}{2}(\overrightarrow{BC} + \overrightarrow{CC_1}) = \frac{1}{2}(\overrightarrow{AC} - \overrightarrow{AB} + \overrightarrow{AA_1})$$

$$\overrightarrow{C_1N} = \frac{2}{3}\overrightarrow{C_1D} = \frac{2}{3}(\overrightarrow{C_1A_1} + \overrightarrow{A_1D}) =$$

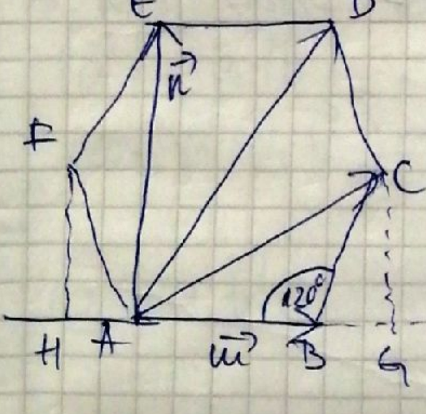
$$= \frac{2}{3}(\overrightarrow{CA} + \frac{1}{2}\overrightarrow{A_1B_1}) = \frac{2}{3}(-\overrightarrow{AC} + \frac{1}{2}\overrightarrow{AB}) = -\frac{2}{3}\overrightarrow{AC} + \frac{1}{3}\overrightarrow{AB}$$

$$\overrightarrow{MN} = \frac{1}{2}\overrightarrow{AC} - \frac{1}{2}\overrightarrow{AB} + \frac{1}{2}\overrightarrow{AA_1} - \frac{2}{3}\overrightarrow{AC} + \frac{1}{3}\overrightarrow{AB}$$

$$\overrightarrow{MN} = -\frac{1}{6}\overrightarrow{AB} - \frac{1}{6}\overrightarrow{AC} + \frac{1}{2}\overrightarrow{AA_1}$$

VEKTOR $\overrightarrow{MN} = (-\frac{1}{6}, -\frac{1}{6}, \frac{1}{2})$ U BAZI $\{\overrightarrow{AB}, \overrightarrow{AC}, \overrightarrow{AA_1}\}$

6. IZRAZITI VEKTORE $\overrightarrow{AC}, \overrightarrow{AD}, \overrightarrow{AF}$ i $\overrightarrow{EF}$ PREKO VEKTORA $\overrightarrow{AB} = \vec{m}$ i $\overrightarrow{AE} = \vec{n}$ AKO JE ABCDEF PRAVILNI 6-UGAO



G - PROJEKCIJA TAČKE C NA PRAVU p(AB)

$\measuredangle ABC = 120^\circ \Rightarrow \measuredangle GBC = 60^\circ$

ΔBGC: $\cos 60^\circ = \frac{|\overrightarrow{BG}|}{|\overrightarrow{BC}|}$

$|\overrightarrow{BG}| = \frac{1}{2}|\overrightarrow{BC}|$

$|\overrightarrow{BG}| = \frac{1}{2}|\overrightarrow{AB}| = \frac{1}{2}\vec{m}$ $\quad \overrightarrow{BG} = \frac{1}{2}\vec{m}$

$\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BG} + \overrightarrow{GC}$

$\overrightarrow{AC} = \vec{m} + \frac{1}{2}\vec{m} + \frac{1}{2}\vec{n}$

$\overrightarrow{AC} = \frac{3}{2}\vec{m} + \frac{1}{2}\vec{n}$

$\overrightarrow{AD} = \overrightarrow{AB} + \overrightarrow{BD} = \vec{m} + \vec{n}$

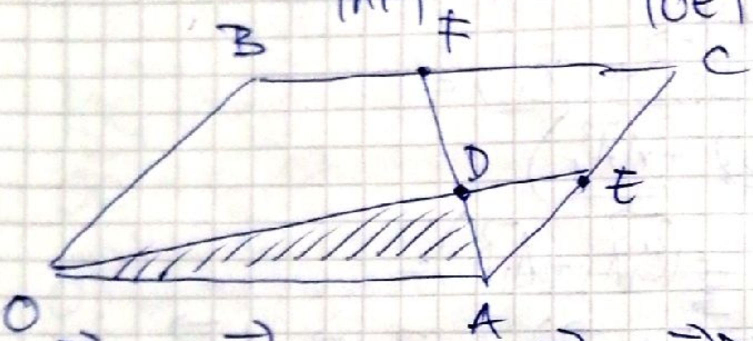
$\overrightarrow{AH} = -\frac{1}{2}\vec{m}$ (SLIČNO KAO $\overrightarrow{BG}$)

$\overrightarrow{AF} = \overrightarrow{AH} + \overrightarrow{HF}$

$\overrightarrow{AF} = -\frac{1}{2}\vec{m} + \frac{1}{2}\vec{n}$

$\vec{EF} = \vec{AF} - \vec{AE} = -\frac{1}{2}\vec{m} + \frac{1}{2}\vec{n} - \vec{n} = -\frac{1}{2}\vec{m} - \frac{1}{2}\vec{n}$

7. DAT JE PARALELOGRAM OACB. NEKA JE E SREDIŠTE STRANICE AC, F SREDIŠTE STRANICE CB A TAČKA D PRESEK DUŽI OE i AF ODREDITI $\frac{|\vec{AD}|}{|\vec{AF}|}$ i $\frac{|\vec{OD}|}{|\vec{OE}|}$



$\vec{OA} = \vec{a}$

$\vec{OB} = \vec{b}$

$\vec{OD} = k \cdot \vec{OE} = k \cdot (\vec{OA} + \vec{AE}) = k(\vec{a} + \frac{1}{2}\vec{b})$

$\vec{AD} = m \cdot \vec{AF} = m(\vec{AC} + \vec{CF}) = m(\vec{b} - \frac{1}{2}\vec{a})$

$\vec{OD} = \vec{OA} + \vec{AD}$

$k(\vec{a} + \frac{1}{2}\vec{b}) = \vec{a} + m(\vec{b} - \frac{1}{2}\vec{a})$

$(k - 1 + \frac{m}{2})\vec{a} + (\frac{k}{2} - m)\vec{b} = \vec{0}$

$\begin{cases} k - 1 + \frac{m}{2} = 0 \\ \frac{k}{2} - m = 0 \end{cases}$

$m = \frac{2}{5}$

$k = \frac{4}{5}$

$\vec{OD} = \frac{4}{5}\vec{OE} \Rightarrow \frac{|\vec{OD}|}{|\vec{OE}|} = \frac{4}{5}$

$\vec{AD} = \frac{2}{5}\vec{AF} \Rightarrow \frac{|\vec{AD}|}{|\vec{AF}|} = \frac{2}{5}$