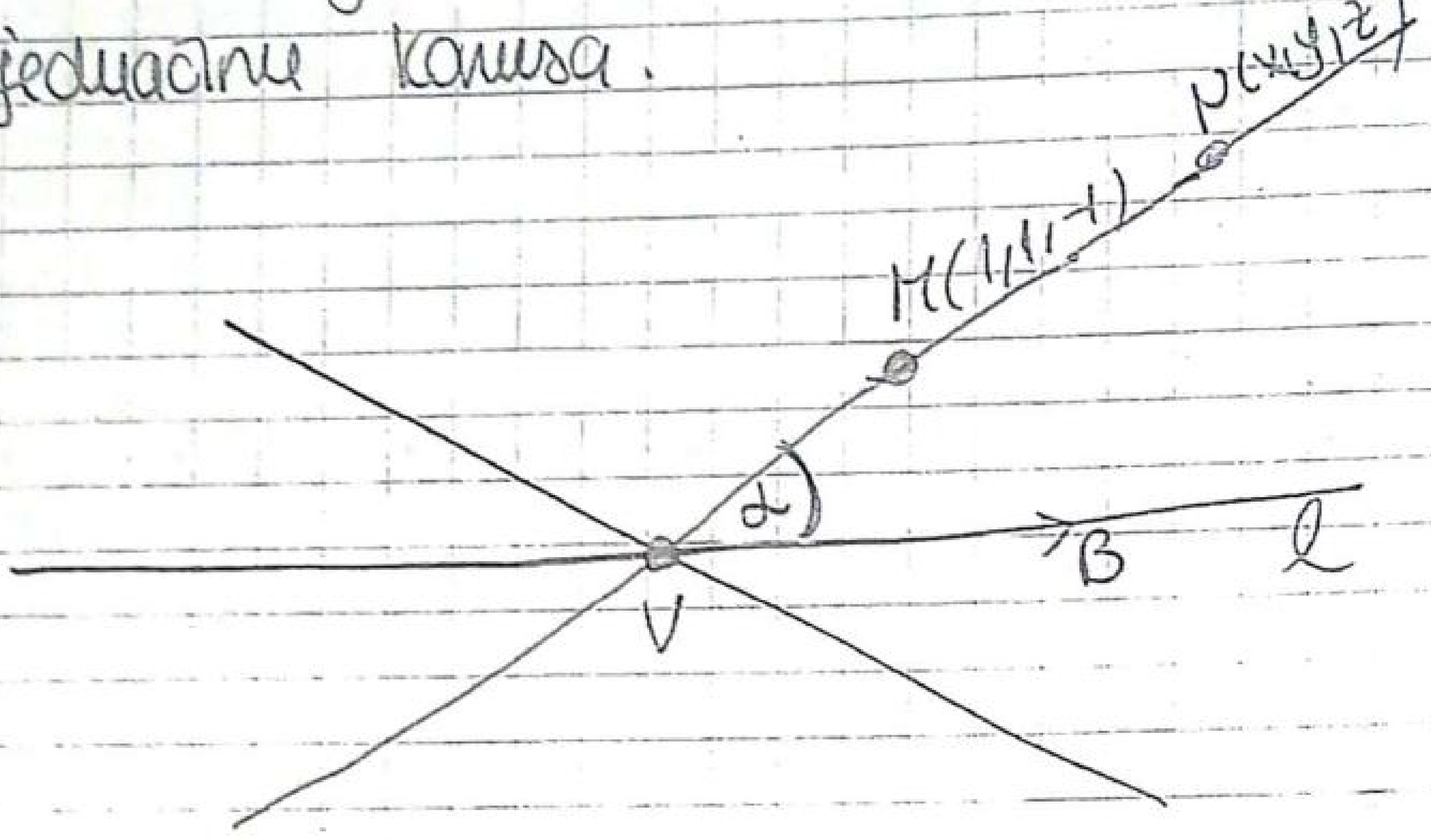
5) Prava l: $\frac{x-2}{2} = \frac{y+1}{-2} = \frac{z+1}{-1}$ je osa kružnog konusa sa vrhom u yOz ravni. Tačka $M(1,1,-1)$ pripada konusu. Naći jednačinu konusa.

R/



Vrh konusa je tačka V koja je presjek prave l i ravni yOz $(x=0)$.

$$l: \begin{cases} x = 2+2t \\ y = -1-2t \\ z = -1-t \end{cases} \qquad \begin{matrix} x=0 \\ y=1 \\ z=0 \end{matrix}$$

$2+2t=0$

$t=-1$

$V(0,1,0)$

$$\vec{VB} = \vec{s_l}$$

$$\cos\alpha = \frac{\vec{VM}\cdot\vec{s_l}}{|\vec{VM}|\,|\vec{s_l}|}$$

$\vec{VM} = (1,0,-1)$

$\vec{s_l} = (2,-2,-1)$

$$\cos\alpha = \frac{3}{\sqrt{2}\cdot\sqrt{9}} = \frac{\sqrt{2}}{2}, \quad \alpha = 45^\circ$$ → Vocth ugao otvora

$N(x,y,z)$ tačka konusa.

α je ugao između $\vec{VN}$ i $\pm\vec{s_l}$

$$\cos\alpha = \pm\frac{\vec{VN}\cdot\vec{s_l}}{|\vec{VN}|\cdot|\vec{s_l}|}$$

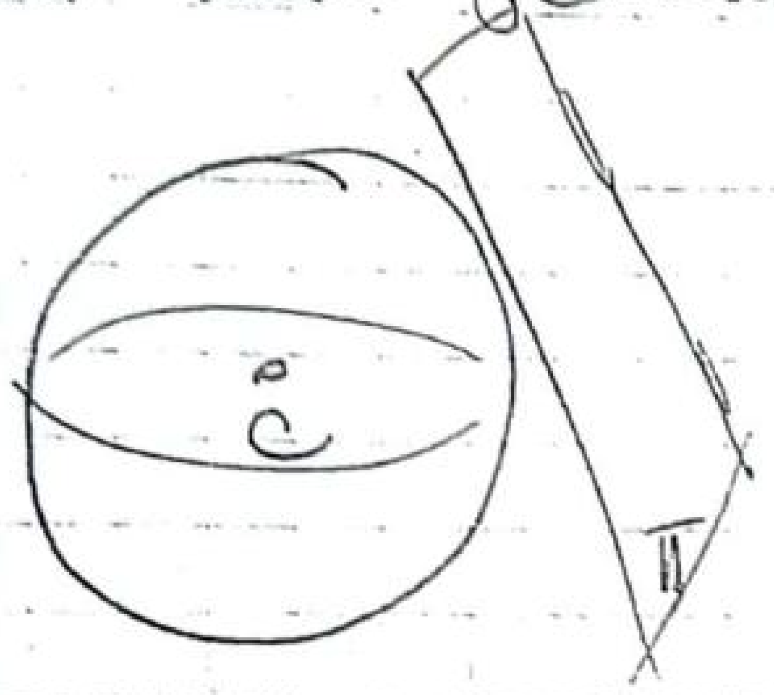
$$\frac{\sqrt{2}}{2} = \frac{2x-2(y-1)-z}{\sqrt{x^2+(y-1)^2+z^2}\;3} \qquad \vec{VN}=(x,y-1,z) \quad \vec{s_1}=(2,-2,-1)$$

$$18\left(x^2+(y-1)^2+z^2\right) = 4\,(2x-2y-z+2)^2$$

$$9\left(x^2+(y-1)^2+z^2\right) = 2\,(2x-2y-z+2)^2 \Rightarrow$$ jedn. konusne površi.

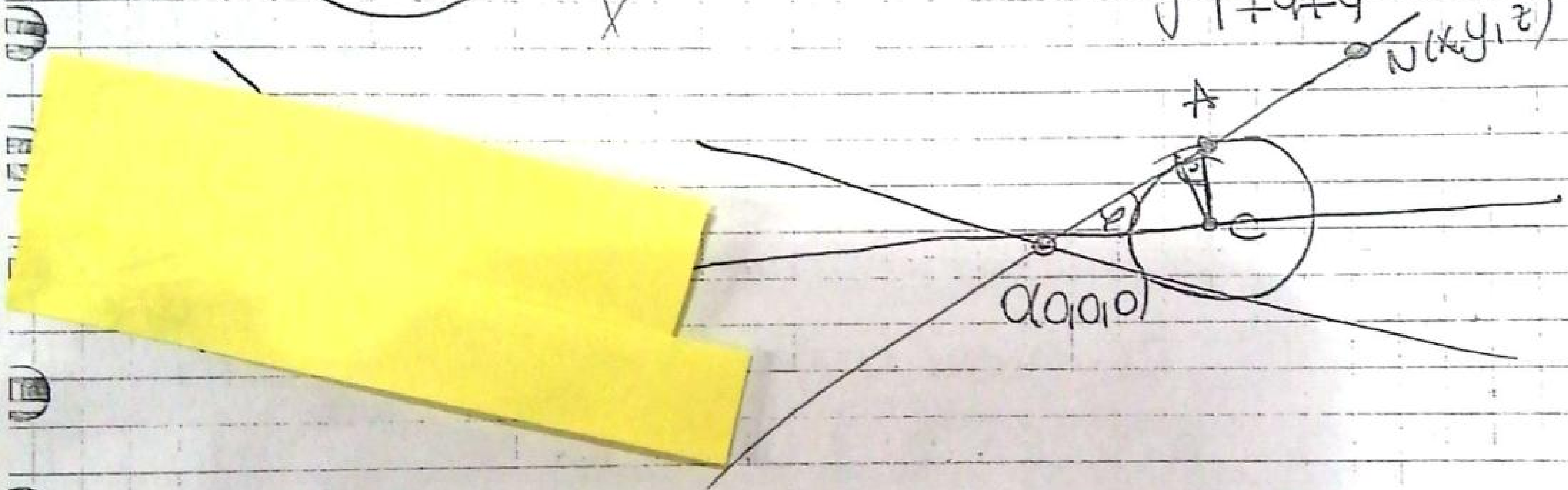
6) Sfera S čiji je centar $C(6,1,1)$ dodiruje ravan $\Pi: x-2y+2z-3=0$. Sfera je osvijetljena zracima čiji se izvor nalazi u koordinatnom početku. Naći jednačinu sjenke sfere u ravni Π.

R||



$d(C,\Pi)=R$

$$R = \frac{|6-2+2-3|}{\sqrt{1+4+4}} = \frac{3}{3} = 1$$



Zraci koji tanguraju sferu formiraju konusnu površ.

A - tačka dodira izvodnice i sfere.

$\triangle OCA \quad \sin\varphi = \frac{|\vec{CA}|}{|\vec{OC}|}$

$\sin\varphi = \frac{R}{\sqrt{38}} = \frac{1}{\sqrt{38}}$

$|\vec{OC}| = \sqrt{36+1+1} = \sqrt{38}$

$N(x,y,z)$ tačka sa konusa.

$\varphi = \measuredangle(\vec{ON}, \pm\vec{OC})$

$\cos\varphi = \sqrt{1-\sin^2\varphi}$

$\cos\varphi = \pm\frac{\vec{ON}\cdot\vec{OC}}{|\vec{ON}||\vec{OC}|} = \pm$

$\cos\varphi = \sqrt{1-\frac{1}{38}}$

$$\sqrt{1-\frac{1}{38}} = \frac{6x+y+z}{\sqrt{x^2+y^2+z^2}\,\sqrt{38}}$$

$$\sqrt{37} = \frac{6x+y+z}{\sqrt{x^2+y^2+z^2}}$$

$$\sqrt{37}\left(\sqrt{x^2+y^2+z^2}\right) = 6x+y+z \;\Big/^2$$

$$37(x^2+y^2+z^2) = (6x+y+z)^2 \rightarrow \text{jednačina konusa}$$

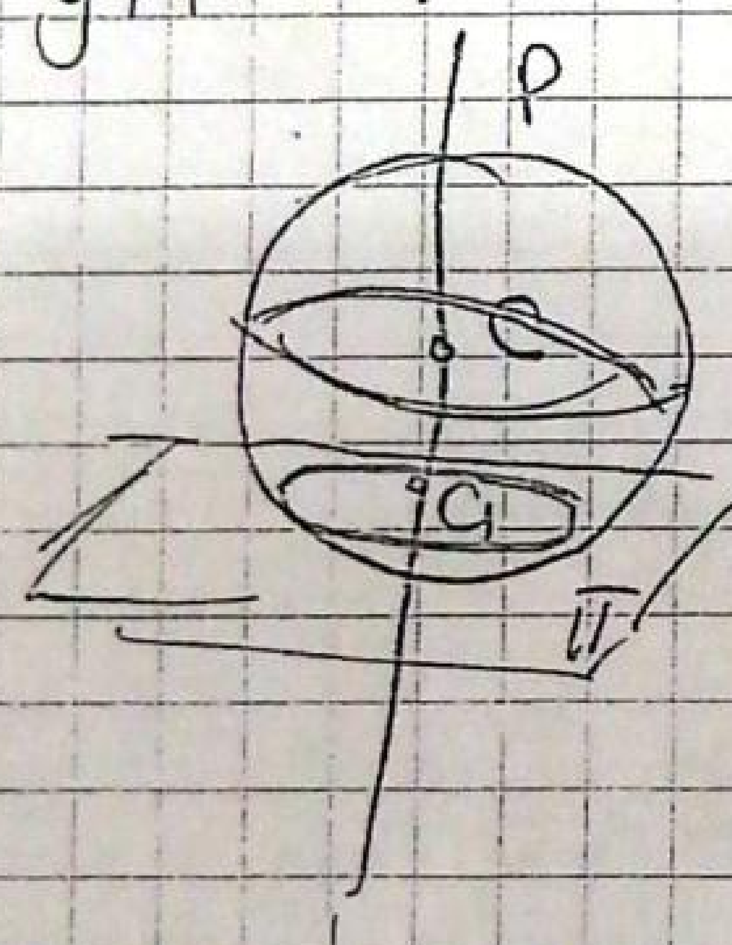
Sijeuka je $\begin{cases} 37(x^2+y^2+z^2) = (6x+y+z)^2 \\ x-2y+2z-3=0 \end{cases}$

7) Odrediti jednačinu konusa čiji je vrh centar kruga

$$(k): \begin{cases} (x-2)^2+(y-2)^2+(z+3)^2=36 \\ \pi : 3x+y-z=0 \end{cases}$$

a direktrisa ortogonalna projekcija kruga K na ravan $y+10=0$.

Rj



$C(2,2,-3)$ – centar sfere, $R=\cancel{36}$

C_1 – centar kruga

p – sadrži C i $\perp$ na π

$\vec{s_p} = \lambda \vec{n_\pi}$ za $\lambda = 1$

$\vec{s_p} = (3,1,-1)$

$$p: \frac{x-2}{3} = \frac{y-2}{1} = \frac{z+3}{-1}$$

$p \cap \pi = \{C_1\}$

$$p: \begin{cases} x = 2+3t \\ y = 2+t \\ z = -3-t \end{cases}$$

$3(2+3t)+2+t+3+t=0$

$11t+11=0$

$t=-1$

$x=-1$
$y=1$
$z=-2$

$\underline{C_1(-1,1,-2)}$

V – tjeme konusa

$V(-1,1,-2)$

Nađimo ortog. projek. kruga k na ravan $\alpha : y+10=0$.
Nađimo jednačinu cilindra kojem je direktrisa krug (k) a vektor pravca generatrise kolinearan sa $\vec{n_\alpha}$, $\vec{n_\alpha}=(0,1,0)$.

$N \in (k)$

$M(x_0, y_0, z_0)$

$\vec{s}=(0,1,0)$

$$g: \frac{x-x_0}{0}=\frac{y-y_0}{1}=\frac{z-z_0}{0}$$

$$g: \begin{cases} x=x_0 \\ y=y_0+t \\ z=z_0 \end{cases} \Rightarrow \begin{matrix} x_0=x \\ y_0=y-t \\ z_0=z \end{matrix}$$

$$N \in (k) \Rightarrow 3x_0+y_0-z_0=0$$

$$3x+y-t-z=0$$

$$t=3x+y-z$$

$x_0=x$

$y_0=-3x+z$

$z_0=z$

$$N \in (k) \Rightarrow (x_0-2)^2+(y_0-2)^2+(z_0+3)^2=36$$

$$(x-2)^2+(-3x+z-2)^2+(z+3)^2=36$$

$\Rightarrow$ jednačina cilindrične površi.

Ortogonalna projekcija kruga k na ravan α:

$$d: \begin{cases} (x-2)^2+(-3x+z-2)^2+(z+3)^2=36 \\ y+10=0. \end{cases}$$

Treba naći jednačinu konusa čiji je vrh tačka $V(-1,1,-2)$ a direktrisa d.

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