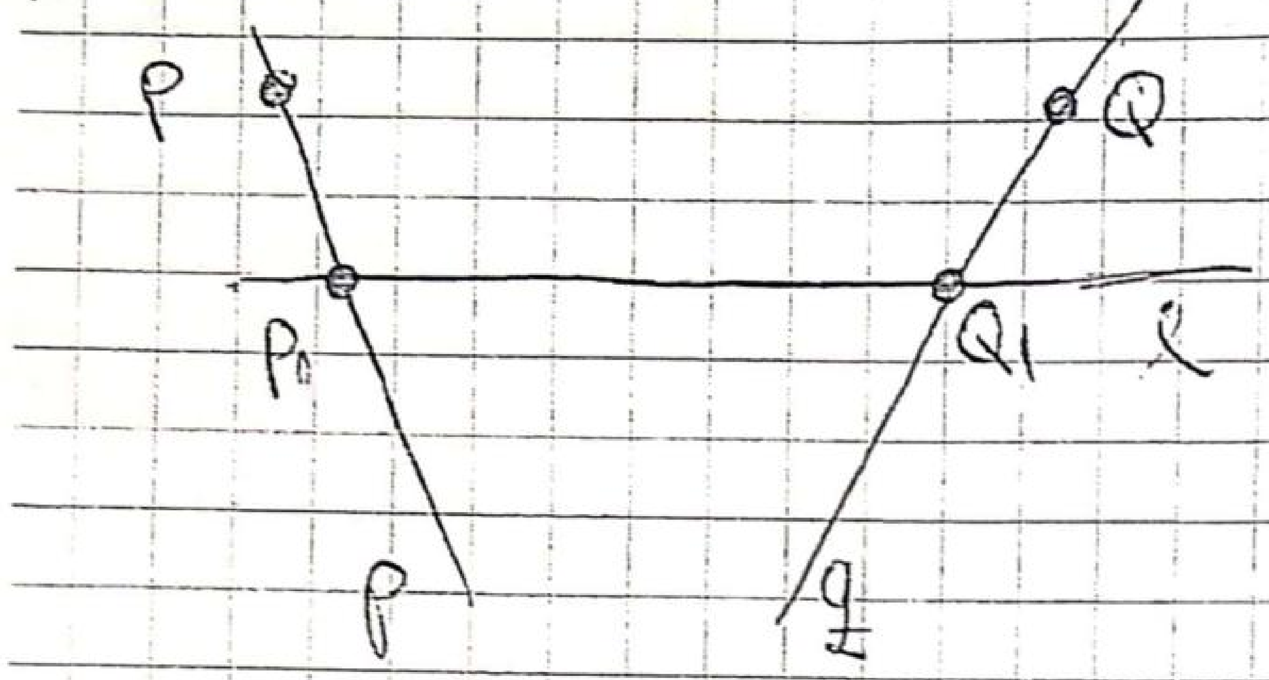
8) Naći zajedničku normalu i kanonske jednačine pravih p i q je su paralelne ravni $\alpha: 2x-y+z=0$, sijeku pravu $l: \frac{x+1}{2}=\frac{y}{-2}=\frac{z}{1}$ prolaze kroz tačke $P(-2,3,5)$ i $Q(2,0,1)$ respektivno.



$l \cap p = \{P_1\}$

$l \cap q = \{Q_1\}$

$p \parallel \alpha \Rightarrow p$ pripada ravni π_1 koja je paralelna ravni α

$\pi_1: 2x-y+z+D=0$

$P \in p \subset \pi_1 \Rightarrow 2(-2)-3+5+D=0$

$\Rightarrow -2+D=0$

$\Rightarrow \underline{D=2}$ ✓

$\pi_1: 2x-y+z+2=0$

$$\left.\begin{array}{l} P_1 \in l \\ P_1 \in p \subset \pi_1 \end{array}\right\} \Rightarrow l \cap \pi_1 = \{P_1\}$$

$$l: \begin{cases} x=-1+2t \\ y=-2t \\ z=t \end{cases} \qquad \begin{array}{l} x=-1 \\ y=0 \\ z=0 \end{array}$$

$-\cancel{2}+4t+2t+t+\cancel{2}=0$

$7t=0$

$\boxed{t=0}$ $\Rightarrow P_1(-1,0,0)$

$p: P_1(-1,0,0),\ P(-2,3,5)$

$p: \frac{x+1}{-1} = \frac{y}{3} = \frac{z}{5}$

$q \parallel \alpha \Rightarrow$ q pripada ravni Π_2 paralelnoj ravni α

$\Pi_2: 2x - y + z + D = 0$

$Q \in q \subset \Pi_2 \Rightarrow 2 \cdot 2 - 0 + 1 + D = 0$

$5 + D = 0$

$\boxed{D = -5}$

$\Pi_2: 2x - y + z - 5 = 0$

$\left.\begin{array}{l} Q_1 \in l \\ Q_1 \in q \subset \Pi_2 \end{array}\right\} \Rightarrow l \cap \Pi_2 = \{Q_1\}$

$-2 + 4t + 2t + t - 5 = 0$

$7t - 7 = 0$

$\boxed{t = 1}$

$x = 1$

$y = -2$

$z = 1$

$\Rightarrow Q_1(1,-2,1)$

$q: Q_1(1,-2,1)\ \ Q(2,0,1)$

$q: \frac{x-1}{1} = \frac{y-2}{2} = \frac{z-1}{0}$ ✓

$\alpha \parallel p$

$\alpha \parallel q$

β - ravan koja sadrži pravu q i ortogonalna je na α.

$\left.\begin{array}{l} \vec{n_\beta} \perp \vec{s_q} \\ \vec{n_\beta} \perp \vec{n_\alpha} \end{array}\right\} \Rightarrow \vec{n_\beta} = \lambda(\vec{s_q} \times \vec{n_\alpha})$

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$$\vec{s_q} \times \vec{n_\gamma} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 2 & 0 \\ 2 & -1 & 1 \end{vmatrix} = 2\vec{i} - \vec{j} + (-5)\vec{k} \quad , \vec{n_\beta} = (2, -1, -5).$$

β: $Q(2, 0, 1)$ $\vec{n_\beta} = (2, -1, 5)$

$2(x-2) - 1(y-0) - 5(z-1) = 0$

$\boxed{\beta}$: $2x - y - 5z + 1 = 0$

$p \cap \beta = \{A\}$ $A \in n$, $n \to$ normala pecum p i q

$p: \begin{cases} x = -1-t \\ y = 3t \\ z = 5t \end{cases}$ $\quad -2-2t-3t-25t+1=0$

$-30t - 1 = 0$

$t = -\frac{1}{30}$

$x = -\frac{29}{30}$

$y = -\frac{1}{10} \Rightarrow A\left(-\frac{29}{30}, -\frac{1}{10}, -\frac{1}{6}\right)$

$z = -\frac{1}{6}$

$\vec{s}$ - vector pentru normale $\urcorner$ Uordi da g

$\left.\begin{matrix} \vec{s} \perp \vec{s_p} \\ \vec{s} \perp \vec{s_q} \end{matrix}\right\} \Rightarrow \vec{s} = \lambda(\vec{s_p} \times \vec{s_q})$

$\vec{s} = \lambda \cdot \vec{n_\alpha}$

$\vec{s} = (2, 1, -1)$

9) Naći jednačinu prave koje sadrži tačku $M(3,-2,-4)$, paralelna je sa ravni $\alpha: 3x-2y-3z-7=0$, a siječe pravu $l: \frac{x-2}{3} = \frac{y+4}{-2} = \frac{z-1}{2}$

R|| p – tražena prava

$\vec{s_p} = (m,n,k) \rightarrow$ njen vektor prava

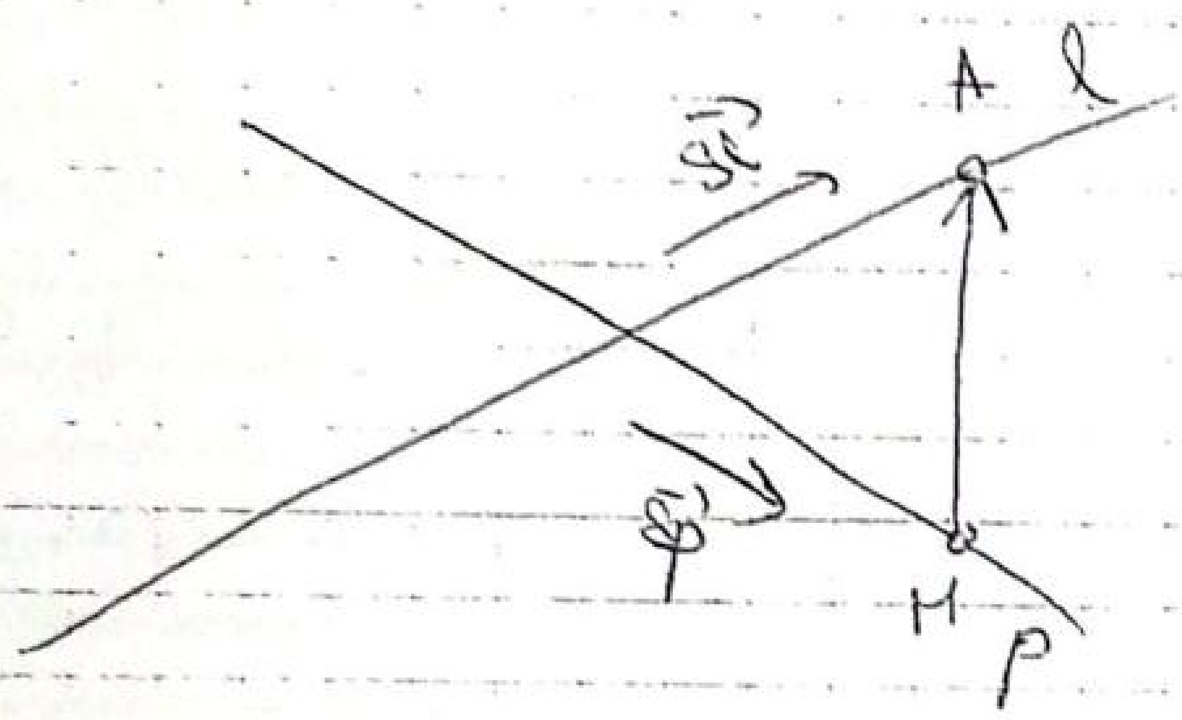
$\vec{s_l} = (3,-2,2)$

$A(2,-4,1) \in l$

$\vec{MA} = (-1,-2,5)$

$\vec{n_\alpha} = (3,-2,-3)$

$p \parallel \alpha \Rightarrow \vec{n_\alpha} \perp \vec{s_p} = \vec{n_\alpha} \cdot \vec{s_p} = 0 \Rightarrow 3m - 2n - 3k = 0 \quad (1)$



prave p i l se sijeku pa su vektori $\vec{s_p}, \vec{s_l}, \vec{MA}$ komplanarni i važi $(\vec{s_p} \times \vec{s_l}) \cdot \vec{MA} = 0$

$$\begin{vmatrix} m & n & k \\ 3 & -2 & 2 \\ -1 & -2 & 5 \end{vmatrix} = 0$$

$-6m - 17n - 8k = 0 \quad (2)$

$$\begin{cases} 3m-2n-3k=0 \quad /2 \\ -6m-17n-8k=0 \end{cases} \qquad \begin{cases} 3m-2n-3k=0 \\ -21n-14k=0 \end{cases}$$

$n = \frac{-2k}{3}$

$m = \frac{2n+3k}{3}$

$m = \frac{-\frac{4k}{3}+3k}{3} = \frac{5k}{9}$

$\vec{s_p} = \left(\frac{5}{9}k, -\frac{2}{3}k, k\right)$

$\vec{s_p} = k\left(\frac{5}{9}, -\frac{2}{3}, 1\right)$

$\vec{s_p} = (5,-6,9) \quad p: \frac{x-3}{5} = \frac{y+2}{-6} = \frac{z+4}{9}$ ✓

10) Date su prave $p: \frac{x+3}{l} = \frac{y-2}{m} = \frac{z-5}{4}$ i $q: \frac{x-2}{1} = \frac{y+1}{2} = \frac{z-3}{-2}$ i $r: \frac{x-9}{3} = \frac{y-3}{1} = \frac{z-5}{2}$

Odrediti vrijednosti parametara l i m tako da prava p prolazi kroz presjek pravih q i r, a zatim naći ugao između prave p i ravni koju određuju prave q i r.

Određimo $q \cap r$

$$q: \begin{cases} x = 2+t \\ y = -1+2t \\ z = 3-2t \end{cases}$$

$$\frac{2+t-9}{3} = \frac{-1+2t-3}{1} = \frac{3-2t-5}{2}$$

$$\frac{-7+t}{3} = \frac{-4+2t}{1} = \frac{-2-2t}{2}$$

$$t-7 = 6t-12$$

$$5t = 5$$

$$\boxed{t=1}$$

$A(3,1,1)$

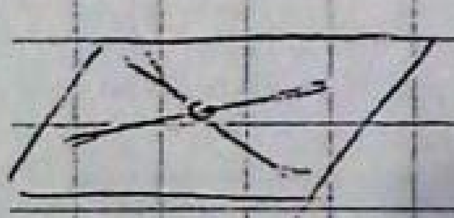
$q \cap r = \{A\}$

$$A(3,1,1) \in p \Rightarrow \frac{3+3}{l} = \frac{1-2}{m} = \frac{1-5}{4}, \quad \frac{6}{l} = \frac{-1}{m} = -1$$

$$\boxed{l=-6, \ m=1}$$

$$p: \frac{x-3}{-6} = \frac{y-2}{1} = \frac{z-5}{4}$$

$\alpha :\rightarrow$ ravan koju određuju q i r



$$\left.\begin{aligned} q \subset \alpha &\Rightarrow \vec{n_\alpha} \perp \vec{s_q} \\ r \subset \alpha &\Rightarrow \vec{n_\alpha} \perp \vec{s_r} \end{aligned}\right\} \vec{n_\alpha} = \lambda(\vec{s_q} \times \vec{s_r})$$

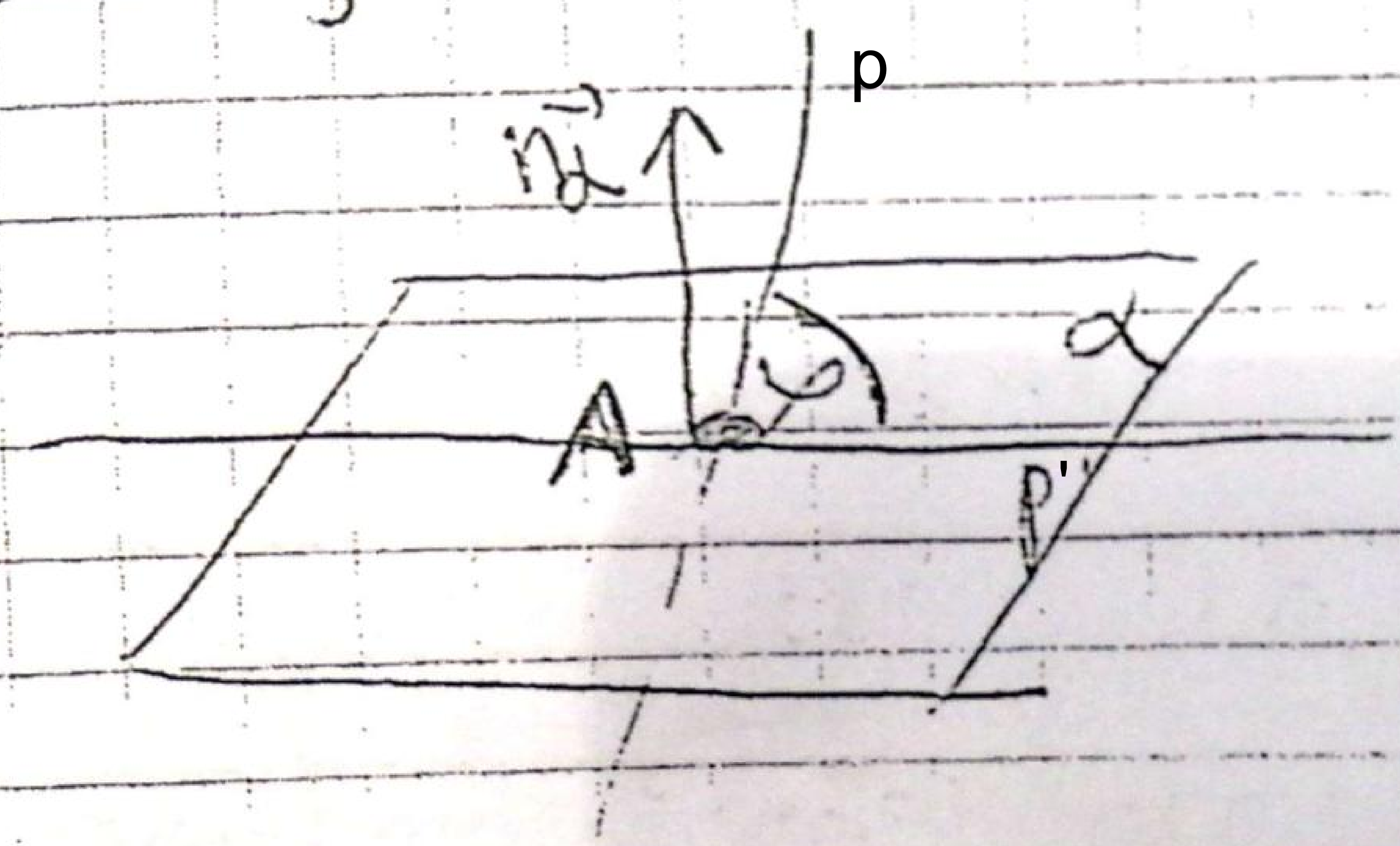
$$\vec{s_q} \times \vec{s_r} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 2 & -2 \\ 3 & 1 & 2 \end{vmatrix} = 6\vec{i} - 8\vec{j} + (-5)\vec{k}$$

Za $\lambda = 1 \Rightarrow \vec{n_\alpha} = (6,-8,-5)$

$\alpha: A(3,1,1) \quad \vec{n_\alpha} = (6,-8,-5)$

$$6(x-3) - 8(y-1) - 5(z-1) = 0$$

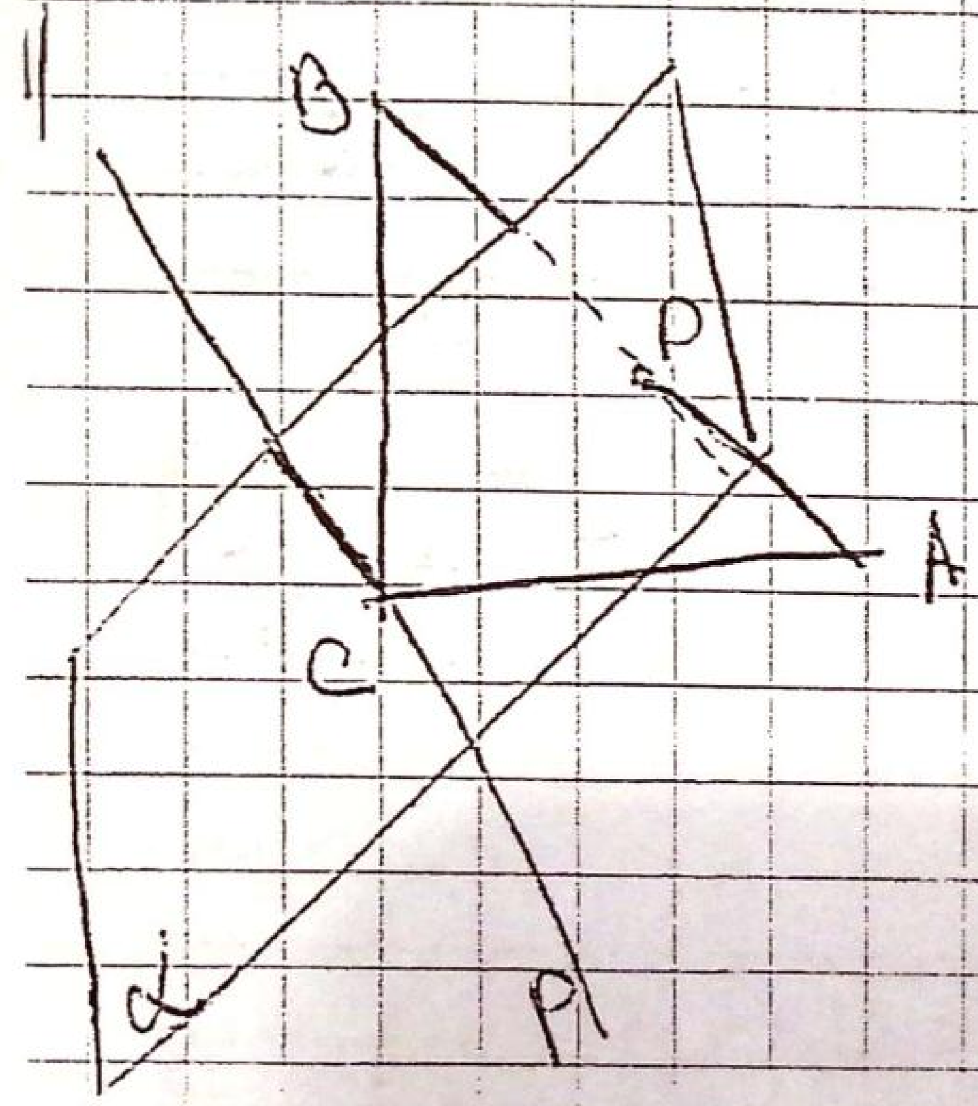
$$6x-8y-5z-5=0$$



$$\sin\varphi=\frac{|\vec{n_\alpha}\cdot\vec{sp}|}{|\vec{n_\alpha}||\vec{sp}|}$$

$$\sin\varphi=\frac{|-36-8-20|}{\sqrt{36+64+25}\,\sqrt{36+1+16}}=\frac{64}{5\sqrt{265}}$$

12) Na pravoj $p: \frac{x-1}{-1} = \frac{y-2}{1} = \frac{z-1}{2}$ naći tačku C jednako udaljeno od tačaka $A(2,1,-2)$ i $B(4,-5,4)$.



D - središte duži AB

α - ravan koja sadrži tačku D i ortogonalna na duž AB.

$p \cap \alpha = \{C\}$.

$D(x,y,z)$

$x = \frac{2+4}{2} = 3$

$y = \frac{1-5}{2} = -2$

$z = \frac{-2+4}{2} = 1$

$D(3,-2,1)$

$\vec{n_\alpha} = \lambda \vec{AB}$

$\vec{AB} = (2,-6,6)$

Za $\lambda = \frac{1}{2}$, $\vec{n_\alpha} = (1,-3,3)$

$\alpha: D(3,-2,1)$, $\vec{n_\alpha} = (1,-3,3)$

$\alpha: x - 3y + 3z - 12 = 0$

$$p: \begin{cases} x = 1 - t \\ y = 2 + t \\ z = 1 + 2t \end{cases}$$

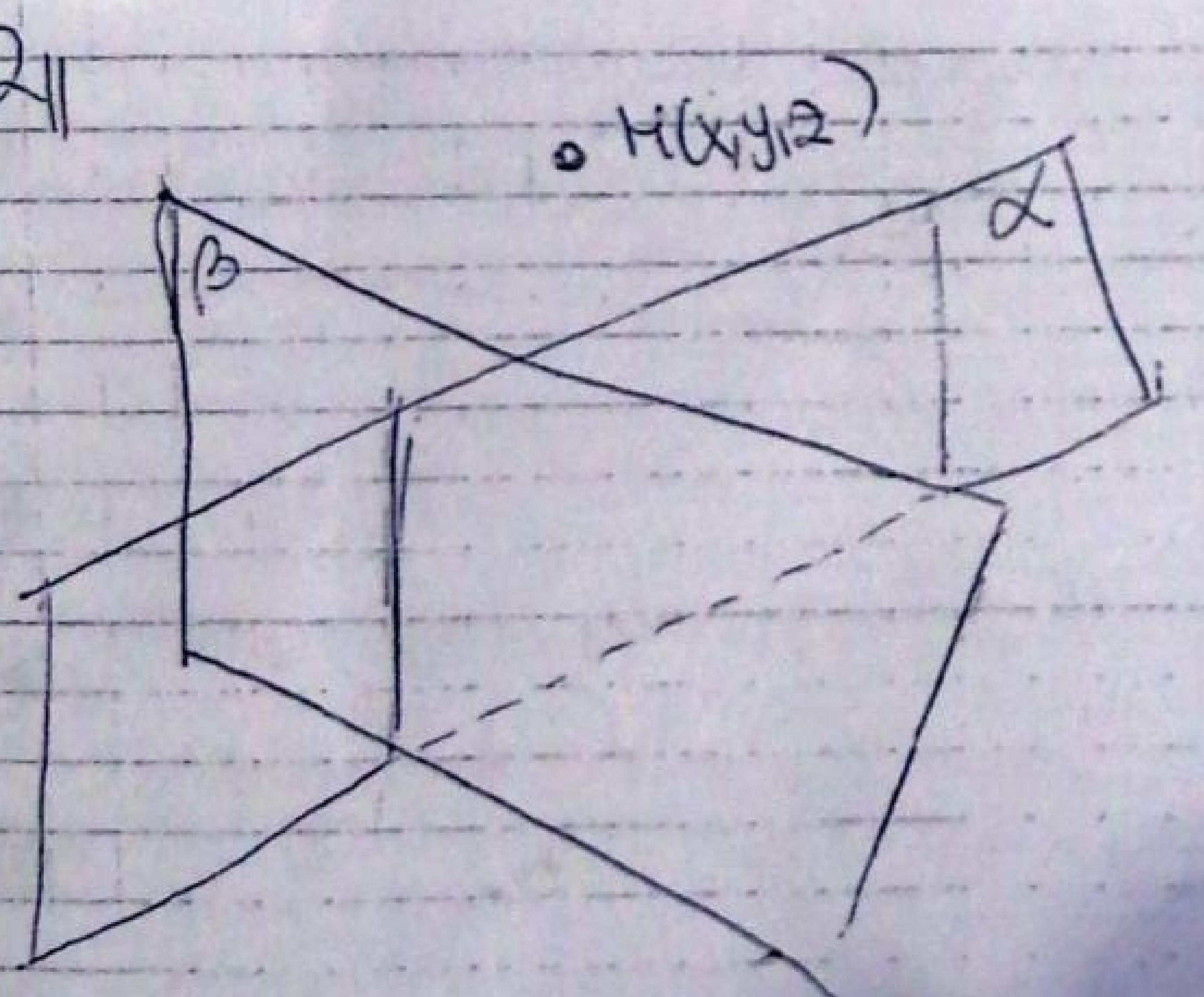
$$1 - t - 3(2+t) + 3(1+2t) - 12 = 0$$
$$2t - 14 = 0$$
$$t = 7$$

$C(-6, 9, 15)$

$$x = -6$$
$$y = 9$$
$$z = 15$$

13) Napisati jednačine ravni koje su simetrale uglova između ravni $\alpha: x+2y+2z-4=0$ i $\beta: 6x-3y+2z+6=0$

R:



Neka je $M(x,y,z)$ proizvoljna tačka simetralne ravni ugla koje grade ravni α i β. Tada je $d(M,\alpha) = d(M,\beta)$.

$$\frac{|x+2y+2z-4|}{\sqrt{1+4+4}} = \frac{|6x-3y+2y+6|}{\sqrt{36+9+4}}$$

$$\frac{|x+2y+2z-4|}{3} = \frac{|6x-3y+2y+6|}{7}$$

$$7|x+2y+2z-4| = 3|6x-3y+2z+6|$$

$$7(x+2y+2z-4) = \pm 3(6x-3y+2z+6)$$

1° $7(x+2y+2z-4) = 3(6x-3y+2z+6)$

$\beta_1: 11x - 23y - 8z + 46 = 0$

2)

$7(x+2y+2z-4) = -3(6x-3y+2z+6)$

$\gamma_2: \ 25x+5y+20z-10=0$

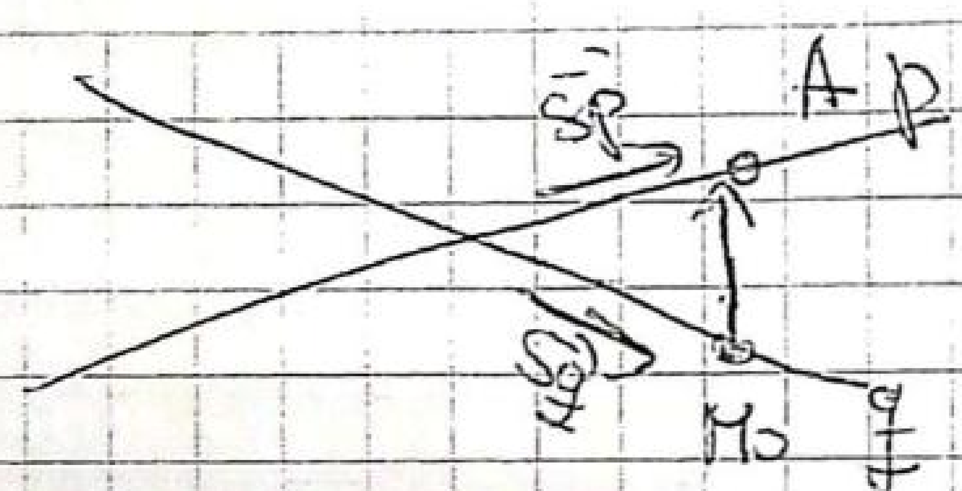
N4) Kroz tačku $M_0(3,-1,-1)$ postaviti pravu koja siječe pravu $p: \ \frac{x-3}{1}=\frac{y+2}{-1}=\frac{z}{2}$ pod uglom $60°$.

// q - tražena prava

$\vec{s_q}=(m,n,k)$ - njen vektor pravca

$\vec{s_p}=(1,-1,2)$

$A(3,-2,0)\in p$



* Kako se prave p i q sijeku to su vektori $\vec{s_q}, \vec{s_p}, \vec{M_0A}$ komplanarni pa je

$(\vec{s_q}\times\vec{s_p})\cdot\vec{M_0A}=0$ (*)

$\vec{M_0A}=(0,-1,1)$

$$(*) \Rightarrow \begin{vmatrix} m & n & k \\ 1 & -1 & 2 \\ 0 & -1 & 1 \end{vmatrix} = 0$$

$m-n-k=0$ (1)

$\alpha \to$ ugao između $\measuredangle(q,p) = \measuredangle(\vec{s_q},\vec{s_p})$

$$\cos 60° = \frac{|\vec{s_q}\cdot\vec{s_p}|}{|\vec{s_q}||\vec{s_p}|} = \frac{|m-n+2k|}{\sqrt{m^2+n^2+k^2}\sqrt{1+1+4}}$$

$$= \frac{|m-n-2k|}{\sqrt{m^2+n^2+k^2}\sqrt{6}}$$

$$\frac{1}{2} \Rightarrow \sqrt{6}\sqrt{m^2+n^2+k^2} = 2|m-n+2k| \quad (2)$$

Dobili smo sistem

$$\begin{cases} m-n-k=0 \\ \sqrt{6}\ \sqrt{m^2+n^2+k^2} = 2|m-n+2k| \end{cases}$$

za $k=1$

$$\begin{cases} m-n-1=0 \\ \sqrt{6}\ \sqrt{m^2+n^2+1} = 2|m-n+2| \ /^2 \end{cases}$$

$m=n+1$

$6\cdot[(n+1)^2+n^2+1] = 4(\cancel{n}+1-\cancel{n}+2)^2$

$3(2n^2+2n+2) = 2\cdot 9$

$n^2+n+1=3$

$n^2+n-2=0$

$n_{1/2} = \frac{-1\pm\sqrt{1+8}}{2}$

$n_{1/2} = \frac{-1\pm 3}{2}$

$\boxed{n_1=1,\ n_2=-2}$

1° $m_1=2$

$\vec{s_q} = (2,1,1)$

$q: M_0(3,-1,-1)\ \ \vec{s_q}=(2,1,1)$

$q: \frac{x-3}{2} = \frac{y+1}{1} = \frac{z+1}{1}$

2° $m_2=-2+1=-1$

$\vec{s_q}=(-1,-2,1)$

$q: \frac{x-3}{-1} = \frac{y+1}{-2} = \frac{z+1}{1}$