VJEŽBE RAVAN I PRAVA

① NAĆI JEDNAČINU RAVNI KOJA SADRŽI TAČKU $M(2,2,-2)$ I PARALELNA JE SA $\alpha: x-2y-3z=0$

π - TRAŽENA RAVAN

$\vec{n_\pi}$ - NJEN VEKTOR NORMALE

$\vec{n_\alpha}=(1,-2,-3)$

$\pi \parallel \alpha \Rightarrow \vec{n_\pi}=\lambda \vec{n_\alpha}$

ZA $\lambda=1$

$\vec{n_\pi}=(1,-2,-3)$

$\pi: M(2,2,-2), \vec{n_\pi}=(1,-2,-3)$

$1\cdot(x-2)-2\cdot(y-2)-3(z+2)=0$

$\pi: \underline{x-2y-3z-4=0}$

II NAČIN

$\pi \parallel \alpha \Rightarrow \pi: x-2y-3z+D=0$

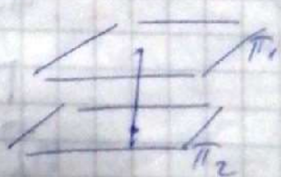
$M(2,2,-2)\in\pi \Rightarrow 2-2\cdot2-3(-2)+D=0$

$\pi: \underline{x-2y-3z-4=0}$ $\qquad \underline{D=-4}$

2. NAĆI UDALJENOST PARALELNIH RAVNI

$\pi_1: 4x+3y-5z-8=0$

$\pi_2: 4x+3y-5z+12=0$



Odredimo jednu tačku ravni π_1.

Npr ZA $y=0$ i $z=0$ iz jednačine ravni π_1

$4x-8=0$

$4x=8$

$\boxed{x=2}$

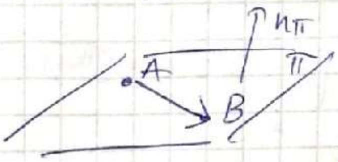
$M(2,0,0)\in\pi_1$

$d(T_1, \Pi_2) = d(M_1, \Pi_2)$

$$= \frac{|4\cdot 2 + 3\cdot 0 - 5\cdot 0 + 14|}{\sqrt{4^2+3^2+(-5)^2}} = \frac{|8+12|}{\sqrt{50}} = \frac{|20|}{5\sqrt{2}} = \frac{4}{\sqrt{2}} = \boxed{2\sqrt{2}}$$

3. NAPISATI JEDNAČINU RAVNI KOJA PROLAZI KROZ TAČKE $A(2,3,-1)$ I $B(1,5,3)$ I NORMALNA JE NA RAVAN α

$\alpha: 3x - y + 3z + 15 = 0$



$\vec{AB} \in \Pi$

$n_\Pi \perp \vec{AB}$

$\Pi \perp \alpha \Rightarrow$

$\vec{n_\Pi} \perp \vec{n_\alpha}$

$\vec{n_\Pi} = \lambda(\vec{AB} \times \vec{n_\alpha})$

$\vec{AB} = (-1, 2, 4)$

$\vec{n_\alpha} = (3, -1, 3)$

$$\vec{AB} \times \vec{n_\alpha} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -1 & 2 & 4 \\ 3 & -1 & 3 \end{vmatrix} = 10\vec{i} + 15\vec{j} - 5\vec{k}$$

$\vec{AB} \times \vec{n_\alpha} = (10, 15, -5)$

ZA $\lambda = \frac{1}{5}$ $\quad \vec{n_\Pi} = (2, 3, -1)$

$A(2,3,-1)$ $\quad \vec{n_\Pi}(2,3,-1)$

$\Pi: 2(x-2) + 3(y-3) - 1(z+1) = 0$

$\Pi: \boxed{2x + 3y - z - 14 = 0}$

4. ODREDITI JEDNAČINU RAVNI KOJA SADRŽI TAČKU $M(3,2,4)$ A NA KOORDINATNIM OSAMA ODSIJECA ODSJEČKE JEDNAKE DUŽINE.

NEKA JE TRAŽENA RAVAN Π I NJENA SEGMENTNA JEDNAČINA DATA SA

$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$

$|a| = |b| = |c|$

$2x + 3y - 4z - 5 = 0$

$2x + 3y - 4z = 5$

$\frac{x}{\frac{5}{2}} + \frac{y}{\frac{5}{3}} + \frac{z}{-\frac{5}{4}} = 1$

$M\left(\frac{5}{2}, 0, 0\right)$ $\quad M_2\left(0, \frac{5}{3}, 0\right)$ $\quad M_3\left(0, 0, -\frac{5}{4}\right)$

$1^\circ \quad a=b=c$

$$\frac{x}{a}+\frac{y}{a}+\frac{z}{a}=1$$

$M(3,2,4) \in \pi$

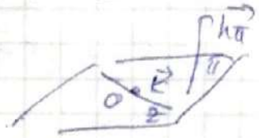
$$\frac{3}{a}+\frac{2}{a}+\frac{4}{a}=1$$

$$\boxed{9=a}$$

$$\pi: \frac{x}{9}+\frac{y}{9}+\frac{z}{9}=1 \quad \text{SEGMENTA}$$

$$x+y+z-9=0$$

5. NAĆI JEDNAČINU RAVNI KOJA SADRŽI OSU Oz A SA RAVNI $\alpha: 2x+y-\sqrt{5}z=0$ OBRAZUJE UGAO OD 60°



$\vec{n_\pi} \perp \vec{k} \Rightarrow \vec{n_\pi}\cdot\vec{k}=0$

$O_z \subset \pi \quad \vec{k}=(0,0,1)$

$C=0$

$\vec{n_\pi}=(A,B,0)$

$\vec{n_\pi}=(A,B,C)$

$\vec{n_\alpha}=(2,1,-\sqrt{5})$

$\varphi=\measuredangle(\pi,\alpha)=60^\circ$

$\varphi=\measuredangle(\vec{n_\pi},\vec{n_\alpha})=60^\circ$

$$\cos 60^\circ=\frac{\vec{n_\pi}\cdot\vec{n_\alpha}}{|\vec{n_\pi}||\vec{n_\alpha}|}$$

$$\frac{1}{2}=\frac{2A+B}{\sqrt{A^2+B^2}\cdot\sqrt{4+1+5}}$$

$$\frac{1}{2}=\frac{2A+B}{\sqrt{10}\sqrt{A^2+B^2}} \quad /\,2\sqrt{10}\cdot\sqrt{A^2+B}$$

$A_1=\frac{1}{3}B$

$A_2=-3B$

$\sqrt{10}\cdot\sqrt{A^2+B^2}=(2A+B)\cdot 2 \,/^2$

$10(A^2+B^2)=(4A^2+4AB+B^2)\cdot 2$

$10A^2+10B^2=(4A^2+4AB+B^2)\cdot 2$

$5A^2+5B^2-(4A^2-4AB-B^2)=0$

$3A^2+8AB-3B^2=0$

$$A_{1,2}=\frac{-8B\pm\sqrt{64B^2+4\cdot 3\cdot 3B^2}}{6}$$

$$A_{1,2}=\frac{-8B\pm\sqrt{100B^2}}{6}$$

$$A_{1,2}=\frac{-8B\pm 10B}{6}$$

1° $A = \frac{B}{3}$ $\qquad \vec{n_\pi} = (B/3, B, 0)$

$\vec{n_\pi} = B(\frac{1}{3}, 1, 0)$

ZA B RECIMO 3

$\vec{n_\pi} = (1, 3, 0)$

$1(x-0) + 3(y-0) + 0\cdot(z-0) = 0$

$\pi: \boxed{x + 3y = 0}$

2° $A = -3B$ $\qquad \vec{n_\pi} = (-3B, B, 0)$

$\vec{n_\pi} = B(-3, 1, 0)$

$B = -1$

$\vec{n_\pi} = (3, -1, 0)$

$3(x-0) - 1(y-0) + 0(z-0) = 0$

$\pi: 3x - y = 0$

DVIJE RAVNI KOJE SE SIJEKU ODREĐUJU PRAMEN (SKUP SVIH RAVNI KOJE SADRŽE NJIHOVU PRESJEČNU PRAVU)

$\pi_1: \quad A_1 x + B_1 y + C_1 z + D_1 = 0$

$\pi_2: \quad A_2 x + B_2 y + C_2 z + D_2 = 0$

JEDNAČINA PRAMENA

$A_1 x + B_1 y + C_1 z + D_1 + \lambda(A_2 x + B_2 y + C_2 z + D_2) = 0$

$(A_1 + \lambda A_2)x + (B_1 + \lambda B_2)y + (C_1 + \lambda C_2)z + D_1 + \lambda D_2 = 0$

6. NAPISATI JEDNAČINU RAVNI KOJA JE NORMALNA NA RAVAN $\alpha: 3x+5y-z=0$ I KOJA JE SIJEČE PO PRAVOJ KOJA LEŽI U RAVNI yOz.

$\vec{n_\alpha} = (3, 5, -1)$

$\beta = yOz$ - ravan

$\beta: x = 0$

π TRAŽENA RAVAN.

$$\left.\begin{array}{l} \pi \perp \alpha \\ \pi \cap \alpha = p \\ p \subset \beta \end{array}\right\} \Rightarrow \alpha \cap \beta = p$$

RAVAN π PRIPADA PRAMENU ODREĐENOM PRAVOM p (ILI RAVNINA α i β) JEDNAČINA PRAMENA JE

$$3x+5y-z+\lambda=0$$

$$(3+\lambda)x+5y-z=0$$

$$\vec{n_\pi} = (3+\lambda, 5, -1)$$

$$\pi \perp \alpha \Rightarrow \vec{n_p} \perp \vec{n_p} \Rightarrow \vec{n_\pi} \cdot \vec{n_\alpha} = 0$$

$$3(3+\lambda)+5\cdot 5+1=0$$

$$9\cancel{\lambda}+3\lambda+26=0$$

$$3\lambda+35=0$$

$$\boxed{\lambda = -\frac{35}{3}}$$

$$\pi: \left(3-\frac{35}{3}\right)x+5y-z=0$$

$$\pi: -26x+15y-3z=0$$

7. NAĆI DVIJE MEĐUSOBNO ORTOGONALNE RAVNI KOJE PROLAZE KROZ PRESJEČNU PRAVU RAVNI $\alpha: x=y$ i xOy RAVNI, AKO JEDNA OD TRAŽENIH RAVNI SADRŽI TAČKU $M(0,4,2)$

$\alpha: x-y=0$

β: xOy- ravan

$z=0$

α i β ODREĐUJU PRAMEN RAVNI. JEDNAČINA PRAMENA JE $x-y+\lambda z=0$

TRAŽENE RAVNI π_1 i π_2 PRIPADAJU PRAMENU.

NEKA $M\in\pi_1$

$0-4+2\lambda=0$

$\boxed{\lambda=2}$

$\pi_1: x-y+2z=0$

$\vec{n_1}=(1,-1,2)$

$\vec{n_2}=(1,-1,\lambda)$

$\pi_1\perp\pi_2 \Rightarrow \vec{n_1}\perp\vec{n_2} \Rightarrow \vec{n_1}\cdot\vec{n_2}=0$

$1+1+2\lambda=0$

$\boxed{\lambda=-1}$

$\pi_2: x-y-z=0$

8. NAĆI JEDNAČINU PRAVE KOJA SADRŽI TAČKU $M(-4,3,0)$ i PARALELNA JE PRAVOJ p:

$$p\begin{cases}\pi_1: x-2y+z-4=0\\ \pi_2: 2x+y-z=0\end{cases}$$

$n_1(1,-2,1)$

$n_2(2,1,-1)$

$$\left.\begin{array}{l} p\subset\pi_1 \Rightarrow \vec{S_p}\perp\vec{n_1} \\ p\subset\pi_2 \Rightarrow \vec{S_p}\perp\vec{n_2}\end{array}\right\} \vec{S_p}=\lambda(\vec{n_1}\times\vec{n_2})$$

$$\vec{n_1}\times\vec{n_2}=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ 1 & -2 & 1\\ 2 & 1 & -1\end{vmatrix}$$